Capra-convexity in sparse optimization

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The ℓ_0 pseudonorm counts the nonzero coordinates of a vector. It is often used in optimization problems to enforce the sparsity of the solution. However, this function is nonconvex and noncontinuous, and optimization problems formulated with ℓ_0 in the objective function or in the constraints are hard to solve in general. Recently, a new family of coupling functions — called Capra (constant along primal rays) — has proved to induce relevant generalized Fenchel-Moreau conjugacies to handle the ℓ_0 pseudonorm. In particular, under a suitable choice of source norm on \mathbb{R}^d — used in the definition of the Capra coupling — the function ℓ_0 is Capra-subdifferentiable, hence is Capra-convex. In this talk, we propose a general overview the hidden structure of some sparse optimization problems in the light of their Capra-convex properties, and discuss perspectives for algorithmic applications.