Recent robust analysis and design results for simple adaptive control

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Simple adaptive control

$$u(t) = K(t)y(t) + w(t) \ , \ \dot{K}(t) = -Gy(t)y^T(t)\Gamma - \phi(K(t))$$

- Passivity-based, Direct of Simple Adaptive Control (SAC)

  [Fradkov, Kaufman et al, Ioannou, Barkana]

- Adaptation does not need parameter measurement or estimation.

- Regulation case (no reference model $y_{ref} = 0$)
- Rectangular uncertain linear systems

  $\dot{x} = A(\Delta)x + B(\Delta)u, \ \ y = C(\Delta), \ \ \Delta \in \mathbf{\Delta}$

  $u \in \mathrm{R}^m, y \in \mathrm{R}^p : p \geq m$

- Properties achieved thanks to closed-loop passification

  (almost passive systems [Barkana])

$$\exists F \ : \ \dot{x} = (A + BFC)x + Bw, \ \ z = GCx \ \text{ passive}$$

Outline

❶ Parallel feedthrough gain for robustness

- LMI formulas for SAC stability analysis

❷ Design of a G matrix

- BMI problem, clues for some heuristics

❸ Guaranteed robust L_2 gain for SAC

- Proves better than some parameter-dependent controllers

❹ Guaranteed robust stability in case of time varying uncertainties

- Convergence to a neigborhood of the origin

Closed-loop stability with SAC

Guaranteed if

$$\exists F \ : \ \dot{x} = (A + BFC)x + Bw, \ \ z = GCx \ \text{ passive}$$

or equivalenty if

$$\exists F, P \ : \ (A + BFC)^T P + P(A + BFC) < 0 \ , \ PB = C^T G^T$$

This condition happens to be LMI+E (for given G):

$$\exists F, P \ : \ A^T P + PA + C^T (G^T F + F^T G)C < 0 \ , \ PB = C^T G^T$$

- Robustness LMI-based techniques may be applied to LMI conditions
- Equality constraint almost impossible to guarantee robustly

$$P(\Delta)B(\Delta) = C^T(\Delta)G^T \ , \ \forall \Delta \in \mathbf{\Delta} \quad !!!$$

❶ Parallel feedthrough gain for robustness

New stability condition [S&CL 2008]

Closed-loop stability with SAC is guaranteed if $\exists F, P, R, D$:

$$\mathcal{L}(F,P,R,D) > 0 \ , \ \begin{bmatrix} R & PB - C^T G^T \\ B^T P^T - GC & \mathbf{I} \end{bmatrix} \geq 0$$

- Includes previous result when $R = 0$
- Related to passivity of closed-loop system with parallel feedthrough

$$\dot{x} = (A + BFC)x + Bw, \ \ z = GCx + Dw \ \text{ passive}$$

(Same passivity propoerty holds for closed-loop with SAC)

- Conditions are all LMI:

  can be used to derive conditions for guaranteed robustness $\forall \Delta \in \mathbf{\Delta}$.

- ⚠ Results only demonstrated for a particular choice of ϕ.

❶ Parallel feedthrough gain for robustness

Simple adaptive control

$$u(t) = K(t)y(t) + w(t) \ , \ \dot{K}(t) = -Gy(t)y^T(t)\Gamma - \phi(K(t))$$

- $-Gyy^T\Gamma$: drives the gain $K(t)$ to stabilizing values
- Choice of Γ: tunes dynamics of $K(t)$, must take into account implementation
- ϕ is dead-zone type, defined by $\phi(K) = \psi(\text{Tr}(K^TK))K\Gamma$ where

$$\begin{cases} \psi(k) = 0 & \forall\, 0 \leq k < \alpha \\ \psi(k) = \frac{k-\alpha}{\beta-k} & \forall\, \alpha \leq k < \beta \end{cases}$$

- ϕ prevents K to grow too large ($\text{Tr}(K^TK) < \beta$)
- α should be large to keep the adaptation free.
- α and β are chosen accordingly to implementation constraints.

