Branch-and-Price-and-Cut for the Vehicle Routing Problem with Transshipment Facilities

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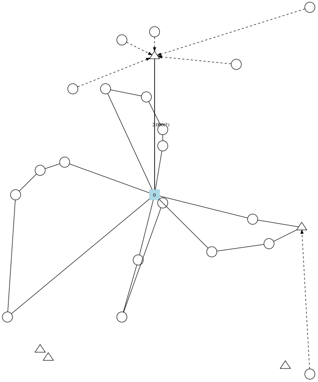
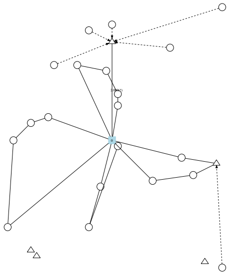
VRPTF Definition



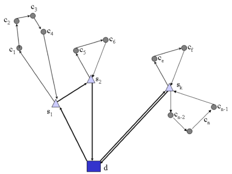
- $|V_C|$ customers, 1 depot, $|V_F|$ transshipment facilities
- Visit each transshipment facility at most once
- Serve each customer with exactly one route (=visit it or assign it to a transshipment facility)
- Route load $\leq$ Vehicle capacity
- m of routes in the network is known and given as an input parameter.

Objective = minimize Total routing + assignment costs

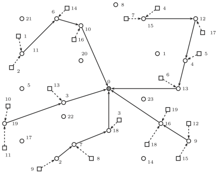
VRPTF vs Literature



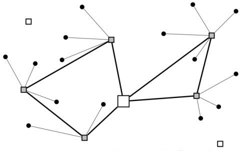
VRPTF



2-echelon VRP



Multiple TSP

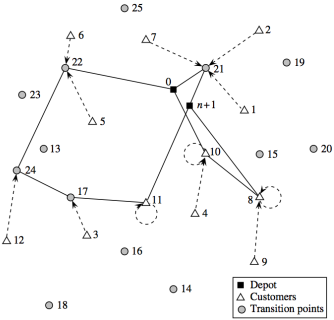


School bus problem

A particular case with unit demands: The CmRSP

CmRSP = Capacitated m-Ring Star Problem:

- introduced by Baldacci et al in 2007
- problem of designing telecommunication networks



The literature review

R. Baldacci, M. Dell'Amico and J. Salazar-González.
The Capacitated m-Ring-Star Problem.
Operations Research, 55(6):1147–1162, 2007.

E. A. Hoshinoa and C. C. de Souza.
A branch-and-cut-and-price approach for the capacitated m-ring–star problem.
Discrete Applied Mathematics, 160:2728–2741, 2012.

A. Mauttone, S. Nesmachnow, A.O.F.Robledo Amoza.
Solving a Ring Star Problem generalization.
CICM2006

A two-Index Formulation

- $x_e \in \{0, 1\}, \forall e \in E \setminus \{\{0, j\} : j \in V'\}$ and
- $x_e \in \{0, 1, 2\}, \forall e \in \{\{0, j\} : j \in V'\}$
- $w_i = 1$ if and only if customer i is assigned to node j, 0 otherwise
- $z_{ij} = 1$ if and only if node i is on a route, 0 otherwise

A two-Index Formulation

$$(TI) \quad \min \sum_{e \in E} r_e x_e + \sum_{(i,j) \in A} d_{ij} z_{ij} \tag{1}$$

$$s.t. \sum_{e \in \delta(0)} x_e = 2m \tag{2}$$

$$\sum_{e \in \delta(i)} x_e = 2w_i, \forall i \in V' \tag{3}$$

$$w_i + \sum_{j \in F_i} z_{ij} = 1, \forall i \in V_C \tag{4}$$

$$\sum_{e \in \delta(S)} x_e \geq \frac{2}{Q} \left(\sum_{i \in V_C(S)} q_i w_i + \sum_{\substack{(i,j) \in A: \\ j \in V_F(S)}} q_i z_{ij} \right), \qquad \forall S \subseteq V' : S \neq \emptyset \tag{5}$$

Simple valid inequalities

$$x_{\{i,j\}} \leq w_j, \quad \forall i \in V_C, \forall j \in V_C, i \neq j \tag{6}$$

$$x_{\{i,j\}} \leq w_j, \quad \forall i \in V_F, j \in V', i \neq j \tag{7}$$

$$x_{\{i,j\}} + z_{ij} \leq w_j, \quad \forall i \in V_C, \forall j \in F_i \tag{8}$$

