Combinatorial optimization with energy constraints

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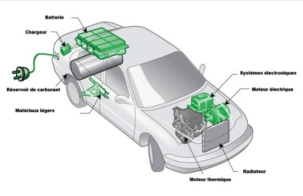
Plan

1. Introduction
2. Literature review
3. Resolution scheme
4. Focus on a scheduling problem with an energy source
5. Conclusion

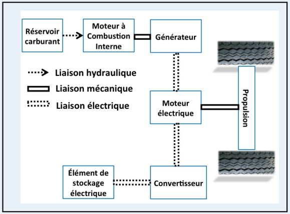
Hybrid-electric vehicles

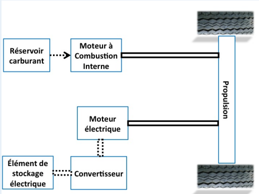


Electric propulsion motor powered by :

- onboard generator :
  - internal combustion engine or
  - hydrogen fuel cell (FC)
- reversible source :
  - battery or
  - supercapacitator (SE)

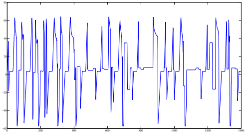
Architectures (hybrid-series, hybrid-parallel, ...)



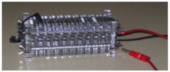


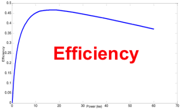
Problem description

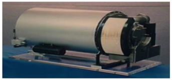
- Given the power request of a driver on a predefined road section ...

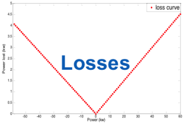


- ... and the characteristics of the energy sources : power limitations (kW), efficiency (%), storage capacity (kWs) ...





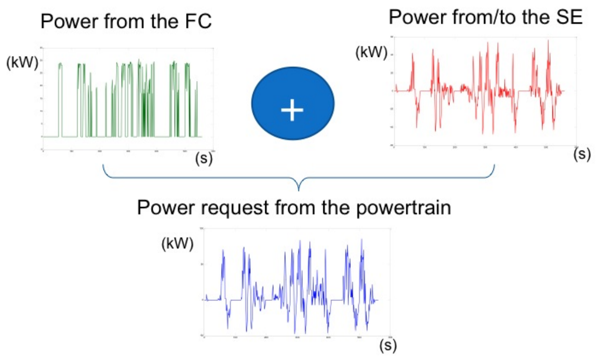




... Find at each instant the optimal power split between the energy sources to minimize the total fuel consumption.

Findings

- Example of solution



- Better modeling hypothesis and efficient reformulations
- 20% improvement over the previous state-of-the-art

Water pumping and desalination process

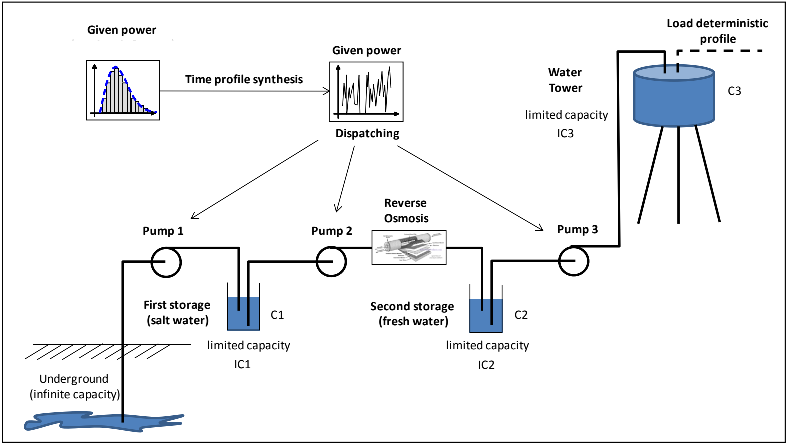


Figure : Source : (Sareni et al., 2012)

Mechanic-hydraulic-electric models

Electrical model

- V_m, I_m : electrical tension, courant
- T_m : motor electromag. torque
- Ω : rotation speed
- k_Φ : torque equivalent coefficient
- r : stator resistance

Electric motor equations (inertia neglected) :

$$V_m = rI_m + k_\Phi \Omega \quad (1)$$

$$T_m = \Phi_m I_m \quad (2)$$

Electrical power needed : $P_e = V_m I_m$.

