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How to solve Time-Varying Semidefinite Programs: Path Following a Burer–Monteiro Factorization

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What is SDP?

Linear Programming

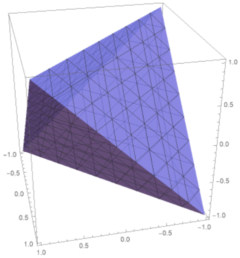
vector variable $x \in \mathbb{R}^n$

$$\min_{x \in \mathbb{R}^n} \ \langle c, x \rangle \quad \text{s.t.} \ \langle a_i, x \rangle = b_i, \quad i = 1, \ldots, m, \quad x \geq 0$$

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$$

$x \geq 0$ iff

$x_i \geq 0, \ \forall i$



Semidefinite Programming

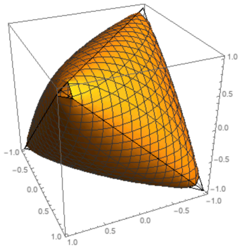
symmetric matrix variable $X \in \mathcal{S}^n$

$$\min_{X \in \mathbb{S}^n} \ \langle C, X \rangle \quad \text{s.t.} \ \langle A_i, X \rangle = b_i, \quad i = 1, \ldots, m, \quad X \succeq 0$$

$$\langle X, Y \rangle = \sum_{i,j=1}^{n} X_{i,j} Y_{i,j}$$

$X \succeq 0$ iff

$v^T X v \geq 0, \ \forall v \in \mathbb{R}^n$



Time-Varying Semidefinite Programming

Time-Varying Semidefinite Programming (TV-SDP) is SDP where the problem data and solutions depends on a time parameter $t \in [0, T]$:

$$\begin{aligned} \min_{X \in \mathbb{S}^n} \quad & \langle C_t, X \rangle \\ \text{s.t.} \quad & \mathcal{A}_t(X) = b_t, \\ & X \succeq 0. \end{aligned} \tag{SDP$_t$}$$

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$$\begin{aligned} \max_{y \in \mathbb{R}^m} \quad & \langle b_t, y \rangle \\ \text{s.t.} \quad & Z + \mathcal{A}_t^*(y) = C_t, \\ & Z \succeq 0. \end{aligned} \tag{D-SDP$_t$}$$

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Assumptions for all $t \in [0, T]$

(A1) The linear operator $\mathcal{A}_t$ is surjective.

(A2) Problems (SDP$_t$) and (D-SDP$_t$) are strictly feasible.

(A3) There exists a solution X_t and a dual solution Z_t such that:

- $\ker X_t = \operatorname{im} Z_t$, *strict complementarity*
- $\ker \mathcal{A} + \mathcal{T}_{X_t} = \mathbb{S}^n$, *primal non-degeneracy*
- $\operatorname{im} \mathcal{A}^* + \mathcal{T}_{Z_t} = \mathbb{S}^n$. *dual non-degeneracy*

(A4) Data $\mathcal{A}_t, b_t, C_t$ are continuously differentiable functions of t.

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Consequences

(C1) (SDP$_t$) has a unique and smooth solution curve $[0, T] \ni t \mapsto X_t$.

(C2) The curve $t \mapsto X_t$ is of constant rank r.

Approaches

A naive strategy: consider the instances of the problem (SDP_{t_k}) for a sequence of times $\{t_k\}_{k \in \{1,\ldots,K\}} \subseteq [0, T]$ and solve them one after another

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$$\begin{cases} X_t \text{ current solution} \\ F_{t+\Delta t}(X_t + \Delta X) = 0 \end{cases} \longrightarrow \quad \Delta x = -\nabla F_{t+\Delta t}(x_t)^{-1} F_{t+\Delta t}(x_t)$$

The Burer–Monteiro factorization

Issues:

How can we deal with the constraint $X \succeq 0$?

Can we reduce the number of variables?

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$$\begin{aligned} \min_{X \in \mathbb{S}^n} \quad & \langle C, X \rangle \\ \text{s.t.} \quad & \langle A_i, X \rangle = b_i, \quad i \in [m] \\ & X \succeq 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \min_{Y \in \mathbb{R}^{n \times r}} \quad & \langle C, YY^T \rangle \\ \text{s.t.} \quad & \langle A_i, YY^T \rangle = b_i, \quad i \in [m] \end{aligned}$$

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Idea: apply a path-following algorithm to the Burer–Monteiro factorization

A new issue

We would like the optimal solutions for the time-varying problem

$$\begin{aligned} \min_{Y \in \mathbb{R}^{n \times r}} \quad & \langle C_t, YY^T \rangle \\ \text{s.t.} \quad & \mathcal{A}_t(YY^T) = b_t \end{aligned} \tag{BM$_t$}$$

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$$Y_t\mathcal{O}_r = \{Y_tQ : Q \in \mathcal{O}_r\}$$

$$\mathcal{T}_{Y_t}Y_t\mathcal{O}_r = \{Y_tS : S^T = -S\}$$

$$\mathcal{H}_{Y_t} := \mathcal{T}_{Y_t}Y_t\mathcal{O}_r{}^{\perp} = \{H : Y_t^TH = H^TY_t\}$$