❷ Design of a G matrix

A non convex problem

▲ In case without parallel feedthrough: take large enough k and solve

$$A^T P + PA - kC^T G^T GC < 0 \ , \quad PB = C^T G^T$$

Some solved cases

- If open-loop system is square and hyper minimum phase: $G = \mathbf{I}$
- If open-loop system such that CB square diagonalizable [Barkana 2006]
- State-feedback
- G may be derived from physical considerations
- G may be imposed by required closed-loop passivity properties

Heuristic for the general case [IEEE-CCA 2009]

▲ -1- Find a stabilyzing SOF gain F (BMI)

▲ -2- For fixed F, find G while minimizing D (LMI)

▲ -3- Perform robust stability analysis for this choice of G (LMI)

❸ Guaranteed robust L_2 gain for SAC

Uncertain linear system with input/output performance signals

$$\dot{x} = A(\Delta)x + B(\Delta)u + B_L(\Delta)w_L$$

$$y = C(\Delta)x \ , \ z_L = C_L(\Delta)x + D_L(\Delta)w_L$$

- Find controller that stabilizes and guarantees

$$\|z_L\|_2 \leq \gamma \|w_L\|_2 \quad , \quad \forall \Delta \in \mathbf{\Delta}$$

- [S&CL 2008] LMI results in case of polytopic parametric uncertainties

$$A(\Delta) = \sum_{i=1}^{\bar{\imath}} \zeta_i A^{[i]}, \ B(\Delta) = \sum_{i=1}^{\bar{\imath}} \zeta_i B^{[i]}, \dots$$

▲ ζ_i are assumed constant in the simplex

$$\zeta_i \geq 0 \ , \ \sum_{i=1}^{\bar{\imath}} \zeta_i = 1$$

❸ Guaranteed robust L_2 gain for SAC

Theorem If $\exists P^{[i]}, F^{[i]}, R^{[i]}, D^{[i]}, \ldots$ solutions to LMI problem $\mathcal{L}_i(P, F, R, D, \ldots) > 0$, $\forall i = 1 \ldots \bar{\imath}$ then

- $u = F(\Delta)y = \sum_{i=1}^{\bar{\imath}} \zeta_i F^{[i]} y$ is a PD SOF such that L_2 gain is guaranteed
- L_2 gain is guaranteed with SAC
- For all Δ: $\|z_{L,SAC}\| \leq \|z_{L,PDSOF}\|$.

Proof Based on the following Lyapunov function

$$x^T(t)P(\Delta)x(t) + \mathrm{Tr}(K(t) - F(\Delta)\Gamma^{-1}(K(t) - F(\Delta))^T$$

▲ LMI conditions, combined to assumptions that $\dot{\zeta} = 0$ and $K(t)$ bounded (due to corrective term $\phi(K)$), prove the derivative of the Lyapunov function to be negative definite whatever admissible ζ. Moreover, for zero initial conditions, one gets that $\|z_L\|_2 \leq \gamma \|w_L\|_2$.

- Note that $\|z_{L,SAC}\| \leq \|z_{L,PDSOF}\|$ whatever choice of w_L, z_L, although SAC does not use any information on these signals.

UAV Example

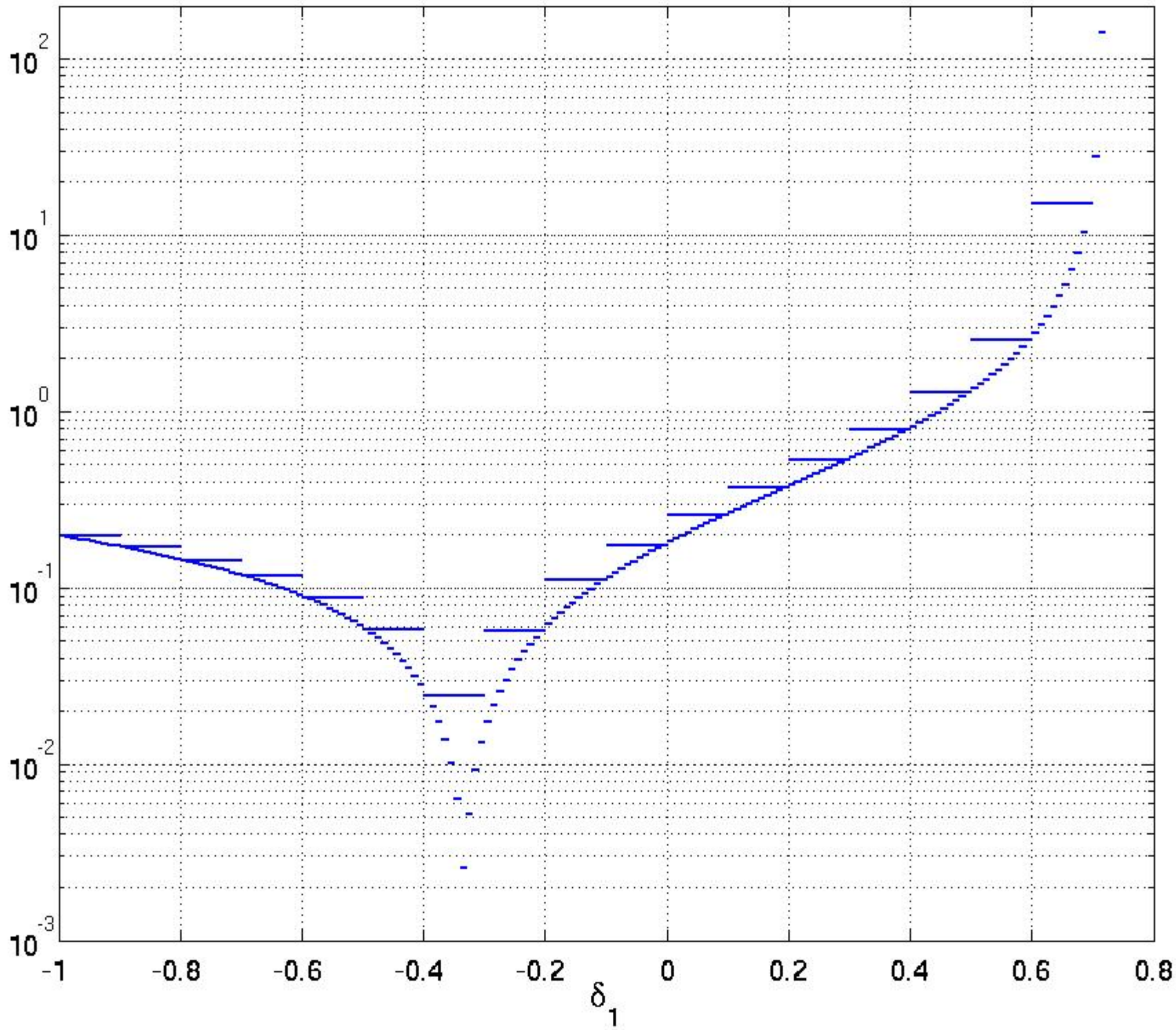
4 states, 2 scalar uncertainties, $\delta_2 \in [\,0\;\;2.5\,]$