Mechanical-Hydraulic conv.

- P_p : output pressure
- q : debit of water
- a, b : non linear girator coefs
- c : hydraulic friction
- p_0 : suction pressure
- $f_p + f_m$: mechanical losses

Static equations of the motor-pump (mechanical inertia neglected) :

$$P_p = (a\Omega + bq)\Omega - (cq^2 + p_0) \quad (3)$$

$$T_m = (a\Omega + bq)q + (f_m + f_p)\Omega \quad (4)$$

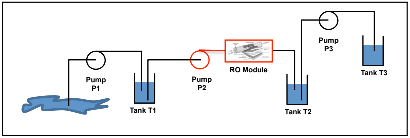
Pressure drop in the pipe

- ΔPipe : pressure drop
- h : height of water pumping
- ρ : water density

Static+Dynamic pressure

$$\Delta\text{Pipe} = kq^2 + \rho g h \quad (5)$$

Efficiency function of pump 2 + RO



The subsystem resulting from the combinaison of pump 2 and the Reverse Osmosis module is modeled with equation :

power required =

$$r * \mathcal{K}(\mathbf{q_c}, \mathbf{h}) + ((f_m + f_p) * \Omega(\mathbf{q_c}, \mathbf{h}) + (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}}) * \mathcal{M}(\mathbf{q_c}, \mathbf{h})) * \Omega(\mathbf{q_c}, \mathbf{h})$$

where

$$\begin{cases} \mathcal{F}(\mathbf{q_c}) = (R_{\mathrm{Mod}} + R_{\mathrm{Valve}}) * \mathbf{q_c}^2 \\ \mathcal{G}(\mathbf{q_c}) = (b * (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}})) \\ \mathcal{M}(\mathbf{q_c}, \mathbf{h}) = a * \Omega(\mathbf{q_c}, \mathbf{h}) + \mathcal{G}(\mathbf{q_c}) \\ \Omega(\mathbf{q_c}, \mathbf{h}) = \frac{-\mathcal{G}(\mathbf{q_c}) + \sqrt{\mathcal{G}(\mathbf{q_c})^2 - 4a*(-(p_0 + \rho g*(\mathbf{h} - l_{\mathrm{out}}) + (k+c)*((\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}})^2) + \mathcal{F}(\mathbf{q_c})))}}{2*a} \\ \mathcal{K}(\mathbf{q_c}, \mathbf{h}) = (((f_m + f_p) * \Omega(\mathbf{q_c}, \mathbf{h}) + (\mathbf{q_c} + \mathcal{F}(\mathbf{q_c})/R_{\mathrm{Me}}) * (a * \Omega(\mathbf{q_c}, \mathbf{h}) + \mathcal{G}(\mathbf{q_c})))/k_\phi)^2 \end{cases}$$

Literature review

Mathematical programming-based resolution methods on similar problems

Camponogara E., De Castro M. P. and Plucenio, A. *Compressor scheduling in oil fields : A piecewise-linear formulation*. IEEE International Conference on Automation Science and Engineering, p. 436 - 441, 2007.

Borghetti A., D'Ambrosio C., Lodi A. and Martello S., *An milp approach for short-term hydro scheduling and unit commitment with head-dependent reservoir*. IEEE Transactions on Power Systems, 23(3), p. 1115 - 1124, 2008.

Generic MINLP resolution methods

Grossmann I.E.. *Review of nonlinear mixed-integer and disjunctive programming techniques*. Optimization and Engineering, 3, p. 227 - 252, 2002.

Hybrid algorithms and frameworks

Polisetty P.K. and Gatzke E.P.. *A decomposition-based minlp solution method using piecewise linear relaxations*. Technical report, Univ. of South Carolina, 2006.

Bonami P., Biegler L.T., Conn A.R., Cornuéjols G., Grossmann I.E., Laird C.D., Lee J., Lodi A., Margot F., Sawaya N., and Wächter A. *An algorithmic framework for convex mixed integer nonlinear programs.* Discrete Optimization, 5(2), p.186 - 204, 2008.

Floudas C.A. and Gounaris C.E.. *A review of recent advances in global optimization*. Journal of Global Optimization, 45, p. 3 - 389, 2009.