Connectivity, Multistar, Rounded Capacity Inequalities

CI

$$\sum_{e\in\delta(S)} x_e \geq 2\left(w_i + \sum_{j\in V_F(S)\cap F_i} z_{ij}\right), \quad \forall S\subseteq V', \forall i\in V_C(S), S\neq\emptyset \tag{9}$$

MI

$$\sum_{e\in\delta(S)} x_e \geq \frac{2}{Q}\left(\sum_{i\in V_C(S)} q_i w_i + \sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij} + \sum_{i\in V_C(\overline{S})}\sum_{j\in S} q_i x_{\{i,j\}}\right), \forall S\subseteq V', S\neq\emptyset \tag{10}$$

RCI

$$\sum_{e\in\delta(S)} x_e \geq 2\left\lceil\frac{\sum_{i\in S:F_i\subseteq S} q_i}{Q}\right\rceil, \quad \forall S\subseteq V', V_C(S)\neq\emptyset \tag{11}$$

Non Linear Rounded Capacity Constraints RCII

$$\sum_{e\in\delta(S)} x_e \geq 2 \left\lceil \frac{\left(\sum_{i\in V_C(S)} q_i w_i + \sum_{\substack{(i,j)\in A:\\ j\in V_F(S)}} q_i z_{ij}\right)}{Q} \right\rceil, \quad \forall S \subseteq V', S \neq \emptyset \qquad (12)$$

These constraints are non linear.

Lemma:
Given three non negative integer values v, y and b, such that $v > b$ et $\mod (v, b) \neq 0$. Therefore

$$\left\lceil \frac{v-y}{b} \right\rceil \geq \left\lceil \frac{v}{b} \right\rceil - \frac{y}{\mod (v, b)}. \qquad (13)$$

Inequalities linearization

The rhs of 12 can be rewritten as follows:

$$q(V_C) - \left(\sum_{i \in V_C(\overline{S})} q_i w_i + \sum_{\substack{(i,j) \in A: \\ j \in V_F(\overline{S})}} q_i z_{ij} \right) \quad ou \quad q(V_C(S)) - \left(\sum_{\substack{(i,j) \in A: \\ i \in V_C(S), \\ j \in V_F(\overline{S})}} q_i z_{ij} - \sum_{\substack{(i,j) \in A: \\ i \in V_C(\overline{S}), \\ j \in V_F(S)}} q_i z_{ij} \right) \tag{14}$$

Using the lemma we can write:

$$\sum_{e \in \delta(S)} \frac{1}{2} x_e \geq \left\lceil \frac{q(V_C)}{Q} \right\rceil - \frac{1}{mod(q(V_C), Q)} \left(\sum_{i \in V_C(\overline{S})} q_i w_i + \sum_{\substack{(i,j) \in A: \\ j \in V_F(\overline{S})}} q_i z_{ij} \right), \tag{15}$$

$$\sum_{e \in \delta(S)} \frac{1}{2} x_e \geq \left\lceil \frac{q(V_C(S))}{Q} \right\rceil - \frac{1}{mod(q(V_C(S)), Q)} \sum_{\substack{(i,j) \in A: \\ i \in V_C(S), j \in V_F(\overline{S})}} q_i z_{ij} \tag{16}$$

There is no dominance relation between 16 and 15

Set partitioning formulation based on classical routes

A *tour* $T = (0, i_1, \ldots, i_r, 0)$, with $r \geq 1$, is a simple cycle in G, with $V(T) \subseteq V$, that respects capacity constraints $\sum_{i \in V(T)} q_i \leq Q$.

- $\mathscr{R}$ index set of all tours
- $E(\ell), \forall \ell \in \mathscr{R}$ set of edges used by the tour
- $a_{i\ell}$, $i \in V'$, $\ell \in \mathscr{R}$ binary coefficient equal to 1 if $i \in V(T_\ell)$, 0 otherwise (we assume that $a_{0\ell} = 1, \forall \ell \in \mathscr{R}$).