Tests on large intervals of δ_1

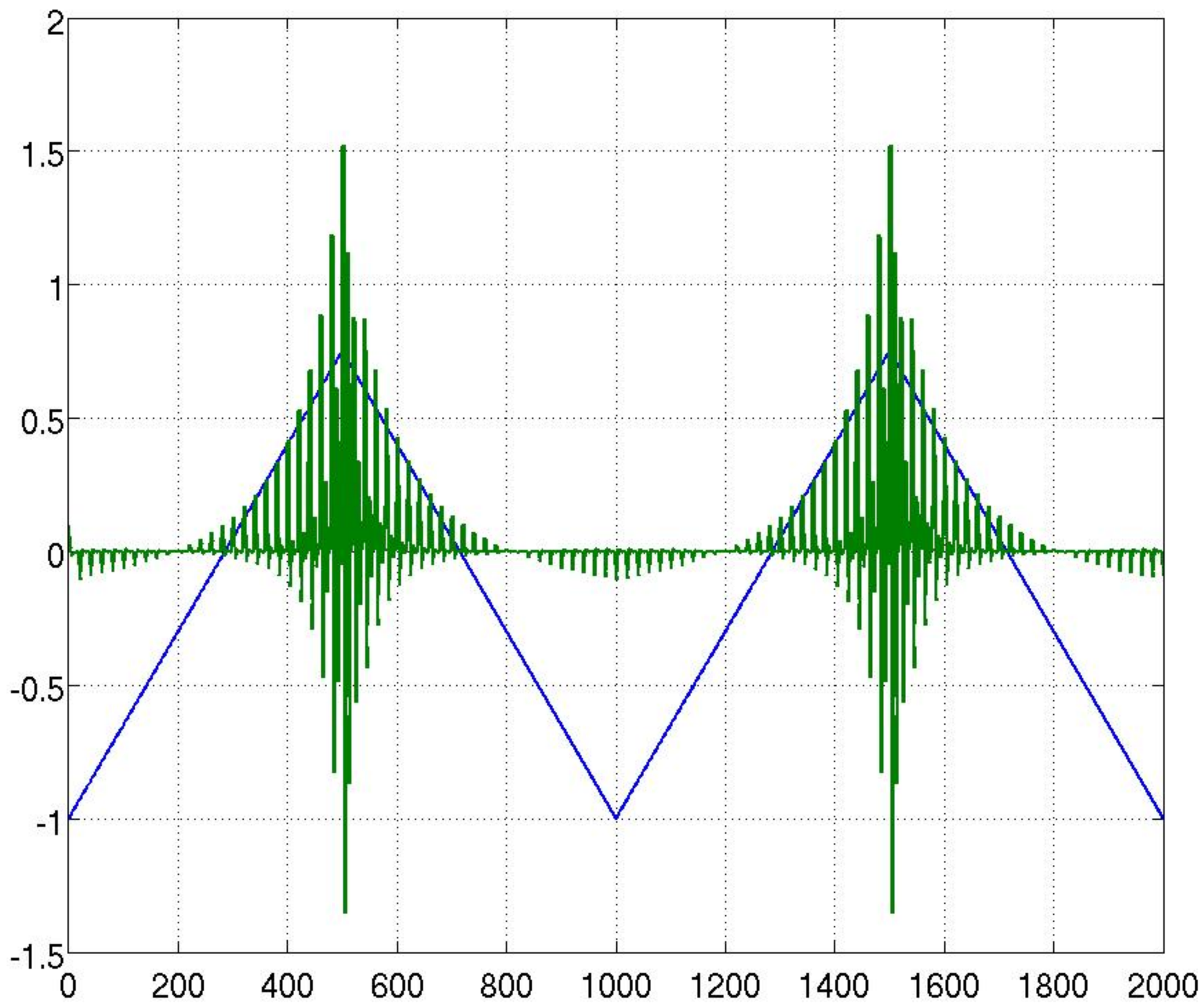
δ_1	min γ	δ_1	min γ	δ_1	min γ
$[\,-1\;\;0\,]$	0.2	$[\,0.7\;\;0.72\,]$	141	$[\,0.72\;\;0.722\,]$	1001
$[\,-1\;\;0.7\,]$	24	$[\,0.7\;\;0.73\,]$	infeas.	0.723	infeas.
$[\,-1\;\;0.72\,]$	infeas.				