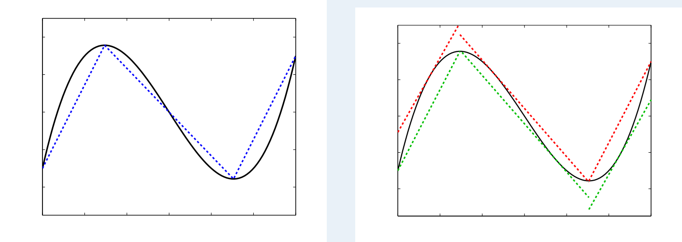
PGMO project OREM

- Previous studies involve multiple energy sources and general non-linear efficiency functions, but no scheduling.
- All our previous work on scheduling under energy constraint considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.
- We want to solve explicitely and in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

http://homepages.laas.fr/sungueve/PGMOOREM.html

Resolution scheme : (Ngueveu et al., 2014)

Step 1 : Piecewise linear bounding of the nonlinear energy transfer/efficiency functions



(a) Linear approximation (b) Piecewise bounding

Step 2 : Reformulation of the problem into two mixed integer problems (MILP)

- the problem is originally a MINLP
- using the pair of bounding functions previously defined

Piecewise bounding (Ngueveu et al., 2014)

Mathematical formulations

Specifities of piecewise bounding with a tolerance ϵ

Proof of optimality

How to perform the bounding

Resulting bounding algorithms proposed

Results on a practical problem

Deeper theoritical analysis required

- What is the impact of the (piecewise linear) energy function on the nature and the structure of a problem ?
- Can it render NP-hard an initially polynomial pb ?
- Can it render polynomial an initially NP-hard problem ?
- If polynomial what is the best algorithm to solve the resulting pb ?
- If NP-hard, what is the best formulation and best approach for the resulting pb ?

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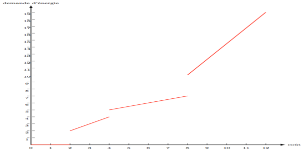
Back to basics : A pre-emptive scheduling problem

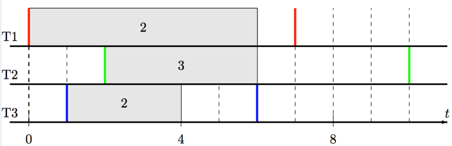
Data

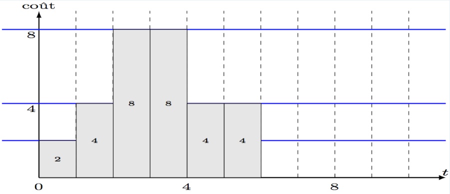
	T1	T2	T3
release date	0	2	1
due date	7	10	6
duration	6	4	3
energy demand	2	3	2

Absolute efficiency function



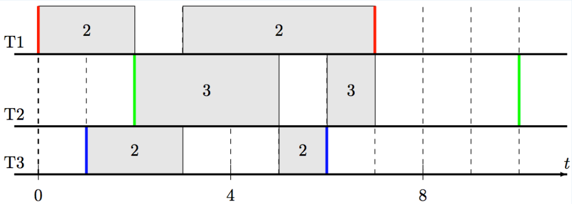
Initial solution : cost = 30





Back to basics : A pre-emptive scheduling problem

Optimal solution : cost = 26



Notations and definition

Data

- Set of time instants T
- Set of activities $\mathcal{A}$
  - r_i, d_i, p_i : release date, due date, duration of activity i
  - b_i : constant instantaneous energy demand of activity i
- Set of non-reversible energy sources $\mathcal{S}$
  - ρ^s : piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$).

Useful constants

- a_{it} : constant term equal to 1 if $t \in [r_i, d_i[$ and 0 otherwise

Decision variable

- x_{it} : binary, $= 1$ iff activity i is ongoing at instant t

Formulation

Minimize the total energy cost

$$(\mathrm{CF})\ \min \quad \sum_{t \in T} \rho^{-1}\Big(\sum_{i \in \mathcal{A}} b_i x_{it}\Big) \tag{8}$$

s.t.
Satisfaction of the demand for each activity

$$\sum_{t \in T} a_{it} x_{it} \geq p_i, \qquad \forall i \in \mathcal{A} \tag{9}$$

Validity domain

$$x_{it} \in \{0, 1\}, \qquad \forall i \in \mathcal{A}, t \in T \tag{10}$$

Equivalence to a single-source problem

Theorem

$\forall$ (P) with $|\mathcal{A}'| > 1$, $\exists$ (P') with $|\mathcal{A}'| = 1$ such that (P) and (P') are equivalent.

