New lower bounds and exact method for the m-PVRP

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1 Introduction

The m-Peripatetic Vehicle Routing Problem (m-PVRP) is defined on a complete undirected graph $G = (V, E)$ where $V = \{0, ..., n\}$ is the node set (node 0 is the depot and $V' = V \backslash \{0\}$) and E is the edge set. Each client $i \in V'$ has a demand d_i and Q is the capacity of vehicles. A cost c_e is assigned to each edge $e \in E$. The objective of the m-PVRP is to identify a set of edge-disjoint routes of minimal total cost over m periods so that each client is served exactly once per period.

The m-PVRP was introduced in [5]. So far its best known upper and lower bounding procedures are the b-matching and the hybrid tabu search of [6]. Applications include money collection, transfer and dispatch when it is subcontracted to specialized companies. For security reasons, peripatetic and capacity constraints ensure that no sequence of clients is repeated during the m periods and the amount of money allowed per vehicle is limited. The m-PVRP can be considered as a generalization of two well-known NP-hard problems: the VRP ($\approx$ 1-PVRP) and the m-Peripatetic Salesman Problem (m-PSP $\approx$ m-PVRP with a single vehicle).

2 Column Generation approach

The new lower bounding procedure described in this section consists in two dual heuristics H1 and H2 that identify good dual feasible solutions for the linear relaxation of the aggregated set partitioning formulation, following the approach used for example in [2] for the VRP. Let $\Re$ be the set of feasible routes, c_r with $r \in \Re$ the cost of route r, $\Re_i$ the set of routes crossing node i, $\Re(e)$ the set of routes crossing edge e and y_r the binary variable equal to 1 if and only if route r is used. The aggregated set partitioning formulation (APF) of the m-PVRP is:

$$\text{(APF)} \quad \min \sum_{r \in \Re} c_r y_r \tag{1}$$

s. t.

$$\sum_{r \in \Re_i} y_r = m, \qquad \forall i \in V' \tag{2}$$

$$\sum_{r \in \Re} y_r \geq m \left\lceil \sum_{i \in V'} \frac{d_i}{Q} \right\rceil \tag{3}$$

$$\sum_{r \in \Re(e)} y_r \leq 1, \qquad \forall e \in E \tag{4}$$

$$y_r \in \{0, 1\}, \qquad r \in \Re \tag{5}$$

This formulation is not suitable for solvers because of its exponential number of variables. Its constraints can be dualized by associating respectively penalties $\lambda_i \in \mathbb{R}$ with $i \in V'$, $\lambda_0 \geq 0$ and $\mu_e \leq 0$ with $e \in E$. The linear relaxation of the resulting model (APF(λ, μ)) still requires an exponential number of variables, justifying the use of a column generation approach.

The first dual heuristic (H1) is based upon three key ideas: the approximation of routes with non-elementary routes called q-routes, the use of a column generation approach to handle the exponential number of variables and the use of dual ascent to estimate the best dual variables values. A route of total load q does not autorise cycles, whereas a q-route does and can be generated with a pseudo-polynomial algorithm. These cycles are then penalized via a Lagrangian relaxation, to obtain a heuristic fast and efficient. H1 does not require the use of Cplex® or any other solver to solve each master-problem obtained after generating q-routes of negative reduced cost. The violations of degree constraints are quantified then used in a subgradient procedure to correct the values assigned to the dual variables, until the improvements become negligible or during a predefined number of iterations. This procedure, known as dual ascent, outputs the best dual variables values that will be used to generate new q-routes of negative reduced cost. If no new q-routes can be generated, then the dual solution found provides a valid lower bound for the m-PVRP. The dual ascent remedies the dual variables stability problems of classical column generation algorithms.

The second heuristic (H2) is applied after two consecutive iterations of H1 without generating any q-route of negative reduced cost. It uses procedures similar to H1's, but generates elementary routes instead of q-routes. Its dual variables must be initialized with H1 to reduce the computing time and memory required, since the generation of elementary routes of negative reduced cost is an NP-complete problem ([3]). H1 is also used to compute lower bounds required by the dominance properties applied within H2 to optimize the generation of elementary routes.

