State estimation by interval analysis for a nonlinear differential aerospace model

Pauline Ribot, Carine Jauberthie and Louise Travé-Massuyès

***Abstract*— This paper deals with guaranteed state estimation in a bounded-error context for an aerospace model. Perturbations are assumed bounded but otherwise unknown. The main tools to solve this problem are the guaranteed techniques for the ordinary differential equations integration and set inversion. The obtained results show the efficiency of the method.**

keywords State estimation; Continuous-time systems; Nonlinear systems; Bounded noise; Interval analysis

I. INTRODUCTION

Complex systems are often subjected to uncertainties that make the modeling task awkward. These uncertainties can be unstructured when the equations of the system are not entirely known or structured when the equations are known but not the values of their parameters. In both cases, it is particularly difficult to get an accurate model of the perturbations and noises acting on the system. This may turn the usual stochastic framework inappropriate.

Guaranteed sate estimation methods are an interesting alternative to stochastic model based estimation when perturbations and noises are assumed to be bounded but otherwise unknown. These methods have received a lot of attention in the last few years and the literature on this topic shows interesting progress [4], [14], and [23].

This paper applies a recently proposed bounded error state estimation method to a highly complex aerospace model. This method follows a classical predictor-corrector approach which makes use of an extended mean value algorithm [23] and is this way succeful in controlling the well-known and undesirable wrapping effect.

This paper is organized as follows. Section III presents the basic tools of interval analysis and the set inversion problem. The notions of interval, ”box”, interval matrix and inclusion function are given. To compute the set inversion, an algorithm is provided. This algorithm is then used in state estimation. Section IV is concerned with the core problem, which is state estimation and provides the algorithms. An algorithm dedicated to solve ordinary differential equations is given. It is based on a classical predictor-corrector approach. Firstly, it verifies the existence and uniqueness of the solution of ordinary differential equations when the initial conditions belong to an interval vector. Secondly, it computes the solution by using a Taylor expansion. Section V presents the results obtained on an aerospace case study. Finally, section VI discusses related work and some conclusions are outlined in section VII.

P. Ribot, C. Jauberthie and L. Travé-Massuyès are with LAAS-CNRS, DISCO Group, University of Toulouse, 7 avenue du Colonel Roche, 31077 Toulouse Cedex 4, France pribot@laas.fr, cjaubert@laas.fr, louise@laas.fr

II. PROBLEM FORMULATION

This paper deals with estimating the unknown state x for a nonlinear dynamic system of the following form:

$$\begin{cases} \dot{x}(t) = f(x(t)), \\ y(t) = g(x(t)), \quad x(0) \in [\mathrm{X}_0]. \end{cases} \tag{1}$$

where $x(t) \in \mathrm{R}^n$ and $y(t) \in \mathrm{R}^m$ denote respectively the state variables and the measured outputs. The initial conditions $x(0)$ are supposed to belong to an initial ”box” $[\mathrm{X}_0]$, the notion of ”box” being described in the following.

Time is assumed to belong to $[0, t_{max}]$. The functions f and g are real and analytic on M, where M is the open set of R^n such that $x(t) \in M$ for every $t \in [0, t_{max}]$). Moreover the function f is assumed to be at least $k-$times continuously differentiable in the domain M.

The output error is assumed to be given by:

$$v(t_i) = y(t_i) - y_m(t_i), \quad i = 1, ..., N. \tag{2}$$

We assume that $\underline{v(t)}$ and $\overline{v(t)}$ are known lower and upper bounds for the acceptable output errors. Such bounds may, for instance, correspond to a bounded measurement noise. The integer N is the total number of sample times.

Interval arithmetic is used to compute guaranteed bounds for the solution of (1) at the sampling times $\{t_1, t_2, ..., t_N\}$.