Proof outline

For all x, $\rho'(x)$ can be defined as the solution cost of the problem :

$$\min(\sum_{i \in \mathcal{S}} \rho^i(y_i)) \tag{11}$$

s.t.

$$\sum_{i \in \mathcal{S}} y_i = x \tag{12}$$

$$y_i \in \mathbb{R}, \quad \forall i \in \mathcal{S} \tag{13}$$

Proof of Complexity

Theorem

(P') is NP-hard.

Proof outline

- Any decisional instance of the discrete bin packing problem can be transformed into a particular decisional instance of (P').
  - Decisional discrete BPP : n items of size $b_i, \forall i \in 1..n$; bin capacity C. Does a solution exists with at most B bins ?
  - equivalent to the following $(\widetilde{\mathrm{P}}')$ problem : n activities, each with energy demand b_i ; an energy source of efficiency function

$$\widetilde{\rho}'(x) = 1 \text{ if } 0 \leq x \leq C \quad \text{ and } B \text{ if } x \geq C$$

Does it exists a solution of $(\widetilde{\mathrm{P}}')$ with a cost not exceeding B ?

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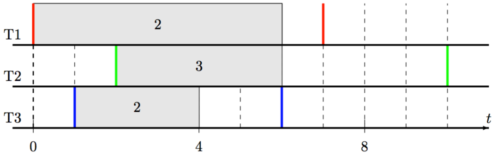
Based on Activity sets

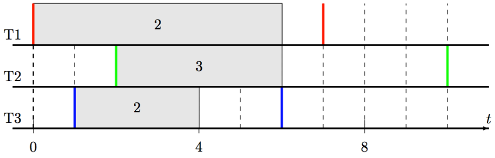


↓

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	t=6	t=7	t=8	t=9
{1}	{1,3}	{1,2,3}	{1,2,3}	{1,2}	{1,2}	∅	∅	∅	∅

	Demand	Release date	Due date	Cost
{1}	2	0	7	2
{3}	2	1	6	2
{1,2}	5	2	7	4
{1,3}	4	1	6	4
{1,2,3}	7	2	6	8

Based on Activity sets



$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	t=6	t=7	t=8	t=9
{1}	{1,3}	{1,2,3}	{1,2,3}	{1,2}	{1,2}	∅	∅	∅	∅

	Demand	Release date	Due date	Cost
{1}	2	0	7	2
{3}	2	1	6	2
{1,2}	5	2	7	4
{1,3}	4	1	6	4
{1,2,3}	7	2	6	8

Additional Notations and definitions

Additional data

- Set of activity sets executable in parallel at any given instant $\mathcal{L}$
- Set of activities belonging to set l $\mathcal{A}_l$:
- $\forall l \in \mathcal{L}$:
  - b_l : energy demand $(= \sum_{i \in \mathcal{A}_l} b_i)$
  - c_l : energy cost $(= \rho^{-1}(b_l))$
- Set of activity sets executable at instant t $\mathcal{L}_t$

Useful constants

- a_{il} : constant term equal to 1 if $i \in \mathcal{A}_l$ and 0 otherwise

Additional decision variables

- y_{lt} : binary, $= 1$ iff activity set l is being executed at time t

Extended formulation

Minimize the total energy cost

$$(EF)\min \sum_{t\in\mathcal{T}}\sum_{l\in L_t} c_l y_{lt} \tag{14}$$

s.t.

Satisfaction of the demand for each activity

$$\sum_{l\in L}\sum_{t=R_l}^{D_l-1} a_{il} y_{lt} \geq p_i, \qquad \forall i\in\mathcal{A} \tag{15}$$

Only one set active at each instant-time

$$-\sum_{l\in\mathcal{L}_t} y_{lt} \geq -1, \qquad \forall t\in\mathcal{T} \tag{16}$$

Validity domain

$$y_{lt}\in\{0,1\} \qquad \forall t\in\mathcal{T}, l\in\mathcal{L}_t \tag{17}$$

Extended formulation

Minimize the total energy cost

$$(EF)\min \sum_{t\in\mathcal{T}}\sum_{l\in L_t} c_l y_{lt} \tag{18}$$

s.t.

