Robust Simple Adaptive Control with Relaxed Passivity and PID control of a Helicopter Benchmark

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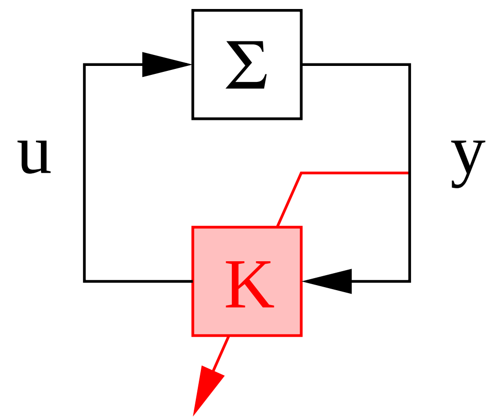
- Considered problem:
  - Stabilization with simple adaptive control

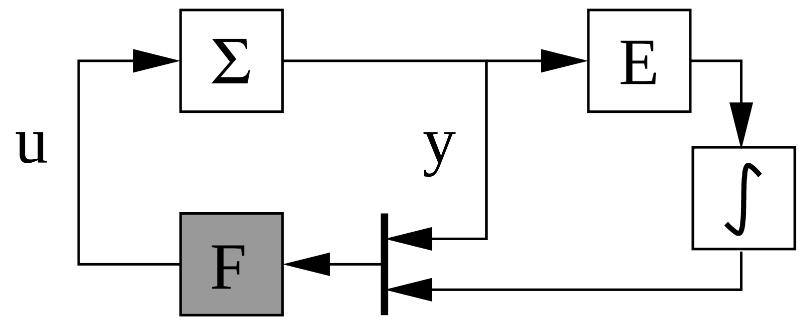
$$u = Ky \ , \ \dot{K} = -Gyy^T\Gamma + \phi$$

  - For MIMO LTI systems

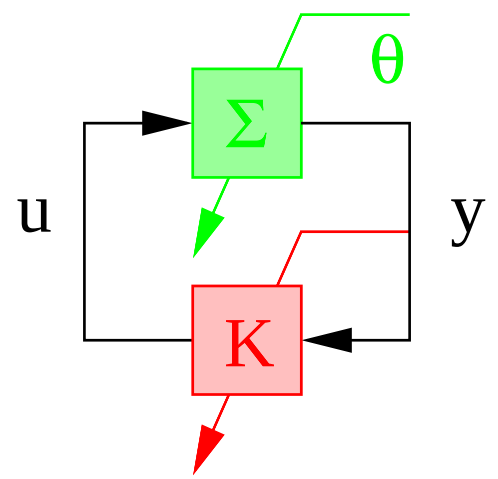


- Assumptions:
  - There exists a (given) stabilizing PI control

$$u = F_P y + F_I \int Ey$$



- Why simple adaptive control?
  - Expected to be more robust
  - No need for estimation $K \neq F(\hat{\theta})$



Outline

- PI structure of the adaptive controller
  - Adaptation that stops as the output reaches the reference

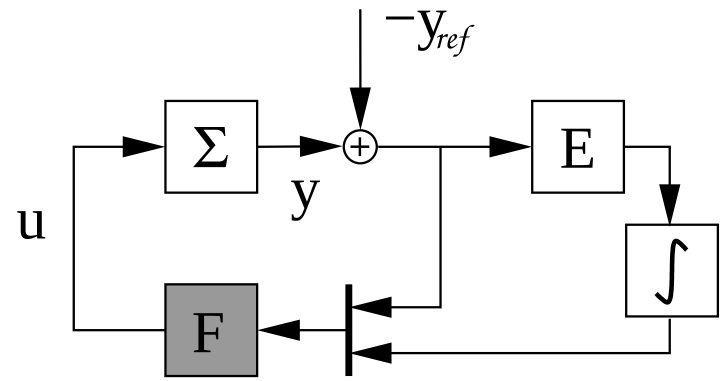
- Design of the adaptive law: $\dot{K} = -Gyy^T\Gamma + \phi$
  - Virtual feedthrough D & barrier function ϕ for bounding K
  - LMI based results $\Rightarrow$ guaranteed robustness