UAV Example

Tests on small intervals of δ_1

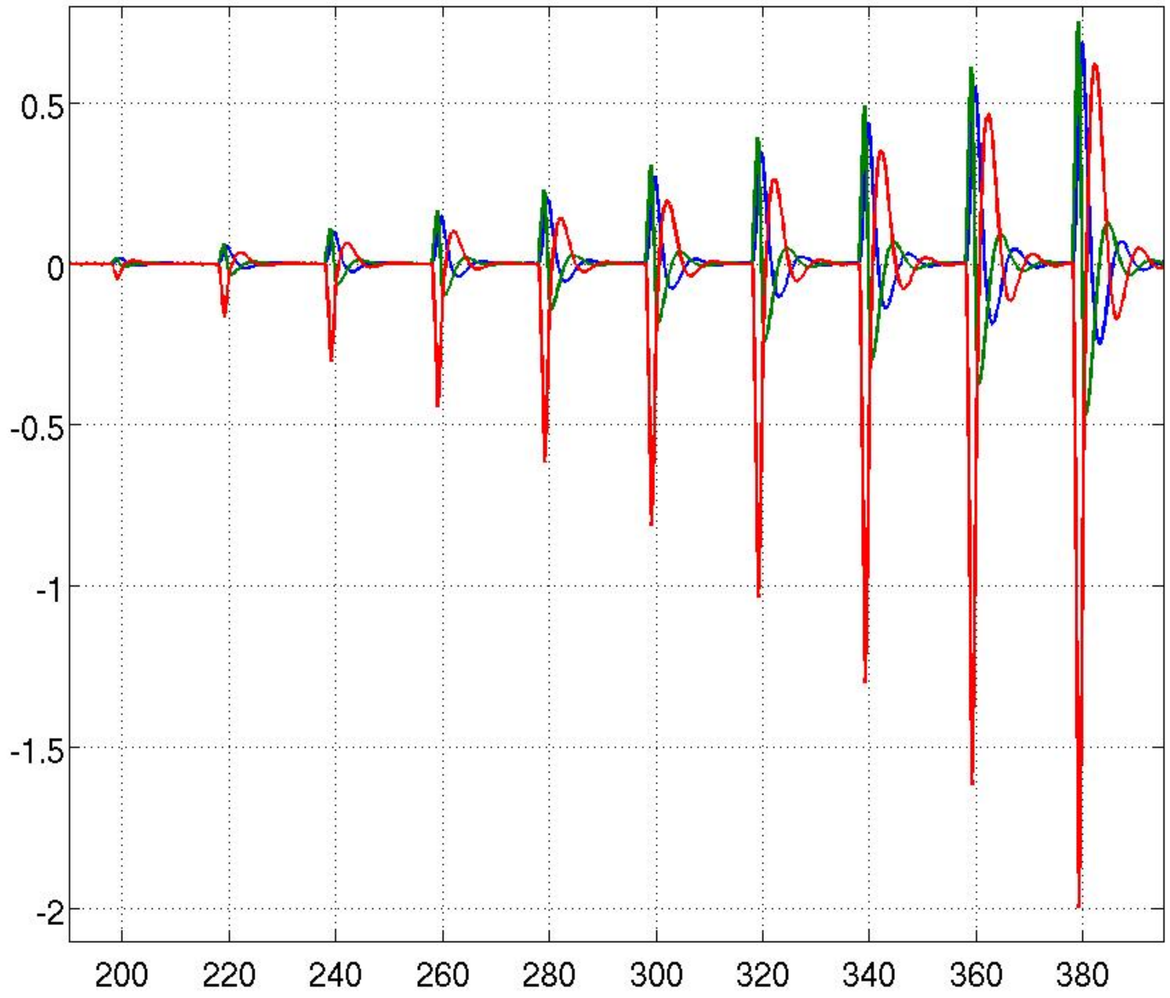


UAV Example SAC simulations with impulse disturbances w_L (every 20s) and slowly