For each tour $\ell \in \mathscr{R}$

- c_ℓ: sum of edge costs
- η_e^ℓ
  - if ℓ is a tour covering node h only, then $\eta_{\{0,h\}}^\ell = 2$ and $\eta_{\{i,j\}}^\ell = 0$, $\forall \{i,j\} \in E \setminus \{0,h\}$;
  - if ℓ is not a single-node tour, then $\eta_{\{i,j\}}^\ell = 1$ for each edge $\{i,j\} \in E(T_\ell)$ and $\eta_{\{i,j\}}^\ell = 0$, $\forall \{i,j\} \in E \setminus E(T_\ell)$.

Set partitioning formulation based on classical routes

Principle: Generate CVRP-like routes

Additional notation:

- $\mathscr{S} = \{S : S \subseteq V', |S| \geq 2, |S \cap V_F| \geq 1\}$ and $\mathscr{R}(S) = \{\ell \in \mathscr{R} : V(T_\ell) \cap S \neq \emptyset\}$.

Decision variables

- ξ_ℓ, $\ell \in \mathscr{R}$ binary variable equal to 1 iff ℓ is in the optimal solution.
- $\{x_e\}$, $\{z_{ij}\}$ and $\{w_i\}$ defined as for formulation TI

Set partitioning formulation based on classical routes

$$(SP1) \quad \min \sum_{\ell \in \mathscr{R}} c_\ell \xi_\ell + \sum_{(i,j) \in A} d_{ij} z_{ij} \tag{17}$$

$$s.t. \sum_{\ell \in \mathscr{R}} a_{i\ell} \xi_\ell = w_i, \forall i \in V' \tag{18}$$

$$\sum_{\ell \in \mathscr{R}} \xi_\ell = m, \tag{19}$$

$$w_i + \sum_{j \in F_i} z_{ij} = 1, \forall i \in V_C \tag{20}$$

$$x_e = \sum_{\ell \in \mathscr{R}} \eta_e^\ell \xi_\ell, \forall e \in E \tag{21}$$

$$\sum_{e \in \delta(S)} x_e \geq \frac{2}{Q} \left(\sum_{i \in V_C(S)} q_i w_i + \sum_{\substack{(i,j) \in A: \\ j \in V_F(S)}} q_i z_{ij} \right), \forall S \in \mathscr{S} \tag{22}$$

New SP formulation

Principle: Include clients assignments in the definition of feasible VRPTF routes

Decision Variables

- ξ_ℓ, $\ell \in \mathscr{R}$, binary variable equal to 1 if and only if route ℓ is in the optimal solution.
- ~~$\{x_e\}$, $\{z_{ij}\}$ and $\{w_i\}$ defined as for formulation TI~~

New SP formulation

Then the formulation is:

$$(SP2) \quad \min \sum_{\ell \in \mathscr{R}} (c_\ell + p_\ell)\xi_\ell \tag{23}$$

$$s.t. \sum_{\ell \in \mathscr{R}} \overline{a}_{i\ell}\xi_\ell = 1, \quad \forall i \in V_C \tag{24}$$

$$\sum_{\ell \in \mathscr{R}} a_{i\ell}\xi_\ell \leq 1, \quad \forall i \in V_F \tag{25}$$

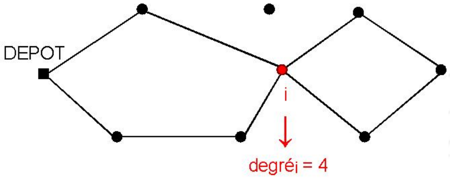
$$\xi_\ell \in \{0,1\}, \quad \forall \ell \in \mathscr{R}. \tag{26}$$

where $\overline{a}_{i\ell} = a_{i\ell} + \sum_{j \in V_F(R_\ell)} b^j_{i\ell}$, $i \in V_C$, $\ell \in \mathscr{R}$.

Classical q-routes and ng-routes

A q-route is a relaxation of route which autorises one or several nodes of the tour (except the depot) to have a degree different than 2.

- Each q-route starts at the depot and visit some nodes of V' before returning to the depot
- A q-route does not necessarily crpsses all nodes of V'
- Except for the depot, any node can be crossed more than once by the same q-route.

