Fast polynomial optimization for Quantum Information

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Summary: The FastQI postdoctoral project aims at modeling and solving important problems issued from quantum information, such as bounding the set of quantum correlations and the ground state energy of many body Hamiltonians. As such systems of interest often involve nonlinear polynomial functions with noncommuting variables (quantum physics operators), we will rely on techniques dedicated to noncommutative polynomial optimization, in particular the so-called Navascues-Pironio-Acin (NPA) hierarchy [3] of convex relaxations to approximate minimal eigenvalue and trace optimization problems. The postdoctoral fellow will pursue research with his two advisors to model various quantum information problems with certain classes of noncommutative polynomials, formed with multiplication of operators with their traces. The project will also focus on exploiting the specific properties of large-scale input problems, including sparsity or symmetry, in order to design and implement efficient relaxation procedures.

In the last decade, researchers in real algebra, optimization and quantum physics have developed frameworks to handle polynomial problems involving noncommuting variables. The underlying motivation of this project is to complement and greatly extend these previous efforts by tackling important applications, including ground-state energies of many body Hamiltonians in condensed matter physics (quantum simulation) or bounds on the maximum violation of Bell inequalities (quantum information theory). The community has acknowledged growing research efforts in polynomial optimization, an emerging field extensively developed in the last two decades. In 2001, Lasserre introduced in [2] a hierarchy of convex relaxations for approximating the minimum of a polynomial under polynomial inequality constraints. One key advantage of these techniques is the ability to model a wide range of problems using optimization formulations, which can be in turn solved with efficient numerical tools. We expect to benefit from this modeling power. However, optimization methods still encompass many major issues on both practical and theoretical side: scalability, unknown complexity bounds, modeling of specific polynomial problems.

Goals: The project is divided in two objectives: modeling quantum information problems with noncommutative optimization (Goal 1) and improving the scalability of relaxations for noncommutative optimization problems (Goal 2).

(1) The first task is to model quantum information problems by using new classes of noncommutative optimization problems, in particular trace polynomials [1]. Then one would like to prove that one can derive a converging scheme to approximate as closely as desired the solution of such noncommutative problems, and potentially analyze the convergence rate of this scheme.

(2) We wish to complement prior research focusing on variable and term sparsity for noncommutative optimization [4] to the case of trace polynomials. Another task is to exploit symmetry arising in the input data. Eventually, one shall apply these sparsity/symmetry exploiting techniques to handle a wide range of instances arising from quantum information theory, such as ground state energy minimization, inflation for quantum correlations and mutually unbiased bases.

Another goal relates to the implementation of the above-mentioned algorithms in an open-source modeling toolbox, such as the Julia layer **NCTSSOS**, dedicated to the modeling of polynomial optimization problems with sparse structure. A significant degree of freedom will be given to the researcher to create and pursue his/her ideas broadly within this scope.

Requirements: A successful candidate will have a strong background in applied mathematics or quantum information, excellent programming skills, as well as a working knowledge of convex optimization. The candidate should be highly motivated and creative.

Funding: This postdoctoral project is part of the FastQI project, funded for 18 months starting from October 1st 2021, by the institute "Quantum technologies in Occitanie". The monthly gross salary is around 4100€. The postdoctoral fellow will be hosted at LAAS in the MAC team, with a co-supervision together with ICFO, Barcelona.

References

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[4] Jie Wang and Victor Magron. Exploiting term sparsity in noncommutative polynomial optimization. *arXiv:2010.06956*, 2020.