Combinatorial optimization with multiple resources and energy constraints

Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 2nd year

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OREM project : context and objectives

Context

The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

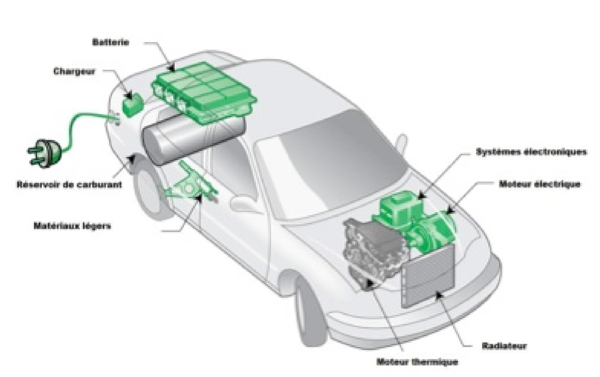
Objectives

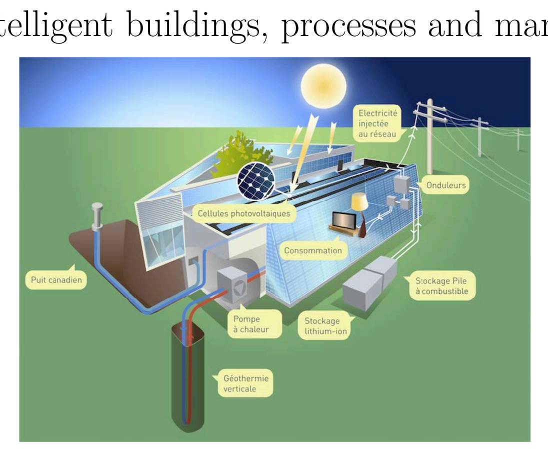
- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve explicitly and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

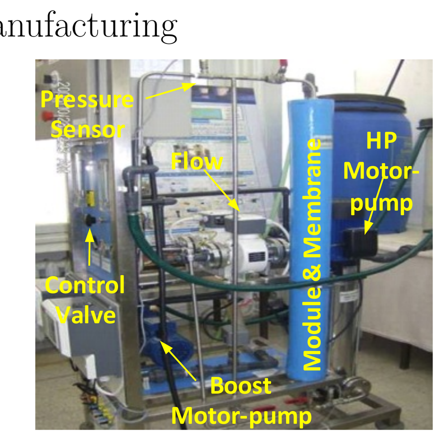
Applications and challenge

Applications

Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing

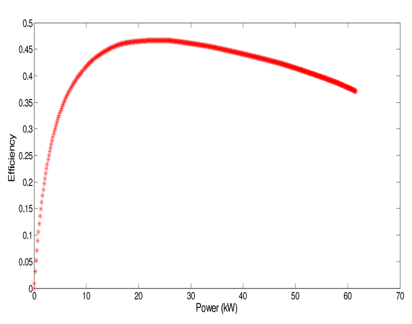


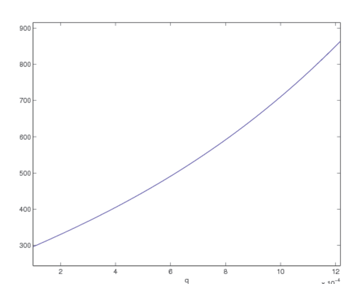




Challenge

Non-linearities come from energy efficiency functions

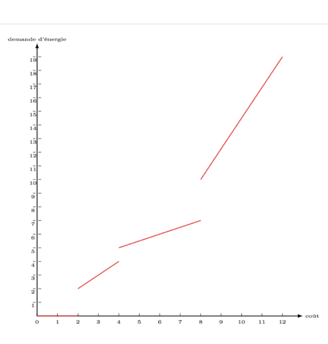




Phase 1 : Scheduling with non-reversible energy sources

Data

- Set of time periods T, Set of activities $\mathcal{A}$,
  - r_i, d_i, p_i : release date, due date, duration of activity i
  - b_i : constant instantaneous energy demand of activity i
- Set of non-reversible energy sources $\mathcal{S}$
  - ρ^s : piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$).
- a_{it} : constant term equal to 1 if $t \in [r_i, d_i[$ and 0 otherwise