varying δ_1 (beyond proved stable values).

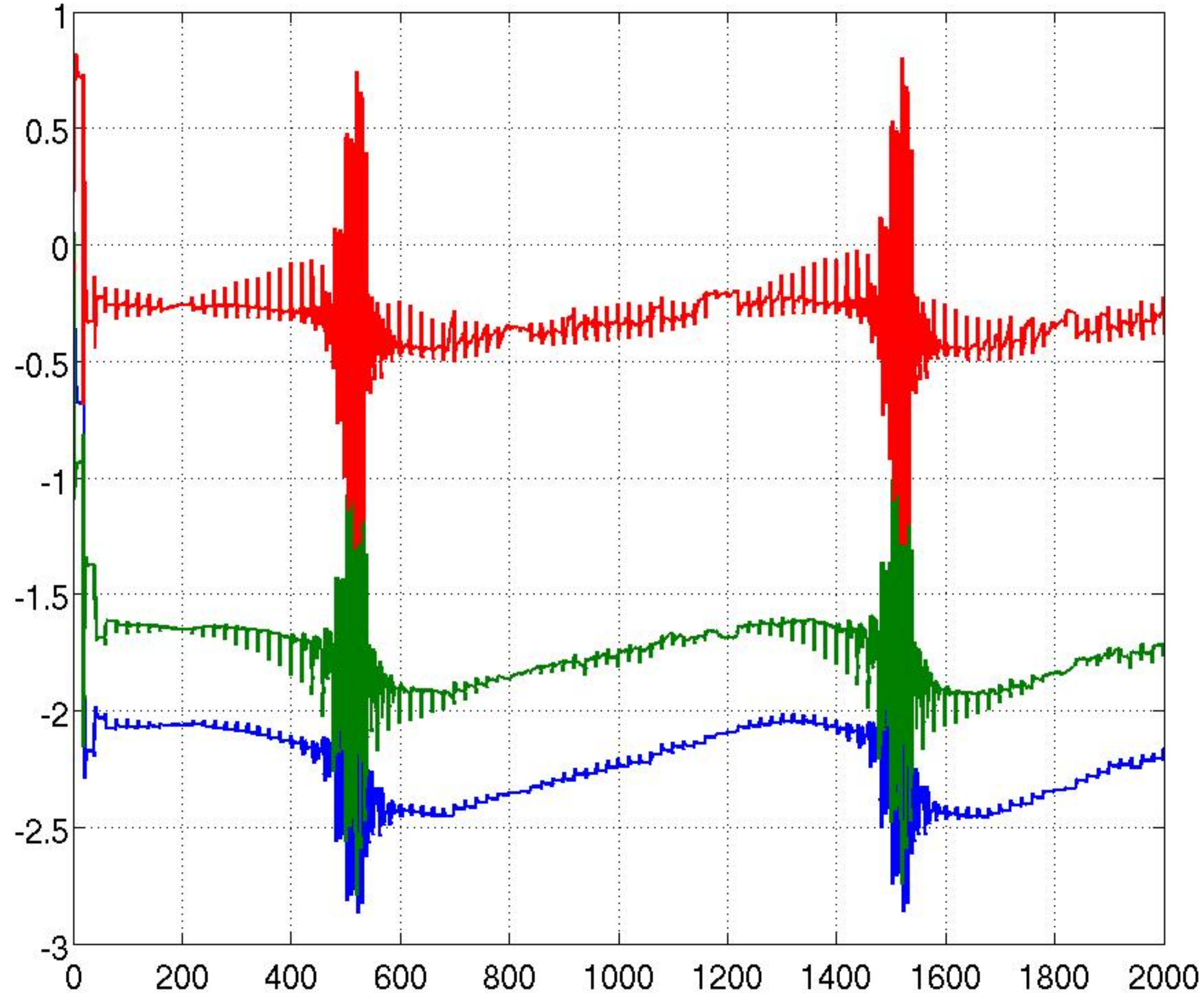


❸ Guaranteed robust L_2 gain for SAC

UAV Example Zoom on the output responses.

