Combinatorial optimization with multiple resources and energy constraints

Optimisation sous contraintes de ressources énergétiques multiples (OREM) - PGMO project (2013-2015) - 1st year

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OREM project : context and objectives

Context

The integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling, yields a combinatorial optimization challenge. It follows that the literature on this subject is sparse. Pre-existing studies involve multiple energy sources and general non-linear efficiency functions, but generally no scheduling. All our previous work on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

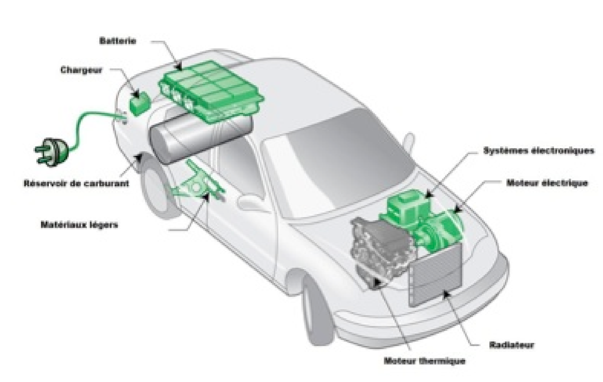
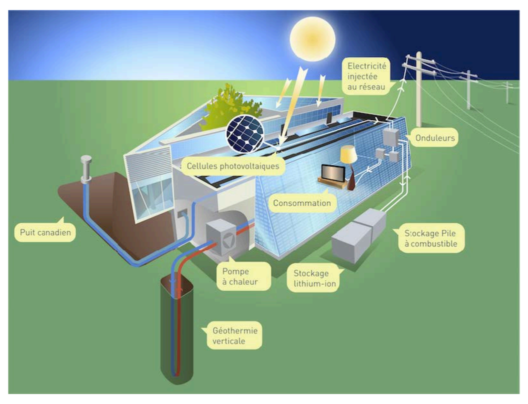
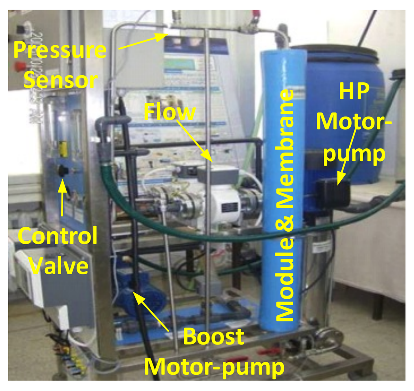
Objectives

- Address the (combinatorial) optimization challenge of integrating energy sources constraints (physical, technological and performance characteristics) in deterministic (scheduling) models.
- Solve explicitly and in an integrated fashion the resulting energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions.

Applications and challenge

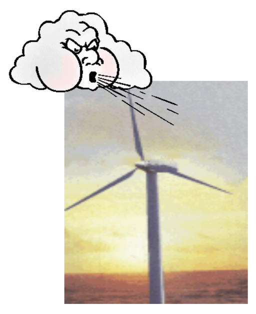
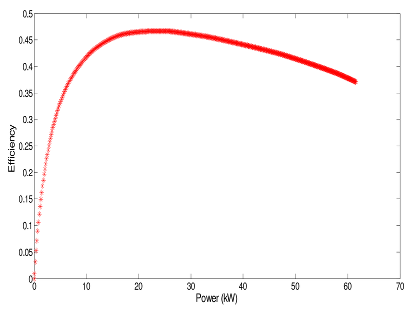
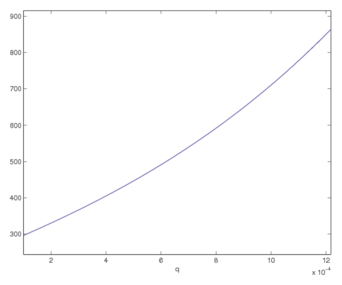
Applications

Scheduling for hybrid electric vehicles, intelligent buildings, processes and manufacturing

