Advanced data-driven techniques for the Lasserre hierarchy

Milan Korda (LAAS-CNRS),
Victor Magron (LAAS-CNRS)
Jean-Bernard Lasserre (LAAS-CNRS)

December 2, 2019

Summary: A large number of problems from diverse fields such as optimization, probability and statistics, dynamical systems or quantum physics can be tackled within the powerful and elegant framework of the Lasserre hierarchy [1, 2], which allows one to solve challenging nonconvex and nonlinear problems by a sequence of convex optimization problems in a unified and very systematic fashion. Additional research [5] investigated the ability of Christoffel-Darboux kernels to capture information about the support of an unknown probability measure; a distinguishing feature of this approach is the ability to infer support characteristics based on the knowledge of finitely many moments of the underlying measure, which is precisely the information obtained from the Lasserre hierarchy. A major open question remains whether the Lasserre hierarchy can be used in a data-driven setting, where the underlying model is unknown and only observed data are available. This project will investigate this direction, building on the very recent work [3]. Progress in this direction would be an enabling factor in bringing the elegant and powerful tools of the Lasserre hierarchy to the realm of the present-day big-data applications, which are currently typically tackled using ad-hoc heuristic techniques with limited mathematical foundation.

Goals: The goal of this project is the extension of the Lasserre hierarchy and Christoffel-Darboux kernels to a data-driven setting, developing new methods furnished with a theoretical analysis (convergence rate, non-asymptotic out-of-sample error etc). The first possible research direction will consist of studying Christoffel-Darboux kernels to extend the approach from [4] for measures supported on specific classes of mathematical varieties. We intend to apply this framework to deep learning network models, for which latent representation correspond to such low-dimensional varieties. Numerical experiments will be performed on several benchmark suites, including MNIST, CIFAR10 or fashion MNIST. Other main steps may include, but are not limited to, the investigation of adaptive sampling techniques and basis choice for the approach developed in [3] as well as extension of the proposed methodology beyond the problem class considered, e.g., to data-driven optimal control. Other directions include complexity reduction based on sparsity, symmetries and other more advanced structural properties of the problem at hand. A significant degree of freedom will be given to the student to create and pursue his/her ideas broadly within this scope.

Requirements: A successful candidate will have a strong background in applied mathematics or physics, having a very good knowledge of probability and statistics as well as a working knowledge of convex optimization, real analysis and basic measure theory. The candidate should be highly motivated and creative.

Funding: This PhD will be funded by IRT Saint Exupéry and co-supervised with University Toulouse 3, Paul Sabatier and LAAS CNRS. The PhD candidate will be hosted by the DEEL team (DEpendable and Explainable Learning), a joint project team between Toulouse and Montréal, in building B612, IRT Saint Exupéry.

References

[1] J.B. Lasserre (2001). Global optimization with polynomials and the problem of moments. SIAM Journal on optimization, 11(3), 796-817.

[2] J.B. Lasserre (2010). Moments, positive polynomials and their applications. World Scientific, 2010.

[3] M. Korda (2019). Data-driven computation of the maximum positively invariant set for nonlinear dynamical systems. FEANICSES workshop, `https://cavale.enseeiht.fr/feanicses/files/W2019/4-korda.pdf`.

[4] E. Pauwels, M. Putinar and J.-B. Lasserre (2019). Data analysis from empirical moments and the Christoffel function. hal-01845137

[5] E. Pauwels and J.-B. Lasserre (2019). The empirical Christoffel function with appication in data analysis. To appear in Advances in Computational Mathematics.