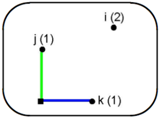


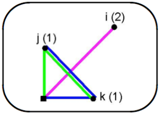
ng-routes

Classical q-routes and ng-routes

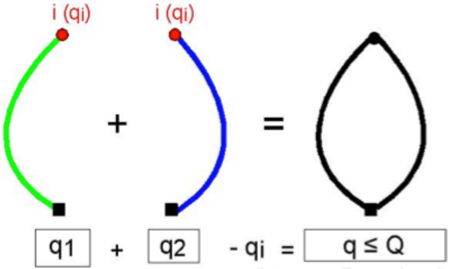
Generate the best q-routes and ng-routes with dynamic programming

- Generate q-paths/ng-paths of load $\leq \lceil Q/2 \rceil$ that ends at the node $i \in V'$





- Merge pairs of q-paths/ng-paths ending at node $i \in V'$



New state-space relaxation: $q*$-routes/$ng*$-routes

How to include assignments of clients in routes relaxations ?

- $\forall k \in V_F$, precompute the best assignments of load $q \in [q_{\min}, Q]$ of customers to k
- In practice, a lower bound is computed as the optimal solution cost of the following knapsack problem $KP(q,k)$

$$(KP(q,k)) \quad lb_k(q) = \min \sum_{i \in V_C} d_{ik}\chi_i \tag{27}$$

$$s.t. \sum_{i \in V_C} q_i \chi_i = q \tag{28}$$

$$\chi_i \in \{0,1\}, \quad \forall i \in V_C. \tag{29}$$

We assume that $lb_k(q) = \infty$ if problem $KP(q,k)$ does not admit a feasible solution for the given pair q and k.

New state-space relaxation: $q*$-routes/$ng*$-routes

Adapted recurrence relations:

$$h(i,j) = \begin{cases} \begin{cases} f(q-q_j,i) + r_{ij}, & \text{if } \pi(q-q_j,i) \neq j \\ g(q-q_j,i) + r_{ij}, & \text{otherwise.} \end{cases} & , j \in V_C \\ \min_{q^{min} \leq w \leq Q} \begin{cases} f(q-w,i) + r_{ij} + lb_j(w), & \text{if } \pi(q-w,i) \neq j \\ q(q-w,i) + r_{ij} + lb_j(w), & \text{otherwise.} \end{cases} & , j \in V_F \end{cases}$$

Then, compute:

$$\begin{cases} f(q,j) = \min_{i \in V' \setminus \{j\}} \{h(i,j)\} \\ \pi(q,j) = i' \end{cases} \quad (31)$$

where i is the node producing the above min

$$\begin{cases} g(q,j) = \min_{i \in V' \setminus \{j,i'\}} \{h(i,j)\} \\ \pi(q,j) = i'' \end{cases} \quad (32)$$

where i'' is the node producing the above min

with

- f(q,i) = cost of the least cost $q*$-path from node 0 to node i
- $\pi(q,i)$: predecessor of i in the least cost path of value $f(q,i)$
- $g(q,i)$ = cost of the second least cost $q*$-path from node 0 to node i ($\gamma(q,i) \neq \pi(q,i)$)
- $\gamma(q,i)$: predecessor of i in the second least cost path of value $g(q,i)$

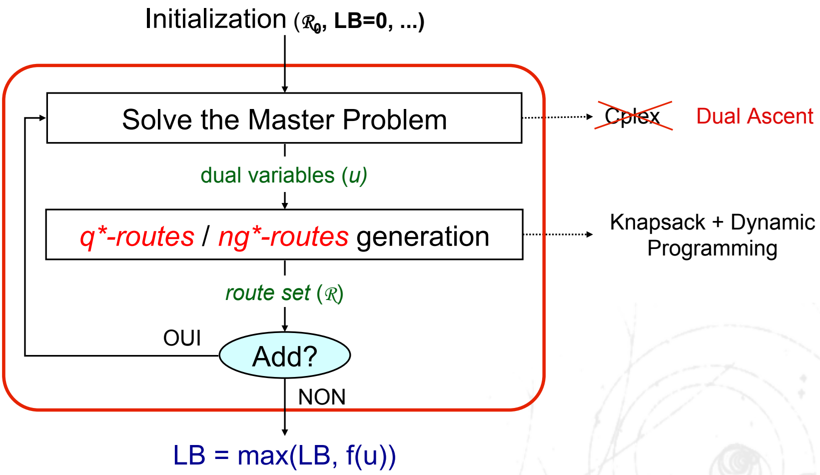
$ng*$-routes

$q*$-routes/$ng*$routes generation

- Generate the best $q*$-paths/$ng*$-paths of load $\leq Q$ that ends at the node $i \in V'$

- Add the edge returning to the depot to each $q*$-path (NO MERGING !)