Satisfaction of the demand for each activity

$$\sum_{l\in L}\sum_{t=R_l}^{D_l-1} a_{il} y_{lt} \geq p_i, \qquad \forall i\in\mathcal{A} \tag{19}$$

Only one set active at each instant-time

$$-\sum_{l\in\mathcal{L}_t} y_{lt} \geq -1, \qquad \forall t\in\mathcal{T} \tag{20}$$

Link variables x and y

$$x_{it} - \sum_{l\in\overline{L}_t} a_{il} y_{lt} = 0, \quad \forall i\in\mathcal{A}, t\in\mathcal{T} \tag{21}$$

Validity domain

$$x_{it} \in \{0,1\} \qquad \forall t\in\mathcal{T}, i\in\mathcal{A} \tag{22}$$

$$y_{lt} \in \mathbb{R} \qquad \forall t\in\mathcal{T}, l\in\mathcal{L}_t \tag{23}$$

Decomposition

The linear relaxation of the master problem

$$(LRMP)\ \min \sum_{t \in T} \sum_{l \in \overline{L}_t} c_l y_{lt} \quad (25)$$

s.t.

$$x_{it} - \sum_{l \in \overline{L}_t} a_{il} y_{lt} = 0, \quad \forall i \in A, t \in T \quad (26)$$

$$\sum_{l \in \overline{L}} \sum_{t=R_l}^{D_l-1} a_{il} y_{lt} \geq p_i, \quad \forall i \in A \quad (27)$$

$$-\sum_{l \in \overline{L}_t} y_{lt} \geq -1, \quad \forall t \in T \quad (28)$$

$$x_{it} \leq 1 \quad \forall i \in A, t \in T \quad (29)$$

$$y_{lt} \geq 0 \quad \forall t \in T, l \in \overline{L}_t \quad (30)$$

$$x_{it} \geq 0 \quad \forall i \in A, t \in T \quad (31)$$

The resulting dual (DLMRP) is :

$$\max \sum_{i \in A} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in A} \sum_{t \in T} z_{it} \quad (32)$$

s.t.

$$\sum_{i \in A} a_{il}(u_i - w_{it}) - v_t \leq c_l, \quad \forall t \in T, l \in \overline{L}_t \quad (33)$$

$$w_{it} + z_{it} \leq 0, \quad \forall i \in A, t \in T \quad (34)$$

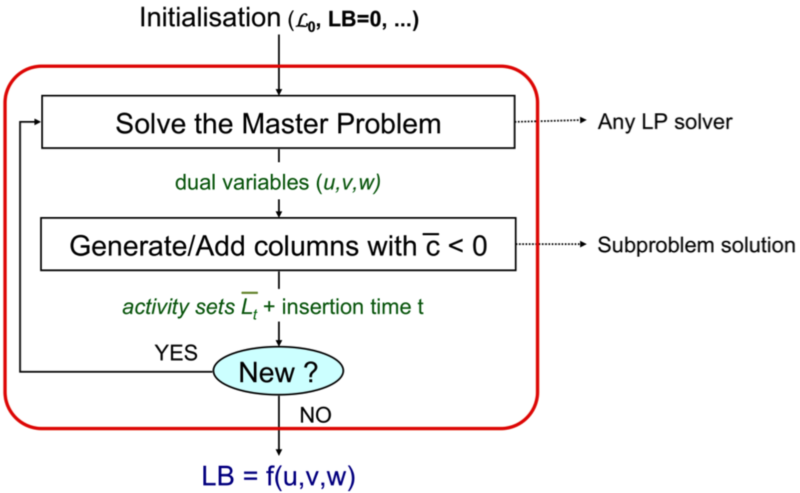
$$w_{it} \in \mathbb{R}, \quad \forall i \in A, t \in T \quad (35)$$

$$u_i \geq 0, \quad \forall i \in A \quad (36)$$

$$v_t \geq 0, \quad \forall t \in T \quad (37)$$

Therefore, the reduced cost of a column y_{lt} is :
$\overline{c} = c_l - \sum_{i \in A} a_{il}(u_i) - v_t$