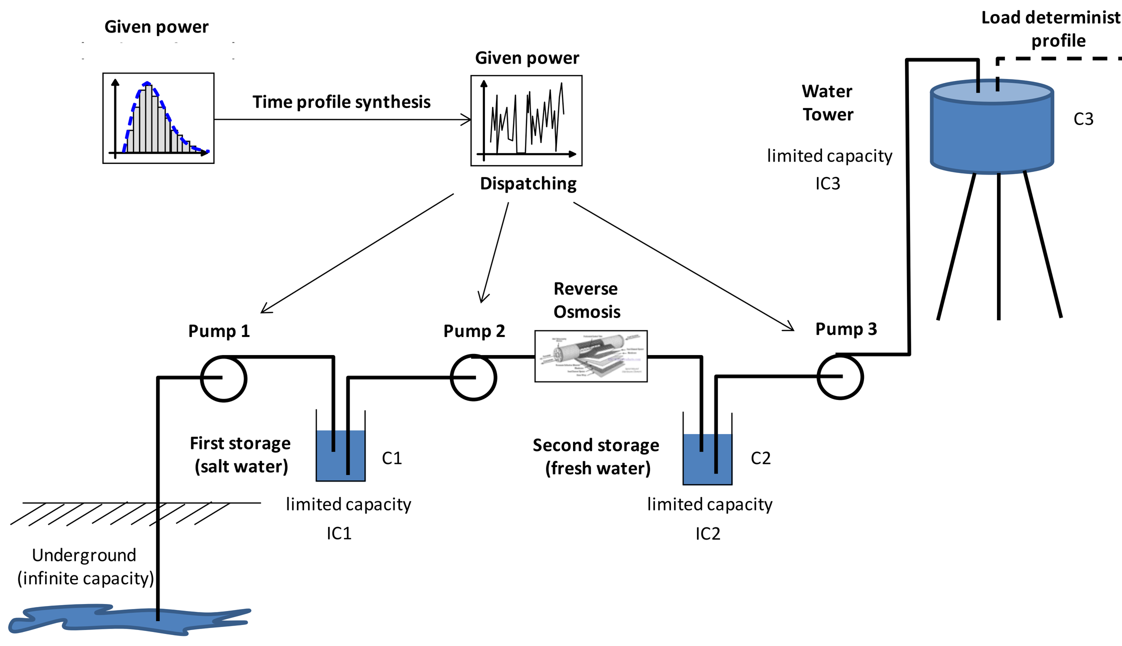
Challenge

Non-linearities come from energy efficiency functions

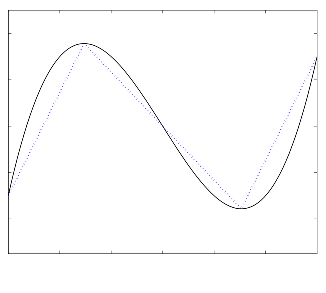
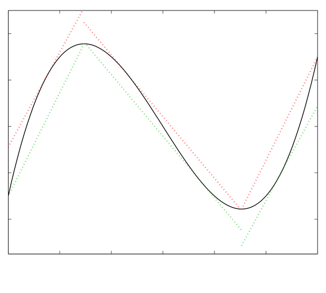
Piecewise linear bounding procedures for the optimal management of water pumping and desalination processes

Proof of concept on a water pumping and desalination system [CLAIO 2014, IFORS 2014]



Solution Method

- Step 1 : Piecewise linear bounding vs classic approx. of the nonlinear energy transfer/efficiency functions

- Step 2 : Reformulation of the problem into two mixed integer problems ($\underline{\text{MILP}}$ and $\overline{\text{MILP}}$)
  - the problem is originally a MINLP
  - using the pair of bounding functions previously defined

Results

Number of sectors per tolerance value

	Pump 1		(Pump 2+RO)		Pump 3	
ϵ	$\overline{n_{p_1}}$	$\underline{n_{p_1}}$	$\overline{n_{p_2}}$	$\underline{n_{p_2}}$	$\overline{n_{p_3}}$	$\underline{n_{p_3}}$
5%	2	2	11	21	3	2
1%	5	5	21	29	8	5
0.5%	8	7	35	62	13	7
0.3%	10	9	43	74	17	9

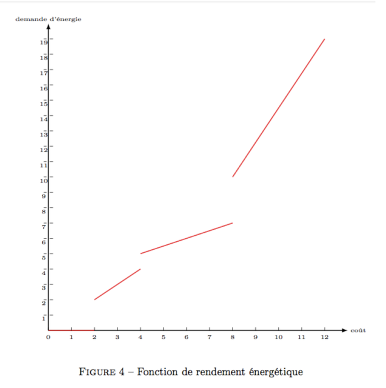
Lower and upper bound obtained

	$\overline{\text{MILP}}$		$\underline{\text{MILP}}$			Gap	opt
ϵ	UB	s	LB	s	UB*	%	
5%	20580	4	19740	15	-	4.25	no
1%	20100	15	19920	140	-	0.9	no
0.5%	20040	178	19980	117	-	0.3	no
0.3%	20040	64	19980	321	19980	0.0	yes

Generic problem : Scheduling with non-reversible energy sources

Data

- Set of time time periods T
- Set of activities $\mathcal{A}$
  - r_i, d_i, p_i : release date, due date, duration of activity i
  - b_i : constant instantaneous energy demand of activity i
- Set of non-reversible energy sources $\mathcal{S}$
  - ρ^s : piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S}$).
- a_{it} : constant term equal to 1 if $t \in [r_i, d_i[$ and 0 otherwise