Restricting $Y \mapsto YY^T$ to the horizontal space $\mathcal{H}_{Y_t}$

Define

$$\mathcal{B}_{Y_t} := \{Y_t + H \in \mathcal{H}_{Y_t} \text{ such that } \|H\|_F \leq \sigma_r(Y_t)\}$$

We have the following facts:

1. The restriction of ϕ to $\mathcal{B}_{Y_t}$ is a local diffeomorphism between a neighborhood of Y_t in $\mathcal{H}_{Y_t}$ and a neighborhood of $X_t = Y_t Y_t^T$ in $\mathbb{S}^n_{+,r}$
2. If $\tilde{X} \in \mathbb{S}^n_{+,r}$ and
$$\|\tilde{X} - Y_t Y_t^T\|_F \leq \frac{2\lambda_r(Y_t Y_t^T)}{\sqrt{r+4} + \sqrt{r}}$$
then there exists a unique $H \in \mathcal{B}_{Y_t}$ such that
$$\tilde{X} = \phi(Y_t + H) = (Y_t + H)(Y_t + H)^T$$
3. For
$$\Delta t < \frac{2\lambda_*}{L(\sqrt{r+4} + \sqrt{r})}$$
there is a unique and smooth solution curve $s \mapsto Y_s$ for the problem (BM_t) restricted to $\mathcal{H}_{Y_t}$ in the time interval $s \in [t, t + \Delta t]$.

The restricted problem

From a current solution Y_t we want a new solution for our problem at $t + \Delta t$.

$$\boxed{\begin{aligned} \min_{Y \in \mathbb{R}^{n\times r}} \quad & \langle C_{t+\Delta t}, YY^T \rangle \\ \text{s.t.} \quad & \mathcal{A}_{t+\Delta t}(YY^T) = b_{t+\Delta t} \\ & {\color{red} Y \in \mathcal{H}_{Y_t}} \end{aligned}} \qquad (\mathrm{BM}_{Y_t, t+\Delta t})$$

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The KKT condition reads:

$$\boxed{\begin{aligned} \nabla_Y \mathcal{L}_{Y_t, t+\Delta t}(Y, \lambda, \mu) = 0, \\ g_{t+\Delta t}(Y) = 0, \\ h_{Y_t}(Y) = 0. \end{aligned}} \tag{KKT}$$

with $\mathcal{L}_{Y_t, t+\Delta t}(Y, \lambda, \mu) := f_{t+\Delta t}(Y) - \langle \lambda, g_{t+\Delta t}(Y) \rangle - \langle \mu, h_{Y_t}(Y) \rangle$.

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The linearization of (KKT) at $(Y_t, \lambda_t, 0)$ leads to the linear system

$$\begin{bmatrix} \nabla_Y^2 \mathcal{L}_{t+\Delta t}(\lambda) & -g'_{t+\Delta t}(Y)^* & -h^*_{Y_t} \\ -g'_{t+\Delta t}(Y) & 0 & 0 \\ -h_{Y_t} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta Y \\ \Delta \lambda \\ 0 \end{bmatrix} = \begin{bmatrix} g'_{t+\Delta t}(Y)^* \lambda_t - \nabla_Y f_{t+\Delta t}(Y_t) \\ g_{t+\Delta t}(Y_t) \\ 0 \end{bmatrix}$$

Proving invertibility of the linearized KKT

We need to show that for Δt small enough the matrix

$$\begin{bmatrix} \nabla_Y^2 \mathcal{L}_{t+\Delta t}(\lambda) & -g'_{t+\Delta t}(Y)^* & -h_{Y_t}^* \\ -g'_{t+\Delta t}(Y) & 0 & 0 \\ -h_{Y_t} & 0 & 0 \end{bmatrix}$$

is invertible when Y_t is a an optimal solution for the non-restricted problem (BM_t).

Proving invertibility of the linearized KKT

We can show that for $\Delta t = 0$ the matrix

$$M = \begin{bmatrix} \nabla_Y^2 \mathcal{L}_t(\lambda) & -g_t'(Y)^* & -h_{Y_t}^* \\ -g_t'(Y) & 0 & 0 \\ -h_{Y_t} & 0 & 0 \end{bmatrix}$$

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Theorem

Let $(X_t = Y_t Y_t^T, Z_t)$ be an optimal primal-dual pair of solutions to (SDP_t)-($\mathrm{D\text{-}SDP}_t$) *which is* ***strictly complementary*** *and such that X_t is* ***primal non-degenerate****.*

Then there exists a unique Lagrange multiplier λ_t for ($\mathrm{BM}_{Y_t, t+\Delta t}$) at $\Delta t = 0$ such that the triple $(Y_t, \lambda_t, 0)$ is a KKT triple for ($\mathrm{BM}_{Y_t, t+\Delta t}$) at $\Delta t = 0$ fulfilling the second-order sufficient conditions. In particular, M is invertible.