3 Branch-and-cut applied on the edge-based formulation

The branch-and-cut algorithm BC_{EF} described in this section is applied on the edge-based m-PVRP formulation (EF) where $\mathbb{K} = \{1, ..., m\}$ is the set of periods of the m-PVRP, $\delta(S)$ is the set of edges having one node in $S \subseteq V'$ and the other node outside. $r(S) = \left\lceil \sum_{i \in S} \frac{d_i}{Q} \right\rceil$ is a lower bound of the number of vehicles necessary to serve the total demand of S and x_e^k is the binary variable equal to 1 if and only if edge e is used within a route during period $k \in \mathbb{K}$.

$$\text{(EF)} \quad \min \sum_{k \in \mathbb{K}} \sum_{e \in E} c_e x_e^k \tag{6}$$

s. t.

$$\sum_{e \in \delta(\{i\})} x_e^k = 2, \qquad \forall k \in \mathbb{K}, \forall i \in V' \tag{7}$$

$$\sum_{e \in \delta(S)} x_e^k \geq 2r(S), \qquad \forall S \subseteq V', S \neq \varnothing, \forall k \in \mathbb{K} \tag{8}$$

$$\sum_{k \in \mathbb{K}} x_e^k \leq 1, \qquad \forall e \in E \tag{9}$$

$$x_e^k \in \{0, 1\}, \qquad e \in E, k \in \mathbb{K} \tag{10}$$

The branch-and-cut algorithm BC_{EF} starts from the linear relaxation of (EF) without the $O(m2^n)$ constraints (8). The valid inequalities sought-after result from the generalization of efficient VRP inequalities applicable on each of the m periods of the m-PVRP: capacity cuts, strengthened combs cuts and multistar cuts. It is important to note that not all VRP cuts are generalizable to the m-PVRP. For example, constraints based on a fixed number of vehicles can not be used for the m-PVRP because its number of routes is not limited and can vary from one period to another (e.g. generalized capacity constraints or hypotour constraints ([1])). The branching is performed on the variable whose value is the closest to 0.5.

4 Branch-and-cut for the aggregated formulation

The aggregated edge-based formulation (AEF) results from the aggregation over k of constraints (7) to (9), the introduction of binary variable $y_e = \sum_{k \in \mathbb{K}} x_e^k$ for each edge $e \in E$ and the deletion of the aggregated constraints (9) because of their redundancy with the aggregated constraints (10).

A solution of (AEF) is a subset of edges which satisfy aggregated degree constraints, but are not specifically assigned to any of the m periods. It may not correspond to a valid m-PVRP solution. In addition, it can be shown that even if this set of edges contained a feasible m-PVRP solution, partitioning the edges between the m periods to extract this solution would be NP-complete. In spite of that, (AEF) is easier and faster to handle than (EF) not only because of the reduced number of variables, but also because of the aggregated constraints which reduce the number of violated valid inequalities to identify.

The branch-and-cut algorithm BC_{AEF} starts from the linear relaxation of (AEF) amputated of the 2^n aggregated capacity constraints. Valid inequalities sought-after are the aggregated capacity cuts, the aggregated strengthened comb cuts and the aggregated multistar cuts. The branching is performed on the variable whose value is the closest to 0.5.

5 Implementation and Results

All algorithms were tested by adding $m \leq 7$ periods to classical instances from the VRP literature (type A, B, P and VRPNC of 19 to 200 nodes). The upper bounds needed were obtained from [6].

H1 is coded in C. The q-route generator of H1 is then replaced with an adaptation of the procedure GENROUTE (in Fortran) from [2] to obtain H2. Results show that H1+H2 complement the b-matching because, contrary to the latter, it performs faster and better for small values of m.

BC_{EF} and BC_{AEF} required a package of separation routines adapted from CVRPSEP [4]. Cplex® was used to solve the linear problems identified after adding valid cuts. With a time limit of 3 hours, the results show an improvement of 5 to 10% of the ratios between best upper bounds and lower bounds known so far, and one third of the instances have been solved to optimality.

References

[1] P. Augerat. *Approche polyédrale du problème de tournées de véhicules.* PhD thesis, Institut National Polytechnique de Grenoble, France, 1995.

[2] R. Baldacci, N. Christofides, and A. Mingozzi. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts. *Mathematical Programming*, 115(2):351–385, 2008.

[3] M. R. Garey and D. S. Johnson. *Computers and Intractability : A Guide to the Theory of NP-Completeness* . W.H. Freeman and Company, New York, 1979.

[4] J. Lysgaard. A package of separation routines for the capacitated vehicle routing problem. http://www.hha.dk/∼lys/, 2003.

[5] S.U. Ngueveu, C. Prins, and R. Wolfler Calvo. Bornes supérieures et inférieures pour le problème de tournées de véhicules m-péripatétiques. In *Actes de la 7ème conférence internationale de Modélisation et Simulation*, vol. 3, pp. 1617–1625, Paris, France, 2008. MOSIM

[6] S.U. Ngueveu, C. Prins, and R. Wolfler Calvo. A hybrid tabu search for the m-peripatetic vehicle routing problem. In T. Stützle and S. Voss, editors, *Matheuristics: Hybridizing Metaheuristics and Mathematical Programming*, Annals of Information Systems Series, 10, pages 253–264. Springer, 2010.