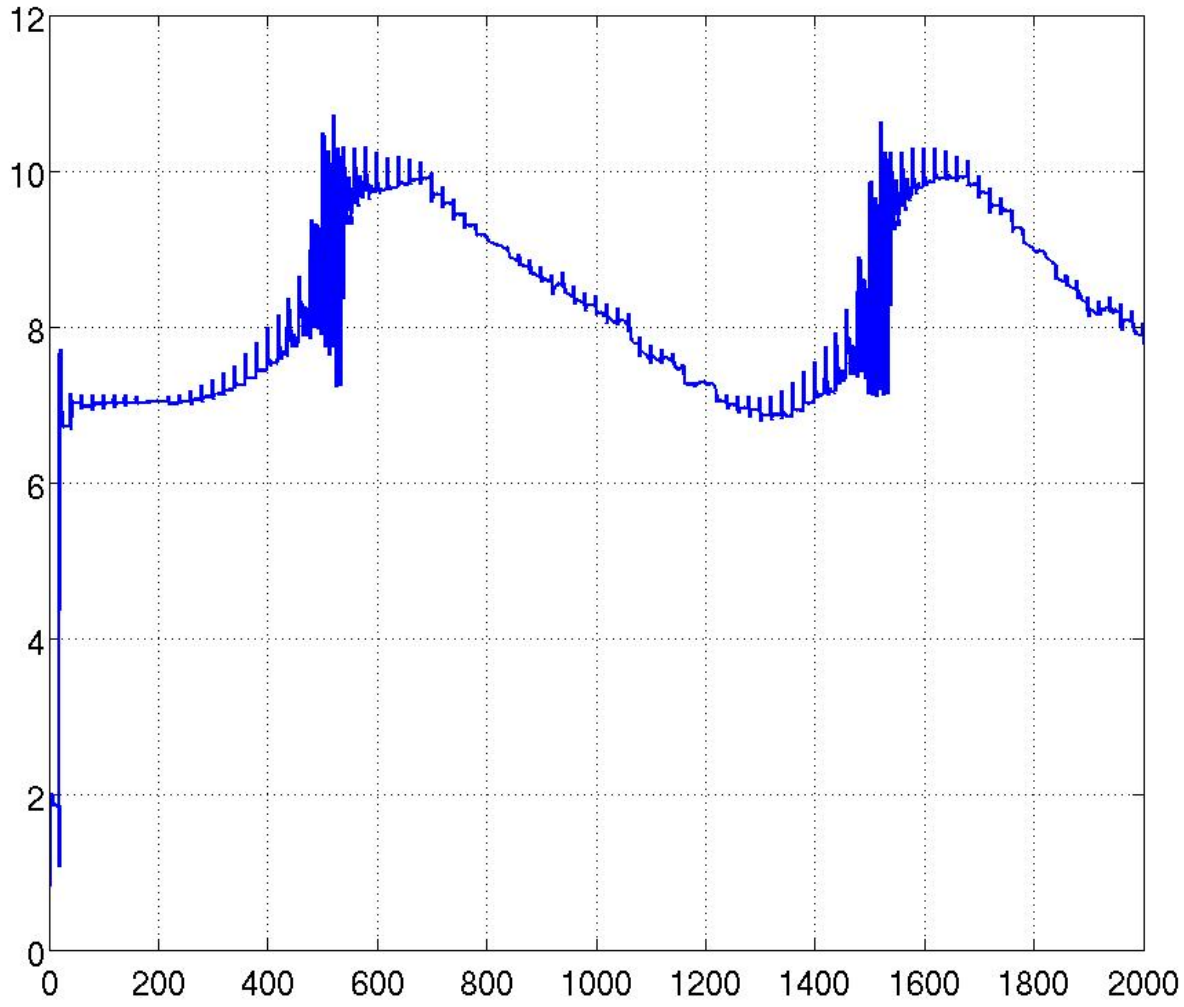


UAV Example Time histories of the SAC gains



❸ Guaranteed robust L_2 gain for SAC

UAV Example $\alpha = 10$, $\beta = 12$: the gains are bounded $\text{Tr}(K^T K) \leq \beta$.