Lower bound H1*



Problem Description and literature review

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New state-space relaxation: $q*$-routes and $ng*$-routes

Two Heuristics

Branch-and-Price-and-Cut

Preliminary computational results

Two Heuristics

A constructive heuristic

Principle: Use CVRP-based heuristic so solve the VRPTF without V_F

1. Initialization (with random seed customers and a G.A.P.)
2. Route construction (with a 3-opt)
3. Local optimization

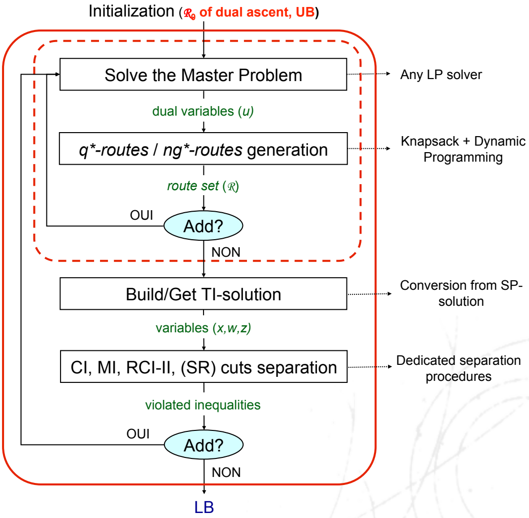
A Lagrangean Heuristic

Principle: Produce feasible VRPTF solutions within the dual ascent

1. Initialization
2. Extract a subset of routes SOL and modify
3. Insert unrouted customers
4. Define the VRPTF solution
5. Local optimization

Branch-and-Price-and-Cut

- Column-and-Cut generation



- Branch-and-Price-and-Cut = Branch-and-Bound + Column-and-Cut

Implementation details and Istances used

Implementation Cutting plane

- C + Cplex
- Intel Core 2Duo, 2.66 GHz
- 4Go de RAM

Dual ascent and heuristics

- C + fortran

Branch-and-Price-and-Cut

- SCIP + C + Cplex

Instances:

- CmRSP: from literature
- VRPTF: class A and B generated using LRP instances. Class RW = real world instances

Cutting plane on CmRSP Instances

instance	LB/UB	s	lit.
A01-n026-m03	100.00	0.15	100.00
A02-n026-m04	100.00	0.09	100.00
A04-n026-m03	99.67	0.96	100.00
A05-n026-m04	100.00	1.61	100.00
A06-n026-m05	99.73	1.44	100.00
A10-n051-m03	100.00	0.61	100.00
A11-n051-m04	100.00	0.17	100.00
A12-n051-m05	100.00	0.12	100.00
A23-n076-m04	98.96	21.5	98.55
A24-n076-m05	97.99	37.84	97.48
A34-n101-m03	100.00	10.02	100
A35-n101-m04	98.80	225.92	99.3
A36-n101-m05	97.99	92.3	100.00

instance	LB/UB	s	lit.
B01-n026-m03	100.00	0.09	100.00
B02-n026-m04	100.00	0.19	100.00
B03-n026-m05	100.00	0.1	100.00
B04-n026-m03	99.38	0.98	100.00
B05-n026-m04	99.58	1.48	100.00
B06-n026-m05	99.85	1.55	100.00
B10-n051-m03	100.00	0.98	100.00
B11-n051-m04	100.00	0.88	100.00
B12-n051-m05	100.00	0.12	100.00
B22-n076-m03	95.87	31.76	96
B23-n076-m04	98.02	25.06	97.17
B24-n076-m05	97.32	33.34	97.17
B34-n101-m03	100.00	344.29	99.85
B35-n101-m04	98.20	320.15	98.79
B36-n101-m05	97.37	152.1	97.73