III. CASE STUDY

The case study that we consider is the longitudinal motion of a glider. The projection of the general equations of motion onto the aerodynamic reference frame of the aircraft and the linearization of aerodynamic coefficients [29] give the following system:

$$\begin{cases} \dot{V} = & -g\sin(\theta-\alpha)-\dfrac{1}{2m}\rho SV^2(C_x^0 \\ & +C_{x\alpha}(\alpha-\alpha_0)+C_{x\delta_m}(\delta_m-\delta_{m_0})), \\ \dot{\alpha} = & \dfrac{2}{2mV+\rho SlVC_{z\dot{\alpha}}}\Big\{mVq+mg\dfrac{\cos(\theta-\alpha)}{V} \\ & -\dfrac{1}{2}\rho SV^2(C_z^0+C_{z\alpha}(\alpha-\alpha_0) \\ & +C_{zq}\dfrac{ql}{V}+C_{z\delta_m}(\delta_m-\delta_{m_0}))\Big\}, \\ \dot{q} = & \dfrac{1}{2B}\rho SlV^2\Big\{C_m^0+C_{m\alpha}(\alpha-\alpha_0)+C_{mq}\dfrac{ql}{V} \\ & +C_{m\dot{\alpha}}\dfrac{2l}{2mV^2+\rho SlV^2C_{z\dot{\alpha}}}\Big[mVq \\ & +mg\dfrac{\cos(\theta-\alpha)}{V} \\ & -\dfrac{1}{2}\rho SV^2(C_z^0+C_{z\alpha}(\alpha-\alpha_0)+C_{zq}\dfrac{ql}{V} \\ & +C_{z\delta_m}(\delta_m-\delta_{m_0}))\Big]+C_{m\delta_m}(\delta_m-\delta_{m_0})\Big\}, \\ \dot{\theta} = & q. \end{cases} \quad (3)$$

In these equations, the state vector x is given by $(V,\alpha,q,\theta)^\top$, the observation y is full (i.e., $y=x$), the input u is δ_m ($u_0=\delta_{m_0}$). The real V denotes the speed of the aircraft, α the angle of attack, α_0 the trim value of α, θ the pitch angle, q the pitch rate, δ_m the elevator deflection angle, ρ the air density, g the acceleration due to gravity, l a reference length and S the area of a reference surface. B represents a moment of inertia. The other coefficients correspond to the dynamic stability derivatives and are supposed to be known.

IV. INTERVAL ANALYSIS FOR GUARANTEED SET ESTIMATION

Interval analysis was initially developed to account for the quantification errors introduced by the rational representation of real numbers in computers and was extended to validated numerics [18].

In the following section, we present some basics tools of interval analysis and an algorithm to compute the outer approximation of sets of arbitrary shape.

A. Basic tools

A real interval $[u]=[\underline{u},\overline{u}]$ is a closed and connected subset of R where $\underline{u}$ represents the lower bound of $[u]$ and $\overline{u}$ represents the upper bound. The width of an interval $[u]$ is defined by $w(u)=\overline{u}-\underline{u}$, and its midpoint by $m(u)=(\overline{u}+\underline{u})/2$.

The set of all real intervals of R is denoted IR.
Two intervals $[u]$ and $[v]$ are equal if and only if $\underline{u}=\underline{v}$ and $\overline{u}=\overline{v}$. Real arithmetic operations are extended to intervals [19].

Arithmetic operations on two intervals $[u]$ and $[v]$ can be defined by:

$$\circ\in\{+,-,*,/\},\quad [u]\ \circ\ [v]=\{x\circ y\mid x\in[u],\ y\in[v]\}.$$

An interval vector (or box) $[X]$ is a vector with interval components and may equivently be seen as a cartesian product of scalar intervals:

$$[X]=[x_1]\times[x_2]\ldots\times[x_n].$$

The set of $n-$dimensional real interval vectors is denoted by IR^n.

An interval matrix is a matrix with interval components. The set of $n\times m$ real interval matrices is denoted by $\mathrm{IR}^{n\times m}$. The width $w(.)$ of an interval vector (or of an interval matrix) is the maximum of the widths of its interval components. The midpoint $m(.)$ of an interval vector (resp. an interval matrix) is a vector (resp. a matrix) composed of the midpoint of its interval components.

Classical operations for interval vectors (resp. interval matrices) are direct extensions of the same operations for punctual vectors (resp. punctual matrices) [19].