Decision variables

- x_{it} : binary, = 1 iff activity i is ongoing at time period t

Formulation

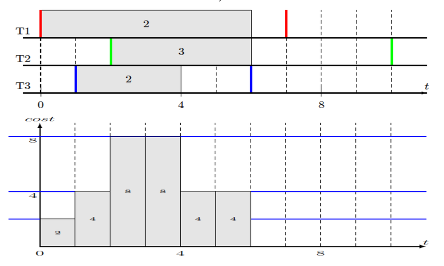
$$
\begin{aligned}
\text{(CF)} \min & \sum_{t \in T} \rho^{-1}\Big(\sum_{i \in \mathcal{A}} b_i x_{it}\Big) \\
\text{s.t.} & \sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in \mathcal{A} \\
& x_{it} \in \{0,1\}, \quad \forall i \in \mathcal{A}, t \in T
\end{aligned}
$$

Complexity and equivalence between single and multiple sources

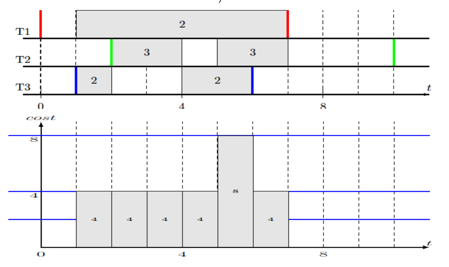
- **Theorem 1** The problem is NP hard by reduction from discrete bin packing
- **Theorem 2** For any problem with multiple sources, there is an equivalent single source problem

Example

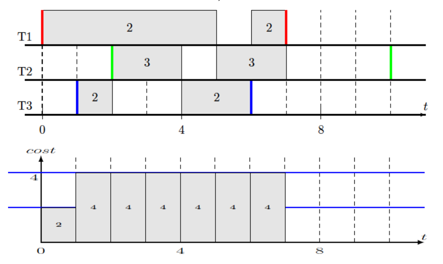
Solution 1, cost = 30



Solution 2, cost = 28



Solution 3, cost = 26



Dantzig-Wolfe decomposition and Branch&Price Method

Extended formulation based on feasible subsets of activities

- Set of activity sets executable in parallel $\mathcal{L}$ (at any given time period $\overline{L}_t$)
- Set of activities belonging to set l : demand b_l, cost c_l, time window $[R_l, D_l]$
- a_{il} : binary constant term equal to 1 iff i belongs to set l
- Variable y_{lt} : binary, = 1 iff activity set l is being executed at time t

The linear relaxation of the master problem

$$
\begin{aligned}
(LRMP) \min & \sum_{t \in T} \sum_{l \in \overline{L}_t} c_l y_{lt} \\
x_{it} - \sum_{l \in \overline{L}_t} a_{il} y_{lt} = 0, & \ \forall i \in A, t \in T \\
\sum_{l \in \overline{L}} \sum_{t=R_l}^{D_l - 1} a_{il} y_{lt} \geq p_i, & \quad \forall i \in A \\
- \sum_{l \in \overline{L}_t} y_{lt} \geq -1, & \quad \forall t \in T \\
x_{it} \leq 1 & \ \forall i \in A, t \in T \\
y_{lt} \geq 0 & \ \forall t \in T, l \in \overline{L}_t \\
x_{it} \geq 0 & \ \forall i \in A, t \in T
\end{aligned}
$$

The resulting dual (DLMRP) is :

$$
\begin{aligned}
\max & \sum_{i \in A} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in A} \sum_{t \in T} z_{it} \\
& \sum_{i \in A} a_{il}(u_i - w_{it}) - v_t \leq c_l, \ \forall t \in T, l \in \overline{L}_t \\
& w_{it} + z_{it} \leq 0, \ \forall i \in A, t \in T \\
& w_{it} \in \mathrm{R}, \ \forall i \in A, t \in T \\
& u_i \geq 0, \quad \forall i \in A \\
& v_t \geq 0, \quad \forall t \in T
\end{aligned}
$$

Therefore, the reduced cost of a column y_{lt} is : $c_l - \sum_{i \in A} a_{il}(u_i) - v_t$

Two column generation schemes

SP1 : all dual values u_i, v_t, w_{it}; problem data a_{it}, b_i, ρ^{-1} → → { best time t'; best task set l' }

SP2 : a predefined time t'; related dual values $u_i, w_{it'}$; problem data a_{it}, b_i, ρ^{-1} → → { best task set l' }

Results

		Extended Model	Compact Model
		Branch & Price	Black Box Solver
#Opt/#Instances		**261**/288	5/288
Ratio	avg (min, max)	**99.81%** (77.47 %, 100%)	84.21 % (57.85 %, 99.99 %)
Time	avg (min, max)	**540 s** (< 1 s, 3603 s)	1034 s (7 s, 1927 s)
Nbnodes	avg (min, max)	171 (1, 9755)	2174044 (57157, 13470373)