Unceratin time-varying linear system

$$\dot{x}(t) = A(\Delta(t))x(t) + B(\Delta(t))u(t) \quad , \quad y = C(\Delta(t))x(t)$$

Stability proof based on the Lyapunov function $V(x, K, \Delta) =$

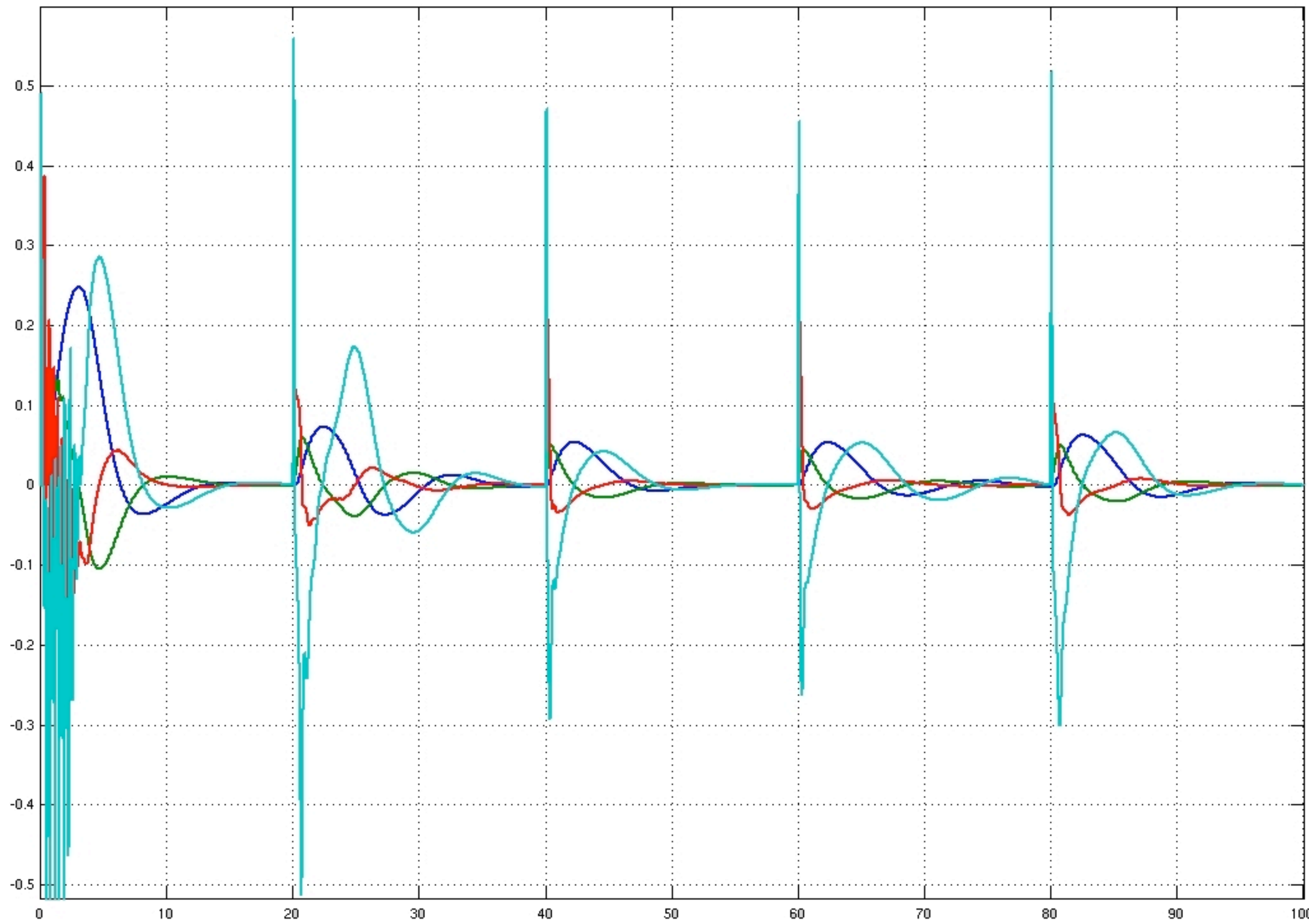
$$x^T(t)P(\Delta(t))x(t) + \text{Tr}(K(t) - F(\Delta(t))\Gamma^{-1}(K(t) - F(\Delta(t)))^T$$

- If $\dot{\Delta}$ is unbounded, then $\dot{V}(x, K, \Delta)$ exists only if:
  $P(\Delta) = P$, $F(\Delta) = F$, are constant
  i.e. the robust stabilisation is solved with constant SOF F.
- If $\dot{\Delta}$ is bounded, then [Auto.R.Ctr'09], LMI conditions for
  $\dot{V}(x, K, \Delta) < 0$ whatever x s.t. $x^T Q x \geq 1$,
  i.e. Lasalle's principle $x^T Q x \leq 1$ attractive set.
- Attractive domain can be made arbitrarily small if $\dot{\Delta} \rightarrow 0$ or $\Gamma \rightarrow \infty$