Let $f:\mathrm{R}^n\to\mathrm{R}^m$, the range of the function f over an interval vector $[u]$ is given by:

$$f([u])=\{f(x)|x\in[u]\}.$$

The interval function $[f]$ from IR^n to IR^m is an inclusion function for f if:

$$\forall[u]\in\mathrm{IR}^n,\quad f([u])\subseteq[f]([u]).$$

An inclusion function of f can be obtained by replacing each occurrence of a real variable by its corresponding interval and by replacing each standard function by its interval evaluation. Such a function is called the natural inclusion function. In practice the inclusion function is not unique, it depends on the syntax of f.

B. Set inversion

Consider the problem of determining a solution set for the unknown quantities u defined by:

$$\mathrm{S}=\{u\in\mathrm{U}|\Phi(u)\in[y]\}=\Phi^{-1}([y])\cap\mathrm{U}, \quad (4)$$

where $[y]$ is known a priori, U is an a priori search set for u and Φ a nonlinear function not necessarily invertible in the classical sense. (4) involves computing the reciprocal image of Φ and is known as a set inversion problem which can be solved using the algorithm **Set Inverter Via Interval Analysis** (denoted **SIVIA**). The algorithm **SIVIA** proposed in [10] is a recursive algorithm which explores all the search space without loosing any solution. This algorithm makes it possible to derive a guaranteed enclosure of the solution set S as follows:

$$\underline{\mathrm{S}}\subseteq\mathrm{S}\subseteq\overline{\mathrm{S}}.$$

The inner enclosure $\underline{\mathrm{S}}$ is composed of the boxes that have been proved feasible. To prove that a box $[u]$ is feasible it is sufficient to prove that $\Phi([u])\subseteq[y]$. Reversely, if it can be proved that $\Phi([u])\cap[y]=\emptyset$, then the box $[u]$ is unfeasible. Otherwise, no conclusion can be reached and the box $[u]$ is said undetermined. The latter is then bissected and tested again until its size reaches a user-specified precision threshold $\varepsilon>0$. Such a termination criterion ensures that **SIVIA** terminates after a finite number of iterations.

The following section concerns the integration of (1). Thus, the aim of this section is to estimate the state vector x at the sampling times $\{t_1, t_2, \ldots, t_N\}$ corresponding to the measurement times of the outputs. We note $[x_j]$ the box $[x(t_j)]$ where t_j represents the sampling time, $j = 1, \ldots, N$ and x_j represents the solution of (1) at t_j.

V. STATE ESTIMATION

When the model is nonlinear like (1), the sets to be characterized may be nonconvex and may even consist of several disconnected components. The interval analysis consists of enclosing such sets in unions of nonoverlapping interval vectors and the usual drawback is to obtain wider and wider interval solution vectors. This is known as the wrapping effect. Thus the wrapping effect leads to very pessimistic results.

The pessimism introduced by the large width of the set can be reduced by using a high-order k for the Taylor expansion and by using mean value forms [22], [24] and matrices preconditioning.

The method proposed recently in [23] is able to control the non desirable wrapping effect. It is based on a classical predictor-corrector approach.

A. *Prediction-correction step*

The prediction step aims at computing the attainability set for the state vector whereas the correction step retains only those parts of the attainability set which are consistent with measurements and prior error bounds as explained in the following figure.

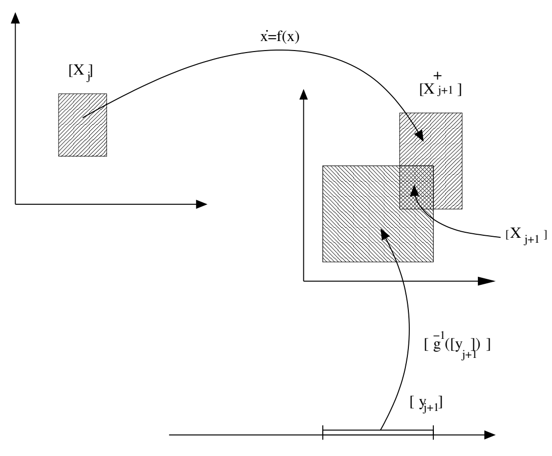

Fig. 1. Prediction-correction step at t_{j+1}.