Column Generation



Subproblem

all dual values u_i, v_t, w_{it}
problem data a_{it}, b_i, ρ^{-1} } $\rightarrow$  $\rightarrow$ { best time t'
best task set I'

Decision variables needed : $\alpha_i, \beta_t, \gamma_{it}, \in \{0, 1\}$

$$\max \sum_{i \in A} \alpha_i u_i - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \rho^{-1} (\sum_{i \in A} \alpha_i b_i) \quad (38)$$

s.t.

$$\alpha_i + \beta_t \leq 1, \qquad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i \quad (39)$$

$$\sum_{t \in T} \beta_t = 1 \quad (40)$$

$$\gamma_{it} - 0.5\alpha_i - 0.5\beta_t \leq 0, \qquad \forall i \in A, t \in T \quad (41)$$

$$\gamma_{it} - \alpha_i - \beta_t \geq -1, \qquad \forall i \in A, t \in T \quad (42)$$

Subproblem

all dual values u_i, v_t, w_{it}
problem data a_{it}, b_i, ρ^{-1} $\}$ $\rightarrow$

$\rightarrow$ $\{$ best time t'
best task set I'

Decision variables needed : $\alpha_i, \beta_t, \gamma_{it}, \in \{0, 1\}$

$$\max \sum_{i \in A} \alpha_i u_i - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \rho^{-1} (\sum_{i \in A} \alpha_i b_i) \quad (38)$$

s.t.

$$\alpha_i + \beta_t \leq 1, \quad \forall i \in A, t \leq r_i - 1 \text{ or t } \geq d_i \quad (39)$$

$$\sum_{t \in T} \beta_t = 1 \quad (40)$$

$$\gamma_{it} - 0.5\alpha_i - 0.5\beta_t \leq 0, \quad \forall i \in A, t \in T \quad (41)$$

$$\gamma_{it} - \alpha_i - \beta_t \geq -1, \quad \forall i \in A, t \in T \quad (42)$$

Subproblem SP2 with fixed t

a predefined time t'
related dual values $u_i, w_{it'}$
problem data a_{it}, b_i, ρ^{-1}

$\rightarrow$  $\rightarrow$ best task set I'

Decision variables needed : $\alpha_i \in \{0, 1\}$

$$\max \quad \sum_{i \in A} \alpha_i(u_i - w_{it'}) - v_{t'} - \rho^{-1}(\sum_{i \in A} \alpha_i b_i) \tag{43}$$

s.t.

$$\alpha_i \leq a_{it'}, \qquad \forall i \in A \tag{44}$$

Note : $v_{t'}$ is constant

Subproblem SP2 with fixed t

Input		Output
a predefined time t' related dual values $u_i, w_{it'}$ problem data a_{it}, b_i, ρ^{-1}	$\rightarrow$  $\rightarrow$	best task set I'

Decision variables needed : $\alpha_i \in \{0, 1\}$

$$\max \quad \sum_{i \in A} \alpha_i (u_i - w_{it'}) - v_{t'} - \rho^{-1} \left(\sum_{i \in A} \alpha_i b_i\right) \tag{43}$$

s.t.

$$\alpha_i \leq a_{it'}, \qquad \forall i \in A \tag{44}$$

Note : $v_{t'}$ is constant

Subproblem SP3

a predefined time t'
a predefined sector s
related dual values $u_i, w_{it'}$
problem data a_{it}, b_i, ρ^{-1}

$\rightarrow$  $\rightarrow$

best task set I'

Decision variables needed : $\alpha_i \in \{0, 1\}$

$$(\mathrm{SP4}_{\widetilde{t},\widetilde{s}})\ \max(\sum_{i \in \mathcal{A}} \alpha_i (u_i - w_{i\widetilde{t}} - \widetilde{a}_{\widetilde{s}} b_i)) - v_{\widetilde{t}} - \widetilde{b}_{\widetilde{s}} \quad (45)$$

s.t.

$$x_{\widetilde{s}}^{\min} \leq (\sum_{i \in \mathcal{S}} b_i \alpha_i) \leq x_{\widetilde{s}}^{\max} \quad (46)$$

$$\alpha_i \leq a_{i\widetilde{t}}, \quad \forall i \in A \quad (47)$$

$$\alpha_i \in \{0, 1\}, \quad \forall i \in A \quad (48)$$

Subproblem SP3

a predefined time t'
a predefined sector s
related dual values $u_i, w_{it'}$
problem data a_{it}, b_i, ρ^{-1}
→  → best task set I'