Computational results on VRPTF RW Instances

Branch-and-Cut

name	n	nc	nf	load	LB	UB	(ratio %)	sec
c_113x9_Q24	123	113	9	894	18431.922	-	(92.13 %)	1816.23 s
c_113x9_Q34	123	113	9	1013	20625.742	-	(92.95 %)	1800.01 s
c_164x12_Q24	177	164	12	1316	25836.151	-	(92.23 %)	1800 s
c_164x12_Q34	177	164	12	1486	28130.263	-	(91.82 %)	1818.39 s
c_74x6_Q24	81	74	6	564	11762.97	-	(95.61 %)	1800 s
c_74x6_Q34	81	74	6	627	12517.792	-	(95.92 %)	1800.3 s
n_103x13_Q24	117	103	13	792	13461.544	-	(92.47 %)	1800.01 s
n_103x13_Q34	117	103	13	890	14949.456	-	(93.75 %)	1800 s
n_142x18_Q24	161	142	18	1134	16800.364	-	(91.44 %)	1802.45 s
n_142x18_Q34	161	142	18	1281	18323.629	-	(90.44 %)	1800 s
n_68x7_Q24	76	68	7	508	8514.436	-	(93.72 %)	1800 s
n_68x7_Q34	76	68	7	556	9330.801	-	(95.04 %)	1801.06 s
s_115x9_Q24	125	115	9	909	19318.448	-	(92.12 %)	1817.2 s
s_115x9_Q34	125	115	9	1035	21012.67	-	(94.57 %)	1813.31 s
s_54x4_Q24	59	54	4	419	10668.481	-	(96.97 %)	1800.01 s
s_54x4_Q34	59	54	4	473	12090.792	-	(95.53 %)	1800.01 s
s_85x7_Q24	93	85	7	643	15632.865	-	(94.44 %)	1800.01 s
s_85x7_Q34	93	85	7	725	17515.198	-	(94.13 %)	1800 s

Preliminary results on VRPTF RW Instances

name	n	nc	nf	load	LB	UB	(ratio %)	time
c_113x9_Q24	123	113	9	894	19612.11	19612.11	(100.00 %)	1720.15 s
c_113x9_Q34	123	113	9	1013	21638.377	21771.92	(99.39 %)	1800 s
c_164x12_Q24	177	164	12	1316	27427.764	27588.87	(99.42 %)	1800.02 s
c_164x12_Q34	177	164	12	1486	29770.812	29833.38	(99.79 %)	1800 s
c_74x6_Q24	81	74	6	564	12213.8	12213.8	(100.00 %)	8.49 s
c_74x6_Q34	81	74	6	627	12930.44	12930.44	(100.00 %)	183.54 s
n_103x13_Q24	117	103	13	792	14396.81	14396.81	(100.00 %)	508.42 s
n_103x13_Q34	117	103	13	890	15510.148	15620.02	(99.30 %)	1800.03 s
n_142x18_Q24	161	142	18	1134	17849.252	17909.39	(99.66 %)	1800 s
n_142x18_Q34	161	142	18	1281	19369.908	-	(95.60 %)	1800.03 s
n_68x7_Q24	76	68	7	508	8893.24	8893.24	(100.00 %)	46.75 s
n_68x7_Q34	76	68	7	556	9652.92	9817.88	(98.32 %)	1800.02 s
s_115x9_Q24	125	115	9	909	20472.416	20708.39	(98.86 %)	1800.14 s
s_115x9_Q34	125	115	9	1035	21733.772	-	(97.82 %)	1800 s
s_54x4_Q24	59	54	4	419	11002.48	-	(100.00 %)	783.8 s
s_54x4_Q34	59	54	4	473	12345.654	-	(97.54 %)	1800.07 s
s_85x7_Q24	93	85	7	643	16202.757	-	(97.88 %)	1800 s
s_85x7_Q34	93	85	7	725	18012.157	18150.55	(99.24 %)	1800.01 s

Preliminary results on VRPTF Class A and B Instances

Branch-and-Price-and-Cut

Class	nb	min LB/UB	avg LB/UB	max LB/UB	100%	$\geq$98%	$\geq$95%
A	52	95.54	98.70	100	26	36	52
B	56	91.01	97.87	100	23	28	50

Conclusion

- Done
  - new state-space relaxations $q*$-routes/$ng*$-routes
  - heuristics
  - real world instances
- Work-in-progress
  - Finalize the code and the paper
- Future work
  - Application and extension of $q*$-routes/$ng*$-routes to other VRP problems with optional facilities