Decision variables

- $x_{it}, \forall i \in \mathcal{A}, \forall t \in T$: binary
- $y_{lt}, \forall l \in \mathcal{L}, \forall t \in T$: binary

Compact Formulation

$$\min \sum_{t\in T} \rho^{-1}\Big(\sum_{i\in\mathcal{A}} b_i x_{it}\Big)$$

$$\text{s.t.} \sum_{t\in T} a_{it}x_{it} \ge p_i, \quad \forall i \in \mathcal{A}$$

$$x_{it} \in \{0,1\}, \quad \forall i \in \mathcal{A}, t \in T$$

Extended Formulation

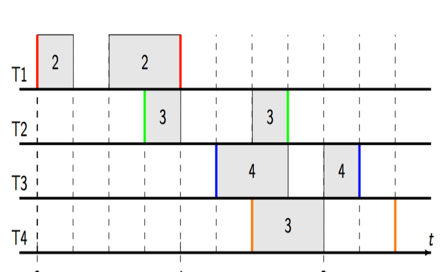
$$(\text{EF}) \min \sum_{t\in T}\sum_{l\in\overline{L}_t} c_l y_{lt}$$

$$x_{it} - \sum_{l\in\overline{L}_t} a_{il} y_{lt} = 0, \ \forall i \in \mathcal{A}, t \in T$$

$$\sum_{l\in\overline{L}}\sum_{t=R_l}^{D_l-1} a_{il} y_{lt} \ge p_i, \quad \forall i \in A$$

$$-\sum_{l\in\overline{L}_t} y_{lt} \ge -1, \quad \forall t \in T$$

Example



(a) Scheduling

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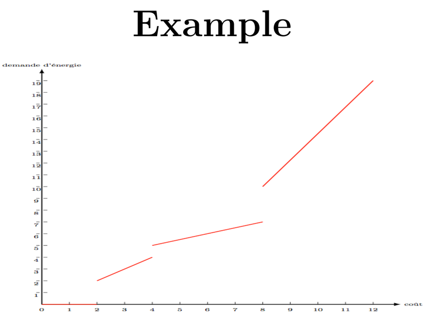

(b) Efficiency

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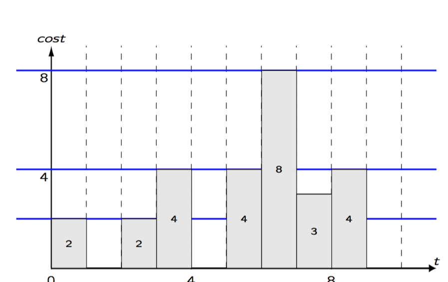


(c) Energy cost

Complexity and equivalence between single and multiple sources

- **Theorem 1** The problem is NP hard by reduction from discrete bin packing
- **Theorem 2** For any problem with multiple sources, there is an equivalent single source problem

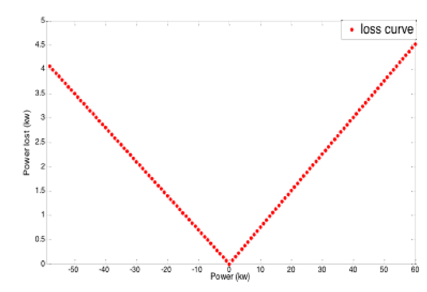
Solution Method : BP

- Dantzig-Wolfe decomposition on the extended formulation and Branch&Price (BP) Method
- Identification of four subproblem formulations and dedicated column generation schemes

Phase 2 : Add reversible energy sources to the scheduling problem with multiple energy sources

