Fast polynomial optimization techniques for optimal power flow

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Summary: Certification and validation of computational results is a major issue for modern sciences raising challenging problems at the interplay of mathematics and computational aspects of computer science. One can emphasize in this context several applications arising in energy networks, such as alternative current optimal power flow (AC-OPF), with a crucial need of certification.

OPF problems can be modeled with polynomial optimization, which consists in computing the infimum of a polynomial function under (in)equality constraints. The emergence of polynomial optimization as an exciting new field goes back to the last two decades and has led to striking developments from a cross fertilization between (real) algebraic geometry, applied mathematics, theoretical computer science and engineering. The backbone of this powerful methodology is the "moment-sums of squares" (moment-SOS) hierarchy [2], relying on semidefinite programming (SDP). Several convex relaxations have been recently provided with the goal of solving AC-OPF instances to global optimality. These efforts led to efficient solution algorithms that can solve many instances found in the literature, which model real-world networks. The concurrent methods usually perform costly domain partitioning and spatial branching on continuous variables. Our framework shall overcome these issues by providing fast yet accurate bounds.

Goals: In view of the current state of SDP solvers, the usual moment-SOS hierarchy does not scale well enough. To overcome these scalability issues, there are two possible remedies: **(1) decreasing the size of the SDP problems** and **(2) improving the speed of SDP resolution**. Researchers from LAAS have recently designed two research frameworks addressing both issues, separately:

(1) In [4], the new **CS-TSSOS** hierarchy allows one to solve large-scale OPF with *sparse* data, and focused on challenging AC-OPF problems, for which the gap is greater than 1.00%, from the AC-OPF library **pglib** [1].

(2) The SDP relaxations in the moment-SOS hierarchy have a nice constant trace property (CTP), which can be exploited to avoid solving the relaxations via interior point methods and rather use ad-hoc spectral methods that minimize the largest eigenvalue of a matrix pencil. The resulting hierarchy of "spectral relaxations" [3] is much more efficient than the usual interior-point methods and can handle matrices of size up to 2 000 with more than 1.5 million constraints in less than a hour on a standard laptop.

The goal of the postdoctoral fellow is to combine both frameworks to solve AC-OPF instances with up to tens of thousand buses with optimality gap close to 0. The idea is to exploit the sparsity of the input data of polynomial optimization problems and the sparsity of the associated spectral relaxations. For this, we will combine the two associated solvers recently developed in LAAS, namely **TSSOS** and **SpectralPOP**, and execute the resulting software library on large-scale instances with the LAAS cluster.

A significant degree of freedom will be given to the researcher to create and pursue his/her ideas broadly within this scope.

Requirements: A successful candidate will have a strong background in applied mathematics or physics, excellent programming skills, as well as a working knowledge of convex optimization. The candidate should be highly motivated and creative.

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References

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