In this figure, we assume that $[X_j]$ is a box which is guaranteed to contain x_j at t_j. Define the outer approximation of the predicted set $[X^+_{j+1}]$ as the validated solution of the differential equation at t_{j+1}. The set $[X^+_{j+1}]$ is computed using the **extended mean value algorithm** (abbreviated **EMV**) given in the next subsection. This set is guaranteed to contain the state at t_{j+1}.

At t_{j+1}, a ”measurement vector” $[y_{j+1}]$ is obtained, corresponding to:

$$[y_j] = [y_j - \underline{v_j},\ y_j - \overline{v_j}], \tag{5}$$

where $\underline{v_j}$ and $\overline{v_j}$ are given in (2).

Then, we compute the set $[g]^{-1}([y_{j+1}])$. This evaluation is obtained by the algorithm **SIVIA**. The solution set at the sampling time t_{j+1} is finally given by $[x_{j+1}] = [X^+_{j+1}] \bigcap [g]^{-1}([y_{j+1}])$.

The procedure for state estimation is summarized in the following algorithm:

for $j = 0$ to $N-1$ do

- Prediction step: compute $[\mathrm{X}^+_{j+1}]$ by using algorithm **EMV**,
- Correction step: compute $[x_{j+1}]$ such that

$$[x_{j+1}] = [X^+_{j+1}] \bigcap [g]^{-1}([y_{j+1}]),$$

where $[y_j] = [y_j - \underline{v_j},\ y_j - \overline{v_j}]$ and $\underline{v_j}$ and $\overline{v_j}$ are the upper and lower bounds for measurement noise.

B. *Extended mean value algorithm*

The most effective methods to solve the state estimation for dynamical nonlinear systems are based on Taylor expansions [5], [18], [20] or [24]. These methods consist in two parts: the first one verifies the existence and uniqueness of the solution by using the fixed point theorem and the Picard-Lindelf operator. At a time t_{j+1}, an a priori box $[\tilde{x}_j]$ containing all solutions corresponding to all possible trajectories between t_j and t_{j+1} is computed. In the second part, the solution at t_{j+1} is computed by using a Taylor expansion, where the remainder term is $[\tilde{x}_j]$.

However, in practice, the set $[\tilde{x}_j]$ often fails to contain the true solution [20]. Thus, the classical technique used consists in inflating this set until it verifies the following inclusion [25]:

$$[x_j] + hf([\tilde{x}_j]) \subseteq [\tilde{x}_j], \tag{6}$$

where h denotes the integration step and $[x_j]$ the first solution.

This method is summarized in the following algorithm, called **Enclosure algorithm** and developed in [16]. The inputs are $[x_j]$ and $\alpha > 0$ and the output is $[\tilde{x}_j]$):

$[\tilde{x}_j] = [x_j]$,

while ($[x_j] + hf([\tilde{x}_j]) \not\subseteq [\tilde{x}_j]$ do)

$[\tilde{x}_j] = \mathrm{inflate}([\tilde{\mathrm{x}}_\mathrm{j}], \alpha)$.

The inflate function for an interval vector $[u] = ([\underline{u}_1, \overline{u}_1], \ldots, [\underline{u}_n, \overline{u}_n])$ consists in inflating all its components, as follows: $([(1-\alpha)\underline{u}_1, (1+\alpha)\overline{u}_1], \ldots, [(1-\alpha)\underline{u}_n, (1+\alpha)\overline{u}_n])$.

The accuracy of the computed set $[\tilde{x}_j]$ depends on the coefficient α.