- Preliminary tests on a 3D helicopter Benchmark
  - Adaptive PID

PI structure of Adaptive Control

- Assumptions:
  - There exists a (given) stabilizing PI control

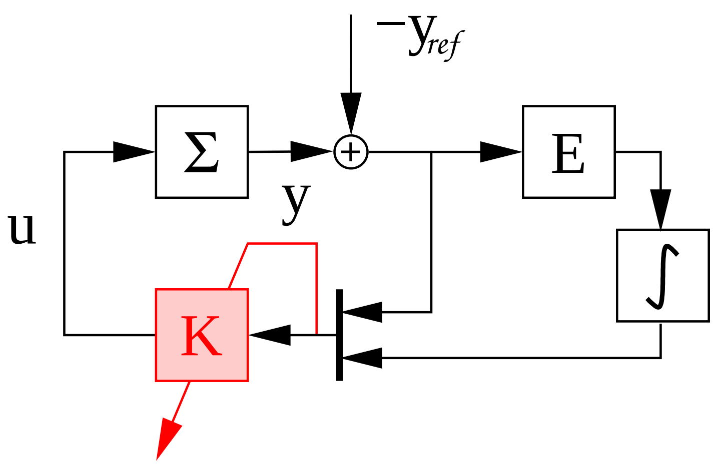
$$u = F_P(y - y_{ref}) + F_I \int E(y - y_{ref})$$



  - Integral term: precision to constant reference

- **Impossible** to apply the following adaptive control:

$$u = K\eta \ , \ \ \dot{K} = -G\eta\eta^T\Gamma + \phi \ , \ \ \eta = \begin{pmatrix} y - y_{ref} \\ \int E(y - y_{ref}) \end{pmatrix}$$



NO! because $\int E(y - y_{ref})$ does not go to 0

PI structure of Adaptive Control

- Proposed PI adaptive control law: $u = K_P(y - y_{ref}) + K_I \int E(y - y_{ref})$

$$\dot{K}_P = -G_P \begin{pmatrix} y - y_{ref} \\ \int E(y - y_{ref}) \end{pmatrix} (y - y_{ref})^T \Gamma_P + \phi_P$$

$$\dot{K}_I = -G_I (y - y_{ref}) \left(\int E(y - y_{ref}) \right)^T \Gamma_I + \phi_I$$

- When $(y - y_{ref})$ goes to zero adaptation stops

- Results applicable to any controller structure of the type

$$u = K_1 y_1 + K_2 y_2 \ , \quad \begin{array}{l} \dot{K}_1 = -G_1 z_1 y_1^T \Gamma_1 + \phi_1 \\ \dot{K}_2 = -G_2 z_2 y_2^T \Gamma_2 + \phi_2 \end{array}$$

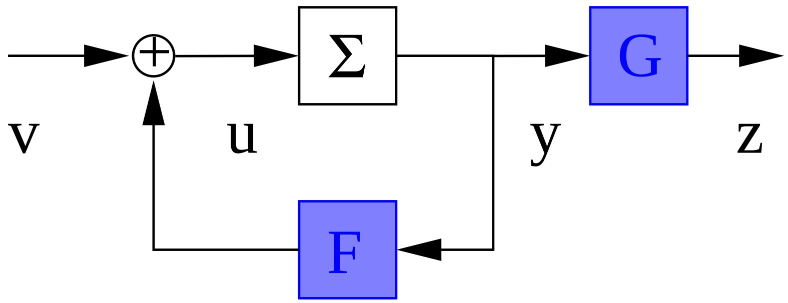
as long as stability and/or tracking imply that y_1 and z_2 go to zero.

