Postdoctoral fellowship: Noncommutative polynomial optimization for quantum network correlations

Advisors: Victor Magron (LAAS CNRS Toulouse)
Igor Klep (University of Ljubljana)

Context: Characterizing quantum network correlations is fundamental to find and quantitatively evaluate network protocols based on quantum nonlocality. The few known tools performing this task, called the quantum inflation hierarchy [1] and its variants, are inefficient: as they require the introduction of multiple copies of the quantum network elements, they quickly become computationally intractable and can in practice only tackle the most elementary networks, and even there with a limited success. Moreover, their eventual convergence is an open question and they might have unknown fundamental limits, beyond the numerical ones [2]. Due to the mathematical formalism of quantum theory, these tools are intrinsically related to noncommutative polynomial optimization [3]. They are adapted from the celebrated Navascués-Pironio-Acín (NPA) hierarchy [4], which can characterize single quantum source correlations through a hierarchy of arbitrarily good – at the cost of increasing computational complexity – bounds formulated as semidefinite programs which can be solved numerically.

Assignment: The research project will focus on methods based on noncommutative polynomial optimization for quantum network correlations. A non-exhaustive list of potential projects is:

- Methods to characterize the set of quantum network correlations which are singled out by these noncommutative optimization methods. One important direction is to extend the existing approaches to complex Hilbert spaces as in [5];
- Provide algorithmic improvements of the quantum network NPA hierarchy, by exploiting the structure they inherently possess, such as sparsity [6];
- Obtain convergence rates for the NPA hierarchy. One proposed track is to extend to the noncommutative setting the recent successful strategy used for the commutative one [7] while relying on noncommutative Christoffel-Darboux kernels [8] and preliminary results [9];
- Investigate conditions for the NPA hierarchy to have finite convergence – the noncommutative analogue of [10] – based on recent work [11] extending Karush-Kuhn-Tucker conditions to noncommutative optimization.

Any other suggested research project in quantum information theory can be discussed (both from the physics, the computer science or the mathematical viewpoint).

Main activities: The position will be funded through a QuantERA project involving a large european consortium: see the project website https://project.inria.fr/compute

It will be achieved by collaborating with Victor Magron, other members of his group (Jean-Bernard Lasserre, Didier Henrion), as well as the consortium members:

- Mariami Gachechiladze (TU Darmstadt, Germany) and David Gross (Cologne)
- Marc-Olivier Renou (INRIA Saclay, Palaiseau, France)
- Igor Klep (University of Ljubljana, Slovenia)
- Antonio Acín (ICFO Barcelona, Spain)

Long stays in the groups of the consortium members will be encouraged.

Application requirements: A successful candidate will have a strong background in applied mathematics and/or quantum physics, a working knowledge of convex optimization as well as strong programming skills. The applicant should hold a PhD degree in computer science, physics, mathematics, or a related field and have an excellent track record of publications in either optimization or quantum information theory. Familiarity with Bell nonlocality, operator algebras, semidefinite relaxations of polynomial optimization problems and/or quantum correlation protocols is a plus.

The candidates are kindly asked to send an email with "Postdoc candidate" in the title, a full CV with a detailed publication track record, as well as a motivation letter to `vmagron@laas.fr` and `igor.klep@fmf.uni-lj.si`. Knowledge of French does not constitute a pre-requisite, but an excellent command of English is demanded.

Working place and salary: The Postdoctoral position is expected to start at any date after October 2024. The Postdoctoral fellow will be hosted at LAAS CNRS Toulouse, in the group of Victor Magron, with the possibility to collaborate and visit the group of Igor Klep at University of Ljubljana. The remuneration is according to the candidate profile.

References

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[4] M. Navascués, S. Pironio and A. Acín. A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations. New Journal of Physics, 10(7), 2008.

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[7] L. Slot. Sum-of-Squares Hierarchies for Polynomial Optimization and the Christoffel-Darboux Kernel. SIAM Journal on Optimization 32(4):2612-2635, 2022.

[8] S. T. Belinschi, V. Magron, and V. Vinnikov. Noncommutative Christoffel-Darboux Kernels. Transactions of the American Mathematical Society, 376(01):181-230, 2023.

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[10] J. Nie. Optimality conditions and finite convergence of Lasserre's hierarchy. Mathematical programming, 146:97-121, 2014.

[11] M. Araújo, I. Klep, T. Vértesi, A. J. Garner and M. Navascués. Karush-Kuhn-Tucker conditions for non-commutative optimization problems. Preprint arXiv:2311.18707.