If the set $[\tilde{x}_j]$ satisfies inclusion (6) then inclusion $x(t) \in [\tilde{x}_j]$ holds for all $t \in [t_j, t_{j+1}]$ and the true solution x_{j+1} of the ordinary differential equation (1) at t_{j+1} is contained, in a guaranteed way, in the interval vector $[x_{j+1}]$ given by the following Taylor expansion [18]:

$$[x_{j+1}] = [x_j] + \sum_{i=1}^{k-1} h^i f^{[i]}([x_j]) + h^k f^{[k]}([\tilde{x}_j]), \qquad (7)$$

where k denotes the order of the Taylor expansion and the coefficients $f^{[i]}$ are the Taylor coefficients of the solution $x(t)$ which are recursively obtained by:

$$f^{[1]} = f, f^{[i]} = \frac{1}{i}\frac{\partial f^{[i-1]}}{\partial x} f, \; i \geq 2. \qquad (8)$$

The inflation of the set $[\tilde{x}_j]$ leads to the increase of its width. The pessimism thus introduced by the large width of the set can be reduced by using a high-order k for the Taylor expansion in expression (7). But the width of the solution always increases even for high orders. To solve this drawback, R. Rihm proposes to evaluate (7) through the **extended mean value algorithm** based on mean value forms [22], [24] and matrices preconditioning. This algorithm will be used to solve the differential equation given by (1).

The inputs of this algorithm are $[\tilde{x}_j]$, $[x_j]$, $\hat{x}_j$, $[v_j]$, p_j, A_j, h and the outputs are $[x_{j+1}]$, $\hat{x}_{j+1}$, $[v_{j+1}]$, $[p_{j+1}]$, A_{j+1}. The variable $\hat{x}_j$ is a midpoint of a certain interval v_j. The initial conditions can be given by $p_0 = 0$, $q_0 = 0$ and $v_0 = x_0$.

1 $[v_{j+1}] = \hat{x}_j + \sum_{i=1}^{k-1} h^i f^{[i]}(\hat{x}_j) + h^k f^{[k]}([\tilde{x}_j])$,

2 $[S_j] = I + \sum_{i=1}^{k-1} J(f^{[i]};[x_j])h^i$,

3 $[q_{j+1}] = ([S_j]A_j)[p_j] + [S_j]([v_j] - \hat{x}_j)$,

4 $[x_{j+1}] = [v_{j+1}] + [x_{j+1}]$,

5 $A_{j+1} = m([S_j]A_j)$,

6 $[p_{j+1}] = A_{j+1}^{-1}([S_j]A_j)[p_j] + (A_{j+1}^{-1}[S_j])([v_j] - \hat{x}_j)$,

7 $\hat{x}_{j+1} = m([v_{j+1}])$.

In the previous algorithm, I represents the identity matrix (with the same dimension as the state vector). $J(f^{[i]};[x_j])$ is the Jacobian matrix of the ith Taylor coefficient $f^{[i]}$ evaluated on $[x_j]$. The variables $\hat{x}_j$ and $[v_j]$ are computed in the previous step (t_{j-1}).

VI. APPLICATION

The state estimation algorithm presented in section IV is now applied to the aerospace case system given by (3). The study has been conducted in simulation. The simulated outputs are perturbed by heavy noise and the noise is bounded by:

$$\mathrm{E} = \begin{bmatrix} -0.1 & 0.1 \\ -0.4 & 0.4 \\ -0.4 & 0.4 \\ -0.4 & 0.4 \end{bmatrix}. \qquad (9)$$

The initial conditions are supposed to belong to:

$$\mathrm{X}_0 = \begin{bmatrix} 28.45 & 28.5 \\ 6.3025 & 6.3730 \\ 0.1719 & 0.2292 \\ 2.2918 & 2.4064 \end{bmatrix}. \qquad (10)$$

The test duration is fixed at one second.

The order of the Taylor expansion is two. The resulting output trajectories are given in figures 2, 3, 4 and 5. In these figures, the full lines represent the measures and the dotted lines represent the reconstruction of the model.