Decision variables needed : $\alpha_i \in \{0,1\}$

$$(\mathrm{SP4}_{\widetilde{t},\widetilde{s}})\ \max(\sum_{i\in\mathcal{A}} \alpha_i(u_i - w_{i\widetilde{t}} - \widetilde{a}_{\widetilde{s}} b_i)) - v_{\widetilde{t}} - \widetilde{b}_{\widetilde{s}} \quad (45)$$

s.t.

$$x_{\widetilde{s}}^{\min} \leq (\sum_{i\in\mathcal{S}} b_i\alpha_i) \leq x_{\widetilde{s}}^{\max} \quad (46)$$

$$\alpha_i \leq a_{i\widetilde{t}}, \qquad \forall i \in A \quad (47)$$

$$\alpha_i \in \{0,1\}, \qquad \forall i \in A \quad (48)$$

Subproblem SP4

a predefined sector s
related dual values $u_i, w_{it'}$
problem data a_{it}, b_i, ρ^{-1} } →  → { best time t' / best task set I'

Decision variables needed : $\alpha_i, \beta_t, \gamma_{i,t} \in \{0,1\}$

$$(\mathrm{SP3}_{\tilde{s}}) \max \sum_{i \in A} \alpha_i (u_i - \tilde{a}_s b_i) - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \tilde{b}_s \quad (49)$$

s.t.

$$x_s^{\min} \leq (\sum_{i \in S} b_i \alpha_i) \leq x_s^{\max} \quad (50)$$

$$\alpha_i + \beta_t \leq 1, \qquad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i \quad (51)$$

$$\sum_{t \in T} \beta_t = 1 \quad (52)$$

Subproblem SP4

a predefined sector s
related dual values $u_i, w_{it'}$
problem data a_{it}, b_i, ρ^{-1}
→  →
best time t'
best task set I'

Decision variables needed : $\alpha_i, \beta_t, \gamma_{i,t} \in \{0, 1\}$

$$(\mathrm{SP3}_{\widetilde{s}})\ \max \sum_{i \in A} \alpha_i(u_i - \widetilde{a}_s b_i) - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \widetilde{b}_s \tag{49}$$

s.t.

$$x_s^{\min} \leq (\sum_{i \in \mathcal{S}} b_i \alpha_i) \leq x_s^{\max} \tag{50}$$

$$\alpha_i + \beta_t \leq 1, \qquad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i \tag{51}$$

$$\sum_{t \in T} \beta_t = 1 \tag{52}$$

Subproblem Variants

Notation a-b-c-d

a) type of subproblem
   1 for fixed t
   2 for variable t

b) column adding policy
   1 at instant t
   2 at all feasible instants
   3 only at feasible instants of negative reduced cost

c) multiple sets ? (only available for a=1)
   0 stop the pricer as soon as one column is added
   1 try to generate multiple columns before exiting

d) time increment (only available for a=1)
   0 restart from $t = 0$
   1 restart from the instant where the last pricing stopped

1 Introduction

2 Literature review

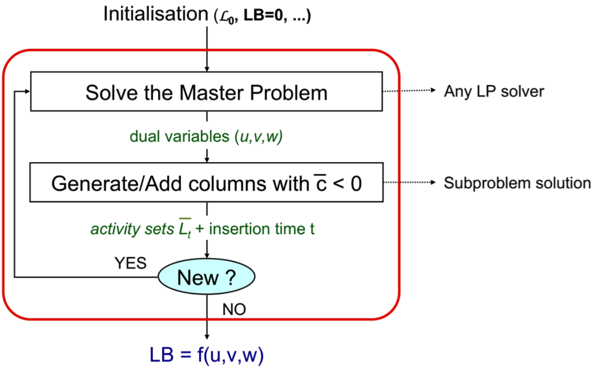
3 Resolution scheme

4 Focus on a scheduling problem with an energy source
- Problem definition
- Dantzig-Wolfe decomposition
- Branch-and-Price
- Results