An algorithm for TV-SDP

Input: an initial approximate primal-dual solution $(\hat{X}_0, Z(\hat{\lambda}_0))$ to (SDP$_0$)–(D-SDP$_0$)
initial stepsize Δt
stepsize tuning parameters $\gamma_1 \in (0, 1)$, $\gamma_2 > 1$
residual tolerance $\epsilon > 0$
Output: solutions $\{\hat{X}_k\}_{k=0,\ldots,K}$ to (SDP$_t$) for $t \in \{0, \ldots, t_k, \ldots, T\}$

1: $k \leftarrow 0$
2: $t_0 \leftarrow 0$
3: $S = \{\hat{X}_0\}$, $r = (\hat{X}_0)$
4: find $\hat{Y}_0 \in \mathbb{R}^{n \times r}$ such that $\hat{Y}_0 \hat{Y}_0^T = \hat{X}_0$
5: **while** $t_k < T$ **do**
6: solve the linearized KKT system with data $\Delta t, t_k, \hat{Y}_k, \hat{\lambda}_k$ and obtain $\Delta Y, \Delta\lambda$
7: **if** $\mathrm{res}_{\hat{Y}_k, t_k+\Delta t}(\hat{Y}_k + \Delta Y, \hat{\lambda}_k + \Delta\lambda) > \epsilon$ **then**
8: $\Delta t \leftarrow \gamma_1 \Delta t$
9: go back to step 6
10: **end if**
11: $(t_{k+1}, \hat{Y}_{k+1}, \hat{\lambda}_{k+1}) \leftarrow (t_k + \Delta t, \hat{Y}_k + \Delta Y, \hat{\lambda}_k + \Delta\lambda)$
12: append $\hat{Y}_{k+1}\hat{Y}_{k+1}^T$ to S
13: $\Delta t \leftarrow \min(T - t_{k+1}, \gamma_2 \Delta t)$
14: $k \leftarrow k + 1$
15: **end while**
16: **return** S

Theorem

Let $\delta > 0$ and $\Delta t > 0$ be small enough such that the following three conditions are satisfied:

$$(2\sqrt{\Lambda_*} + \delta)\delta + L\Delta t < \frac{2\lambda_*}{\sqrt{r+4} + \sqrt{r}}, \tag{1}$$

$$\delta < \frac{2}{3}\frac{m}{M}, \tag{2}$$

$$\left[\frac{1}{\lambda_*}((2\sqrt{\Lambda_*} + \delta)\delta + L\Delta t)^2 + \sqrt{r}(2\sqrt{\Lambda_*} + \delta)\delta + L\Delta t\right]^2 + (\delta + K\Delta t)^2 \leq \frac{2}{3}\frac{m}{M}\delta. \tag{3}$$

Assume for the initial point $(\hat{Y}_0, \hat{\lambda}_0)$ that

$$\min_{Q \in \mathcal{O}_r} \|(\hat{Y}_0, \hat{\lambda}_0) - (Y_0 Q, \lambda_0)\| \leq \delta. \tag{4}$$

Then the path-following algorithm is well-defined and for all $t_{k+1} = t_k + \Delta t$ the iterates satisfy

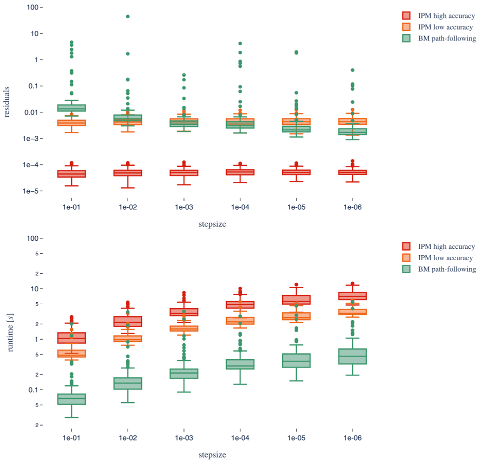
$$\min_{Q \in \mathcal{O}_r} \|(\hat{Y}_k, \hat{\lambda}_k) - (Y_{t_k} Q, \lambda_{t_k})\| \leq \delta.$$

It then holds that

$$\|\hat{X}_k - X_{t_k}\|_F \leq (2\sqrt{\Lambda_*} + \delta)\delta$$

for all t_k.

Numerical experiments



We tested the algorithm on SDP relaxations of 110 instances of time-varying Max-Cut problem with 100 nodes.

- As the stepsize decreases the accuracy of the solutions increases
- The runtime is competitive compared to using IPMs iteratively

Conclusions

- We proposed an algorithm for solving time-varying SDPs based on a path-following scheme for the Burer–Monteiro factorization.
- The restriction to a horizontal space ensures the invertibility of the linearized KKT conditions under standard regularity assumptions on the TV-SDP problem.
- We proved that if the initial precision and the time-step are small enough the tracking error is bounded
- Preliminary numerical experiments suggest that our algorithm is competitive both in terms of runtime and accuracy when compared to the application of standard IPMs

References

A. Bellon et al., *TV-SDP: Path Following a Burer–Monteiro Factorization*, `arxiv.org/abs/2210.08387` (2022)

A. Bellon et al., *TV-SDP: Geometry of the Trajectory of Solutions*, `arxiv.org/abs/2104.05445` (2021)

Thank you!

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