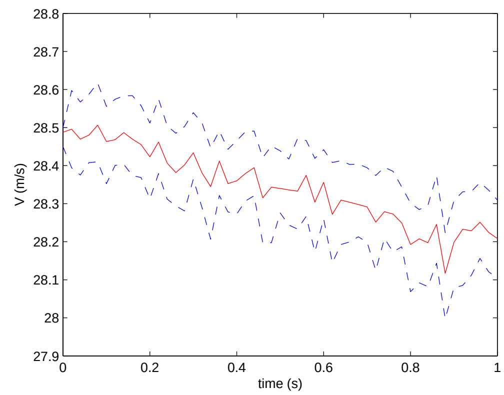

Fig. 2. Speed reconstruction

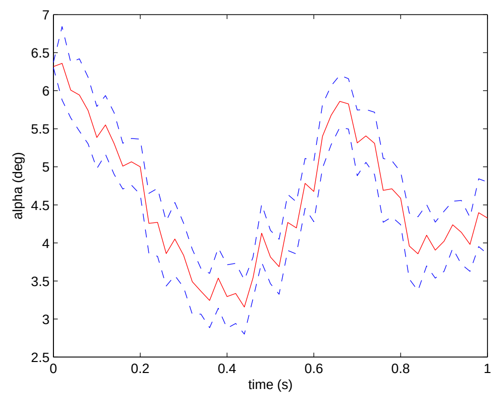

Fig. 3. Angle of attack reconstruction

In these figures, the obtained enveloppes for state variables show that uncertainty is appropriately controlled by the estimation algorithm in spite of heavy noise. The estimated enveloppes are smooth in the case of the pitch rate reconstruction, and the fact that they are somehow perturbated for the other three variables indicates that the estimation is quite tight. This points out the efficiency of the proposed method based on high-order Taylor expansion to solve the state equations allied with centred forms and matrices preconditioning. This method really leads to good predictions.

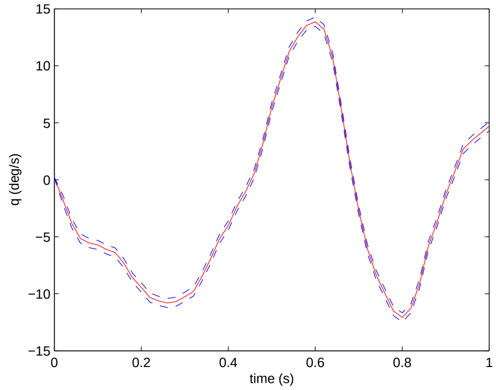

Fig. 4. Pitch rate reconstruction

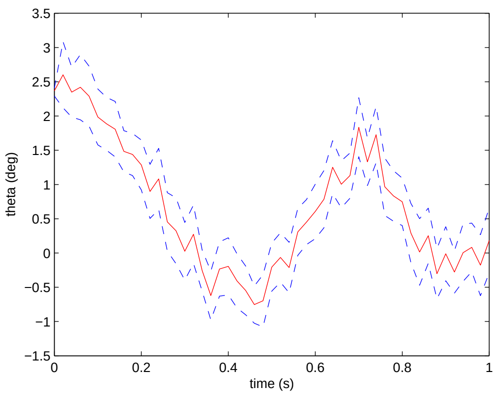

Fig. 5. Pitch angle reconstruction

VII. RELATED WORK IN STATE ESTIMATION

Processes modelling can be tricky due to the presence of noises and perturbations. State estimation problems from experimental data are usually solved by probabilistic methods [27] when these noises and perturbations can be reasonably assumed to be random variables. In this statistical context, state estimation problems are solved through optimization after the choice of an appropriate criterion.

However, in practice, it is often the case that an explicit characterization of noise and perturbation variables is not available, making difficult to assess proper stochastic hypotheses.

An alternative approach consists in assuming that uncertain variable values belong to sets, hence modeling bounded uncertainty. Thus, state estimation problems are now placed into a bounded-error context. Bounded-error approaches permit the characterization of the set of all values of the state vector that are consistent with the measured data, the model structure and the prior known error bounds. Available methods based on set-membership approaches exist for linear and non linear models.

Numerous approaches have been investigated for the case of linear models. We can characterize the solution set by a convex polyhedron. But in practice, this set is very difficult to obtain. Thus it may be preferable to compute other geometric shapes, such as ellipsoids [7], [15], [17], parallelotopes, polytopes or zonotopes [2] guaranteed to contain the exact solution set. More recently, a state estimator based on zonotopes was presented in [9]. The computation of ellipsoids alternates prediction and correction phases, like the algorithm used in this paper to solve the state estimation problem.