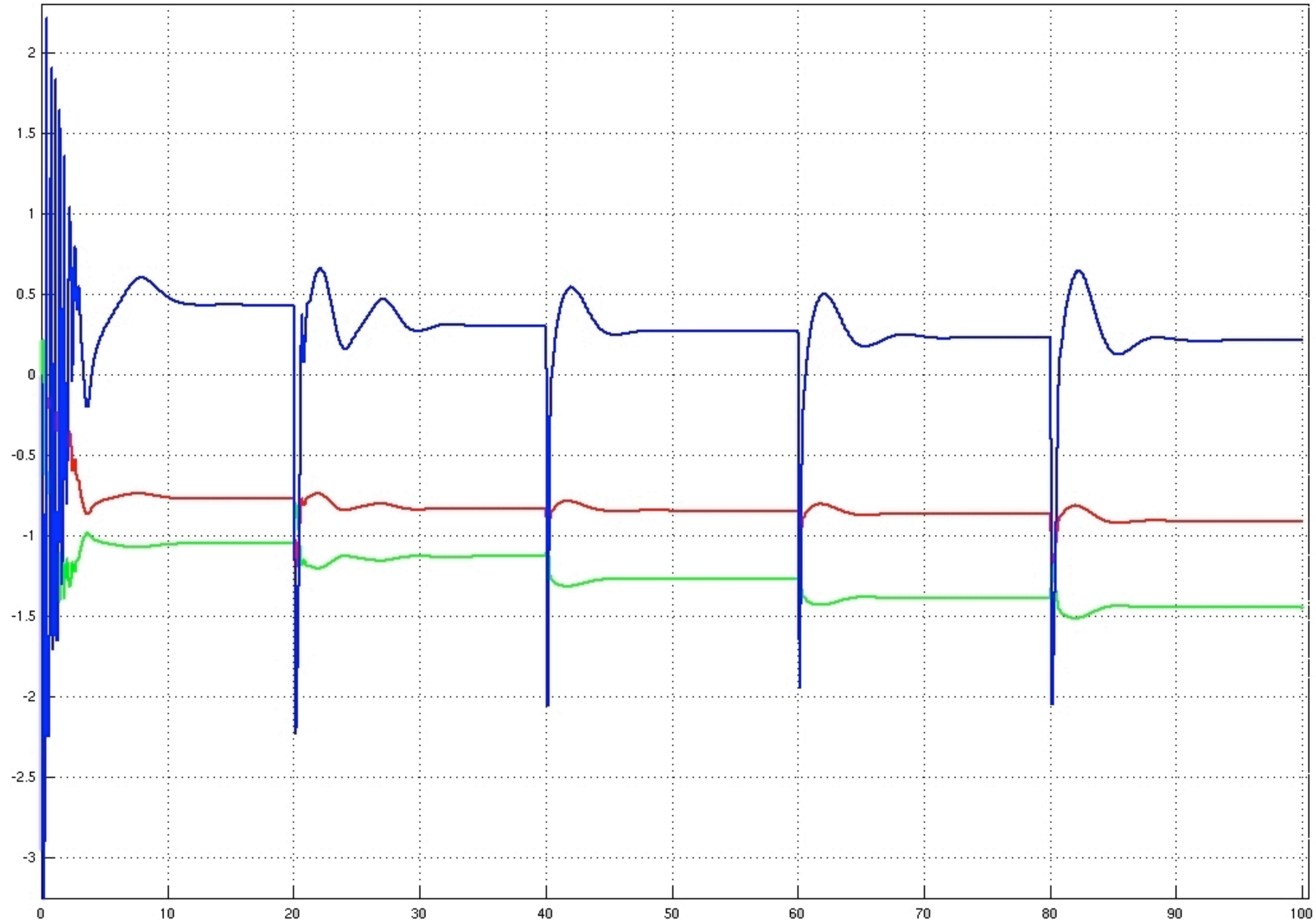
$$u(t) = K(t)y(t) + w(t) \quad , \quad \dot{K}(t) = -Gy(t)y^T(t)\Gamma - \phi(K(t))$$

Example State of the UAV for input impulses every 20s and

$$\delta_1(t) = 0.75\sin(0.125t + 3\pi/2) + 0.1\sin(49t + 3\pi/2) - 0.15 \leq 0.7$$



Example Gains of SAC:



Conclusions

Novel robustness results

- LMI-based: use of efficient numerical tools [YALMIP, SeDuMi...]
- Guaranteed robustness $(A(\delta), B(\delta), C(\delta))$
- Estimated attraction domain in case of time-vaying uncertainties

Future work

- Validations of the theoretical results on examples
- Heuristics for the design of G matrix
- SAC applied to dynamic output-feedback
- ...