Data

- Data from Phase 1 : time periods, activities, non-reversible energy sources with piecewise-linear efficiency functions, a_{it} terms
- Set of reversible energy sources $\mathcal{RS}$
  - q^s_{init} : initial stock for source $s \in \mathcal{RS}$
  - $q^s_{\max}$: maximum storage capacity for source $s \in \mathcal{RS}$
  - initial assumptions :
    - negligeable losses (identity efficiency functions) for reversible sources
    - $q^s_{\min} = 0$ and final stock $\ge$ initial stock
- a_{it} : constant term equal to 1 if $t \in [r_i, d_i[$ and 0 otherwise



Decision variables

- x_{it} : binary, $= 1$ iff activity i is ongoing at time period t
- $p^s_t \ge 0$: energy produced by non-reversible energy source $s \in \mathcal{S}$ at time period t
- $r^s_t \in \mathbb{R}$: energy produced or received by reversible energy source $s \in \mathcal{RS}$ at time period t
- $q^s_t, \in [0, q^s_{\max}]$: energy stored in reversible energy source $s \in \mathcal{RS}$ at time period t

Compact Formulation with $|\mathcal{S}| = 1$ and $|\mathcal{RS}| = 1$

$$(\text{P}) \min \sum_{t\in T} \rho^{-1}(p_t)$$

$$\text{s.t.} \sum_{t\in T} a_{it}x_{it} \ge p_i, \forall i \in \mathcal{A} \quad | \quad q_{t+1} - q_t - r_t = 0, \forall t \in T$$

$$p_t = \sum_{i\in\mathcal{A}} b_i x_{it} + r_t, \forall t \in T \quad | \quad 0 \le q_t \le q_{\max}, \forall t \in T$$

$$q_0 = q_{\text{init}} \le q_{|T|} \quad | \quad x_{it} \in \{0,1\}, \forall i \in \mathcal{A}, t \in T$$

$$r_t \in \mathbb{R}, \forall t \in T$$

Properties identified for the problem (P)

- **P1** In (P), multiple reversible energy sources with identity efficiency functions can be agregated into an equivalent single reversible energy source
- **P2** In (P), two reversible energy sources with efficiency functions different from identity can not be agregated into an equivalent single energy source
- **P3** In (P), a reversible and a non reversible energy source can not be agregated into an equivalent single energy source

Relation with lot sizing problems and resulting upper and lower bounding procedures

A strong relation with lot sizing problems when $|RS| \neq 0$

- If x_{it} values are fixed in (P), then for the resulting subproblem :
  - the energy demand at each time period t is known and can be denoted $\bar{b}_t$
  - the goal = find the quantity of energy to produce/store from/to energy sources at each time period
  - the resulting subproblem is a lot sizing problem with bounded inventory

  $\rightarrow$ NP-hard if the production capacity is bounded, polynomial otherwise.

Lower bounds computation

- relax the integrality constraints on x_{it} and solve the resulting lot sizing problem with demand time windows

Iterative upper bounding scheme

Initialization (see ⚙)

1. Step 1 : Apply the efficient BP solution procedure from Phase 1 which solves the scheduling problem with only non-reversible energy sources
2. Step 2 : Fix the energy demand profile $\bar{b}$ using the solution from Step 1 and solve a lot sizing problem to obtain the best storage profile $\bar{r}$ for $\bar{b}$.
3. Step 3 : Update the piecewise linear function of the non-reversible energy source using the storage profile $\bar{r}$ from Step 2 : $\bar{\rho}_t(\sum_{i\in\mathcal{A}} b_i x_{it}) = \rho(\max\{0, \sum_{i\in\mathcal{A}} b_i x_{it} + \bar{r}_t\}), \forall t \in T$
4. Return to Step 1 to solve the scheduling problem without $\mathcal{RS}$, but with updated piecewise linear efficiency function $\bar{\rho}$ for the non-reversible energy source

NB : Stopping criterion = local optima

⚙ $\rightarrow$ { set $\bar{r} = 0, \forall t \in T$; set $\bar{r}$ using the solution of the lower bounding procedure }

Perspectives

- Extension of the bounding methods to handle general efficiency functions for reversible energy sources
- Combine mathematical and constraint programming to solve exactly the problem with reversible energy sources