When the model is nonlinear, the set of values of the state vector to be characterized is usually non convex and may consist of several disconnected components. Few results are available in this field. The previous methods are no longer relevant and other algorithms based on interval analysis have been developed [11]. Interval analysis provides tools for guaranteed non linear state estimation in a bounded error context [14]. An example is handled with the application for the localization of a mobile robot in [13].

Actual systems are often described by ordinary differential equations. Interval analysis and a first-order enclosure of the solution of the ordinary differential equation allow one to compute guaranteed solutions to the state estimation problem. Then, validated numerical methods for solving the ordinary differential equation are applied. These methods use high-order interval Taylor models [6], [20], [23] to compute intervals which are guaranteed to contain the solution of the ordinary differential equation. This paper has used a method of this type and shows their efficiency on the a complex aerospace case study.

VIII. CONCLUSION

In this contribution, a procedure for state estimation in a bounded-error context is pointed out. This method combines high-Taylor models and interval analysis. It has been applied to study the guaranteed state estimation for the longitudinal motion of a glider successfully and this paper reports about this experiment. We plan to apply this method to the complete aircraft model in the near future. It was shown that using high-order Taylor expansion to solve the state equations allied with centred forms and matrices preconditioning makes the use of bissections unnecessary to obtain a good prediction.
This makes it possible to study state estimation for systems with high dimensions.

The method has potential for being used for fault detection and diagnosis problems in continuous-time systems or hybrid systems. Fault detection mechanisms using bounded uncertainty models present the advantage to guarantee absence of false alarms [3]. They have been investigated in the last few years and some comparative analysis works exist [4], [26]. The drawback of this methods is the missing alarms problem, which is due to overestimated results. However, recent methods such as the one used in this paper or [1], [21] should provide significant improvement in this direction.

Another direction of future work consists in extending state estimation in a bounded-error context to parameter estimation for nonlinear systems. It can shown that set inversion combined with validated integration of ordinary differential equations is able to solve the parameter estimation problem.

This opens new perspectives for fault diagnosis, for instance using faults models [8].

References

[1] O. Adrot, S. Ploix, ”Fault detection based on set-membership inversion”, *in Proceedings of SAFEPROCESS'06*, Beijing, China, 2006, pp 619-624.

[2] T. Alamo, J. Bravo and E. Camacho, Guaranteed state estimation by zonotopes, *Automatica*, vol. 41, 2005, pp 1035-1043.

[3] J. Armengol, L. Travé-Massuyès, J. Vehi, J.Ll. de la Rosa, A survey on interval model simulators and their properties related to fault detection, *Annual Reviews in Control*, vol. 24, No. 1. ISSN: 1367-5788. Elsevier Science. Oxford, UK., 2000, pp. 31-39.

[4] J. Armengol, J. Vehi, L. Travé-Massuyès, M.A. Sainz, Application of modal intervals to the generation of error-bounded envelopes, *Reliable Computing*, vol 7, n 2, pp. 171-185. ISSN : 1385-3139. Kluwer Academic Publishers, The Netherlands, 2001.

[5] M. Berz, K. Makino, Verified integration of ODEs and flows using differential algebraic methods on high-order Taylor models, *Reliable Computing*, vol. 4(4), 1998, pp 361-369.

[6] G. Corliss, ”Guaranteed error bounds for ordinary differential equations”, *Lectures notes presented at the VI-th SERC Numerical Analysis Summer School*, 1994.

[7] C. Durieu and E. Walter, *Identification des systèmes*, Chapitre Estimation ellipsoidales à erreur bornée, Hermès, Paris; 2001.

[8] T. Escobet, L. Travé-Massuyès, ”Parameter estimation methods for fault detection and isolation”, *in Proceedings of Bridge Workshop and DX'01*, 2001.

[9] P. Guerra, B. Puig, A. Ongimundarson and M. Witczak, ”Robust fault detection with unknown input set-membership state estimators and interval models using zonotopes”, *in Proceedings of Safeprocess'06*, Beijing, China, 2006, pp 1117-1122.