5 Conclusion

Branch-and-Price

- Column Generation



- Branch-and-Price = Branch-and-Bound + Column Generation

1. Introduction
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Preliminary results and best parameters identification

Settings	NbObs
1-1-1-0	91
1-1-1-1	91
1-3-1-1	90
1-2-1-1	85
1-3-1-0	85
1-2-0-1	82
1-2-1-0	82
1-3-0-1	82
1-1-0-1	68
2-2-0-0	63
2-3-0-0	63
2-1-0-0	55
1-2-0-0	53
1-3-0-0	52
1-1-0-0	41

- Basic settings : 2-1-0-0 (subproblem 1) and 1-1-0-0 (subproblem 2)
- All alternatives produced improvements over their respective basic settings (i.e. all 2-X-X-X are better than 2-1-0-0. All 1-X-X-X are better than 1-1-0-0)

- 100 instances among 288
- time limit = 600 s

Preliminary results and best parameters identification

		Settings		
		1-1-1-0	1-1-1-1	1-3-1-1
Ratio	min	92.80 %	**92.96 %**	81.41 %
	max	**100 %**	**100 %**	**100 %**
	avg	**99.79 %**	**99.78 %**	99.62 %
NbCols	min	**113**	116	139
	max	26507	**17327**	138782
	avg	3236.3	**2870.1**	9726.4
NbPrice	min	**6**	**6**	8
	max	**761**	1176	1114
	avg	78.0	**72.96**	78.5
Nbnodes	min	**1**	**1**	**1**
	max	488	**331**	698
	avg	35.6	**27.8**	36.81

- 100 instances among the 288, time limit = 600 s

Final results

		1-1-1-1	Compact	
		Branch & Price	Root	Branch & Cut
NbObs		288	288	288
NbOpt		**261**	0	5
Ratio	min	77.47 %	57.82 %	57.85 %
	max	100%	83.54 %	99.99 %
	avg	**99.81%**	69.60 %	84.21 %
Time	min	$<$ 1 s	$<$0.1 s	7 s
	max	3603 s	2 s	1927 s
	avg	**540 s**	$<$ 1 s	1034 s
Nbnodes	min	1	1	57157
	max	9755	1	13470373
	avg	171	1	2174044

- All 288 instances, time limit = 3600 s + node limit

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Conclusion

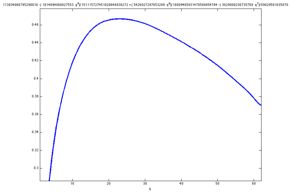
Done

- Energy sources characteristics in (combinatorial) optimization pbs.
- Resolution scheme : piecewise bounding and integer programming
- Impact of non reversible sources functions : agregability, ...
- Complexity analysis, Extended formulation, Dantzig-Wolfe Decomposition, Branch-and-price

What next ?

- Reversible energy sources
  - Previous theorems no longer valid and no direct adaptation
  - No clustering ! Time horizon ?
- Non-energy-related problems ?

Integration of a real-world efficiency function



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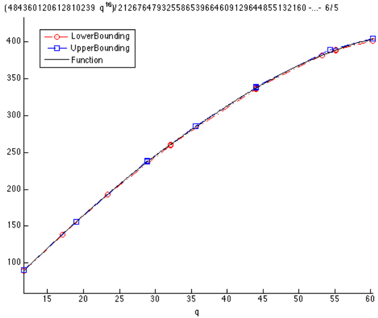

References and Acknowledgements

M. Guemri, S. Caux, S.U. Ngueveu and F. Messine,
Heuristics and lower bound for energy management in hybrid-electric vehicles.
Under review

Y. Gaoua, S. Caux and P. Lopez,
A combinatorial optimization approach for the electrical energy management in a multi-source system.
Under review

X. Roboam, B. Sareni, D.-T. Nguyen, and J. Belhadj,
Optimal system management of a water pumping and desalination process supplied with intermittent renewable sources.
Proceedings of PPPSC 2012, vol 8, p. 369-374, 2012.

S.U. Ngueveu, B. Sareni, S. Caux,
Piecewise bounding and Integer Linear Programming for the optimal management of a water pumping and desalination process
Technical report, 14 pages, 2014.

S.U. Ngueveu, C. Artigues, P. Lopez,
Scheduling under multiple energy resources
PGMO-COPI'14 accepted.