- Can also be extended to more than 2 terms ($u = K_1 y_1 + K_2 y_2 + K_3 y_3 + \ldots$)

- Passivity-Based Adaptive Control [Fradkov 1974, 2003]

  & Simple Adaptive Control [Kaufman, Barkana, Sobel 94]

- Let $\Sigma \sim (A, B, C, D)$ be a MIMO system with m inputs / $p \geq m$ outputs.
- If $\exists\ (G, F) \in (\mathrm{R}^{p\times m})^2$ such that the following system is passive



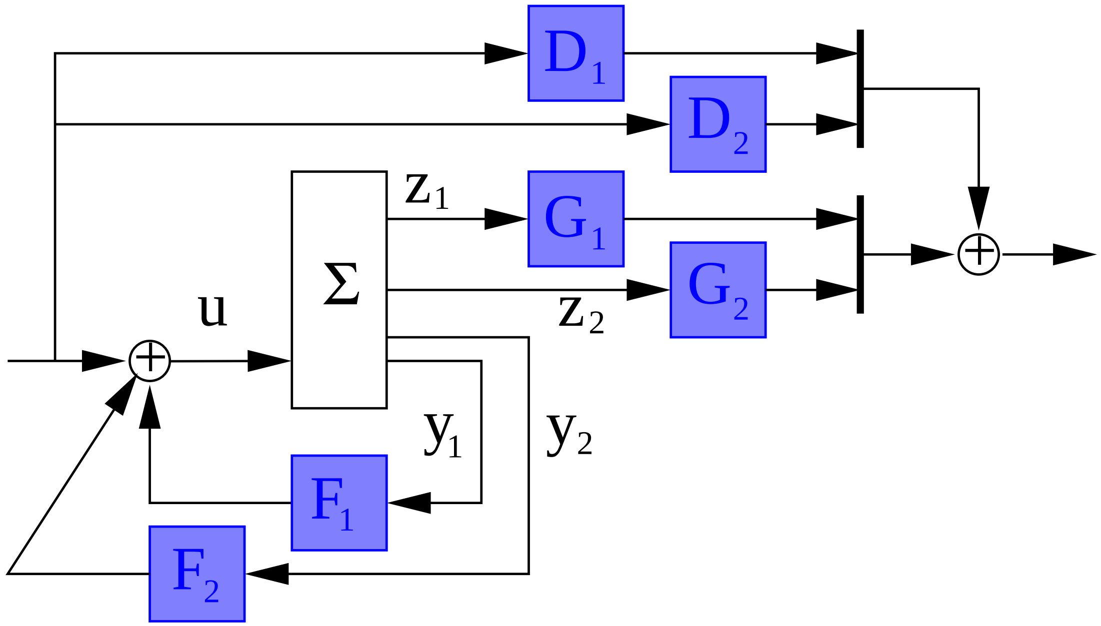
- then the following adaptive law stabilizes the system for all $\Gamma > \mathbf{0}$

$$\dot{K} = -Gyy^T\Gamma \ ,\ \ u = Ky$$

Design of the adaptive law

■ Proposed result

- If $\exists\ (G_1, G_2, F_1, F_2, D_1, D_2)$ such that the system below is passive



- then the following adaptive law stabilizes the system for all $\Gamma_1 > \mathbf{0}$, $\Gamma_2 > \mathbf{0}$

$$u = K_1 y_1 + K_2 y_2 \quad , \quad \begin{array}{l} \dot{K}_1 = -G_1 z_1 y_1^T \Gamma_1 - \phi_1(K_1) \\ \dot{K}_2 = -G_2 z_2 y_2^T \Gamma_2 - \phi_2(K_2) \end{array}$$

where $\phi_1(K_1)$ and $\phi_1(K_2)$ are to be determined (depend of D_1 and D_2).