[10] L. Jaulin and E. Walter, Set inversion via interval analysis for nonlinear bounded-error estimation, *Automatica*, vol. 29(4), 1993, pp 1053-1064.

[11] L. Jaulin, M. Kieffer, O. Didrit, E. Walter, *Applied Interval Analysis, with examples in parameter and state estimation, Robust control and robotics*, Springer, Londres; 2001.

[12] M. Kieffer, E. Walter, ”Interval Analysis for guaranteed nonlinear parameter estimation”, *In A.C. Atkinson, L. Pronzato, H.P. Wynn, Advances Model-Oriented Data Analysis and Experimental Design*, Physica-Verlag, Heidelberg, 1998, pp 115-125.

[13] M. Kieffer, L. Jaulin, E. Walter and D. Meizel, Robust robot localization and tracking using interval analysis, *Reliable Computing*, vol. 6, 2000, pp 337-362.

[14] M. Kieffer, L. Jaulin and E. Walter, Guaranteed recursive nonlinear state bounding using interval analysis, *International Journal of Adaptative Control and Signal Processing*, vol. 6, 2002, pp 191-218.

[15] S. Lesecq, A. Barraud and K. Tran Dinh, ”Numerical accurate computations for ellipsoidal state bounding”, *in Proceedings of MED'03*, Rhodos, Greece, 2003.

[16] R. Lohner, ”Enclosing the solution of ordinary initial and boundary value problems”, *in E. Kaucher, U. Kulisch, and C. Ullrich (eds)Computer arithmetic: Scientific Computation and Programming Languages*, BG Teubner, Stuttgart, Germany, 1987, pp 255-286.

[17] M. Milanese and A, Vivino, Estimation theory for nonlinear models and set membership uncertainty, *Automatica*, vol. 27, no. 2, 1991, pp 403-408.

[18] R.E. Moore, *Automatic error analysis in digital computation*, Technical report LMSD-48421 Lockheed Missiles and Space Co, Palo Alto, CA; 1959.

[19] R.E. Moore, *Interval Analysis*, Prentice Hall, New Jersey; 1966.

[20] N.S. Nedialkov, K.R. Jackson, ”Methods for initial value problems for ordinary differential equations”, *In U. Kulisch, R. Lohner and A. Facius, editors, Perspectives on Enclosure Methods*, Springer Verlag, Vienna, 2001, pp 219-264.

[21] H. Niemann, ”Active fault diagnosis in closed-loop uncertain systems”, *in Proceedings of SAFEPROCESS'06*, Beijing, China, 2006, pp 631-636.

[22] A. Neumaier, *Interval Methods for Systems of Equations*, Cambridge University Press, Cambridge, UK; 1990.

[23] T. Raïssi, N. Ramdani, Y. Candau, Set membership state and parameter estimation for systems described by nonlinear differential equations, *Automatica*, vol 40, 2004, pp 1771-1777.

[24] R. Rihm, ”Interval methods for initial value problems in ODEs”, *In J. Herzberger (Ed.), Topics in validated computations: Proceedings of the IMACS-GAMM international workshop on validated computations*, University of Oldenburg, Elsevier Studies in Computational Mathematics, Amsterdam, New York: Elsevier, 1994.

[25] N.S. Nedialkov, K.R. Jackson, J.D. Pryce, An effective high-order interval method for validating existence and uniqueness of the solution of an IVP for an ODE, *Reliable Computing*, vol 7(6), 2001, pp 449-465.

[26] S. Tornil, T. Escobet, L. Travé-Massuyès, Robust fault detection using interval models, *12th European Control Conference ECC'03*, Cambridge, UK, 2003.

[27] E. Walter and L. Pronzato, *Identification de modèles paramétriques à partir de données expérimentales*, Masson; 1994.

[28] M. Milanese, J.P. Norton, H. Piet-Lahanier and E. Walter, *Bounding approaches to system identification*, Plenum, New York; 1996.

[29] J.C. Wanner, *Dynamique du vol et pilotage des avions*, Ecole Nationale Supérieure de l'Aéronautique et de l'Espace; 1984.