M2 internship: approximation hierarchies for quantum entanglement detection

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Context: Tensor products model quantum state systems consisting of multiple subsystems. Detecting whether a quantum state is entangled (or separable) is a fundamental question in quantum information theory, which boils down to studying density matrices on tensor products of Hilbert spaces. One needs to develop tractable approximations for tackling this computationally hard problem.

Objectives of the internship: Tight links have been recently established between an approximation hierarchy of Doherty, Parrilo and Spedaglieri [1] for separable states (based on state extension) and the classical moment and sum-of-squares approach for polynomial optimisation [2, 3]. In this project we will investigate further methods from conic and polynomial optimisation, aiming to design dedicated tools for some well-structured classes of (bipartite) quantum states. This includes the states that satisfy invariance properties under certain group actions, which were recently characterized as arising from pairwise completely positive matrices [4, 6]. The cone of completely positive matrices is well-studied in optimisation and algebraic matrix theory, but the more general cone of pairwise completely positive matrices much less. A natural question is developing approximation hierarchies for these cones and investigating how they can be used to design better approximations for the corresponding quantum states. Understanding the links between the ranks of completely positive factorizations and the separable rank of quantum states is also a relevant question in this context. Hence, this project revolves around the interplay between conic approximations for matrix properties and their use for a better understanding of quantum entanglement for well-structured bipartite quantum states. Exploiting the structure of the initial quantum information problem, such as correlative [5] or term sparsity [7] will be decisive to improve the scalability of the related hierarchies of conic programming formulations.

The goal of the internship is to familiarize the candidate with this research area, through literature study, and investigating some first research questions.

Application requirements: A successful candidate will have a strong background in applied mathematics and/or quantum physics, a working knowledge of convex optimization as well as strong programming skills. The candidates are kindly asked to send an email with "M2 candidate" in the title, a CV, detailed course records, as well as a motivation letter to `M.Laurent@cwi.nl` and `vmagron@laas.fr`. Knowledge of French does not constitute a pre-requisite, but an excellent command of English is demanded.

Working place and salary: The M2 internship is expected to start during first-half of 2024. The M2 intern will be hosted at LAAS CNRS Toulouse, in the group of Victor Magron, with the possibility to collaborate and visit the group of Monique Laurent at CWI Amsterdam. The monthly stipend for M2 internships in France is around 600 euros, please visit this link for more details.

PhD opportunity: Upon successfull completion of the master internship, a PhD position may be a possibility in the context of the Marie Skłodowska-Curie Doctoral Network TENORS (Tensor modEliNg, geOmetRy and optimiSation), funded for the period 2024-2027. This PhD position would be located at CWI, and joint with LAAS CNRS (with two years at CWI, 10 months of secondment at LAAS CNRS and 2 months of secondment at the company Quantinuum).

Tensors are nowadays ubiquitous in many domains of applied mathematics, computer science, signal processing, data processing, machine learning and in the emerging area of quantum computing. TENORS aims at fostering cutting-edge research in tensor sciences, stimulating interdisciplinary and intersectoriality knowledge developments between algebraists, geometers, computer scientists, numerical analysts, data analysts, physicists, quantum scientists, and industrial actors facing real-life tensor-based problems.

References

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