- 2 step LMI design procedure

- Step 1: Given stabilizing F_1, F_2 solve LMI problem $\mathcal{L}_1$ to get (G_1, G_2, D_1, D_2):

$$\mathcal{L}_1 : \quad \begin{bmatrix} A^T(F_1,F_2)P + PA(F_1,F_2) & PB - {C_1}^T {G_1}^T & PB - C_2^T {G_2}^T \\ B^T P - G_1 C_1 & -2D_1 & \mathbf{0} \\ B^T P - G_2 C_2 & \mathbf{0} & -2D_2 \end{bmatrix} < \mathbf{0}.$$

- Step 2: Given $(G_1, G_2, F_1, F_2, D_1, D_2)$ solve LMI problem $\mathcal{L}_2$ (see paper)

  to get α_1 and α_2 that define the functions ϕ_1, ϕ_2:

  dead-zone: $\phi_i(K_i) = 0 \quad$ if $\quad \text{Tr}\left((K_i - F_i)^T D_i (K_i - F_i)\right) \leq \alpha_i$

  barrier: $\phi_i(K_i) \to +\infty \quad$ if $\quad \text{Tr}\left((K_i - F_i)^T D_i (K_i - F_i)\right) \to \alpha_i \beta$

Design of the adaptive law

■ Properties of the LMI design procedure

- Applicable as soon as there is a given stabilizing control F_1, F_2
- The TV gains K_1, K_2 are guaranteed to be bounded in

$$\mathrm{Tr}\left((K_i(t) - F_i)^T D_i (K_i(t) - F_i)\right) < \alpha_i \beta$$

- It is possible to maximize the domain of admissible adaptation values by

  minimizing $\mathrm{Tr}(D_i)$ in the first LMI step

  maximizing α_i in the second LMI step

- Proof of stability with adaptive control using Lyapunov function

$$V(x, K_1, K_2) = x^T Q x + \sum_{i=1}^{2} \mathrm{Tr}\left((K_i - \hat{F}_i)\Gamma_i^{-1}(K_i - \hat{F}_i)^T\right)$$

where Q, $\hat{F}_1$ and $\hat{F}_2$ are solutions to the LMI problem $\mathcal{L}_2$

Design of the adaptive law

- Robustness properties of the LMI design procedure

- In case of polytopic uncertainties

$$\left[\begin{array}{cc} A(\zeta) & B(\zeta) \end{array}\right] = \sum_{j=1}^{N} \zeta_j \left[\begin{array}{cc} A^{[j]} & B^{[j]} \end{array}\right] \ , \ \sum_{j=1}^{N} \zeta_j = 1 \ , \ \zeta_j \geq 0$$

- Robust LMI version of second step (based on slack variables technique)

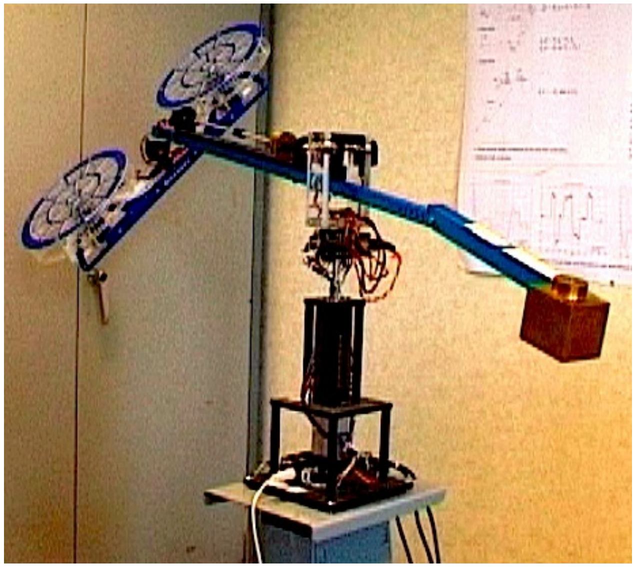
  proves stability with parameter-dependent Lyapunov function

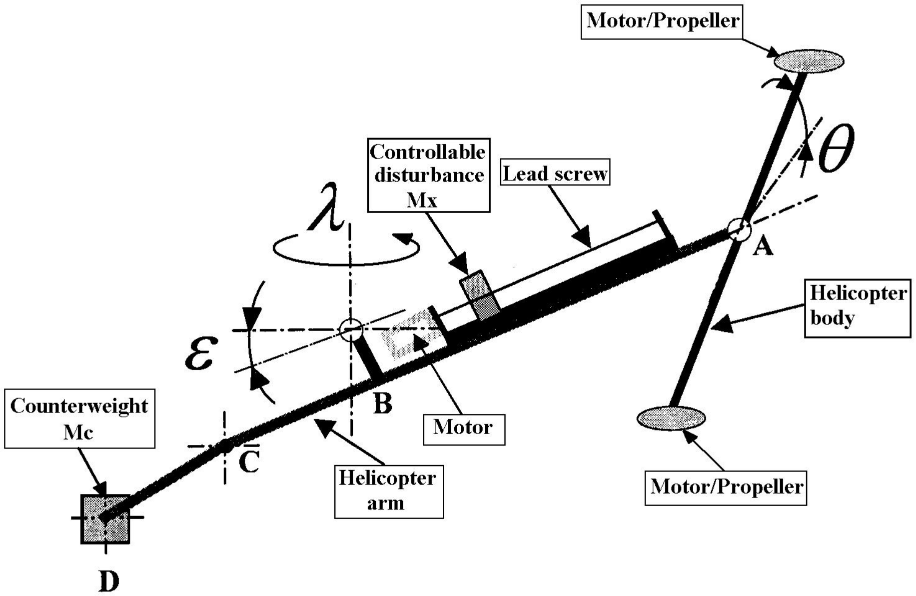
$$V(x, K_1, K_2, \zeta) = x^T Q(\zeta) x + \sum_{i=1}^{2} \text{Tr}\left((K_i - \hat{F}_i(\zeta)) \Gamma_i{}^{-1} (K_i - \hat{F}_i(\zeta))^T \right)$$

where $Q(\zeta) = \sum_{j=1}^{N} \zeta_j Q^{[j]}$ and $\hat{F}_i(\zeta) = \sum_{j=1}^{N} \zeta_j \hat{F}_i^{[j]}$.

3DOF Helicopter control

- 3DOF helicopter from Quanser© - LAAS configuration

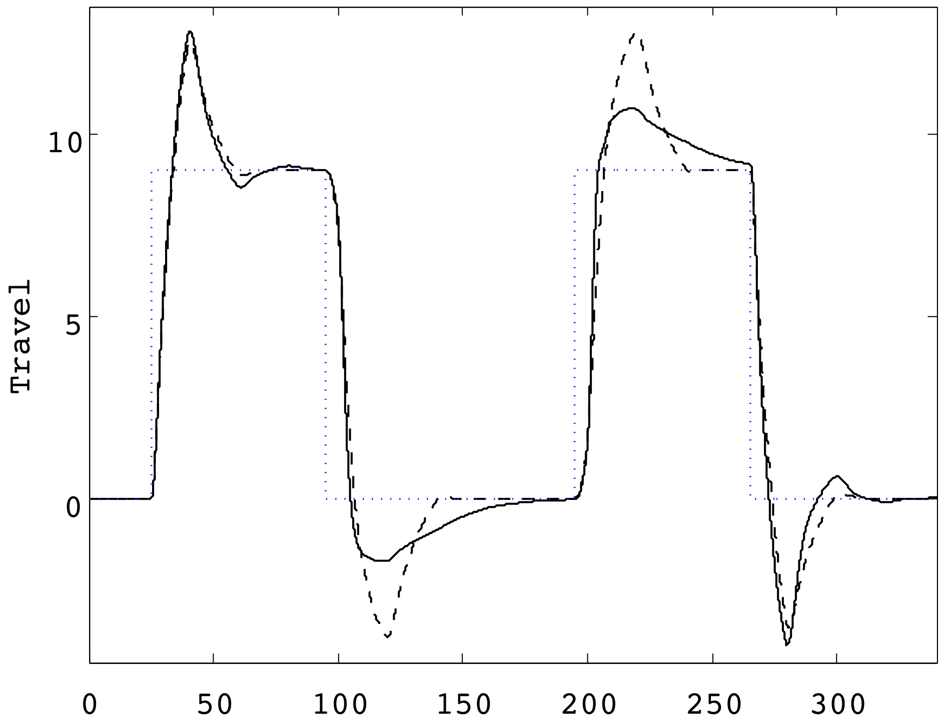


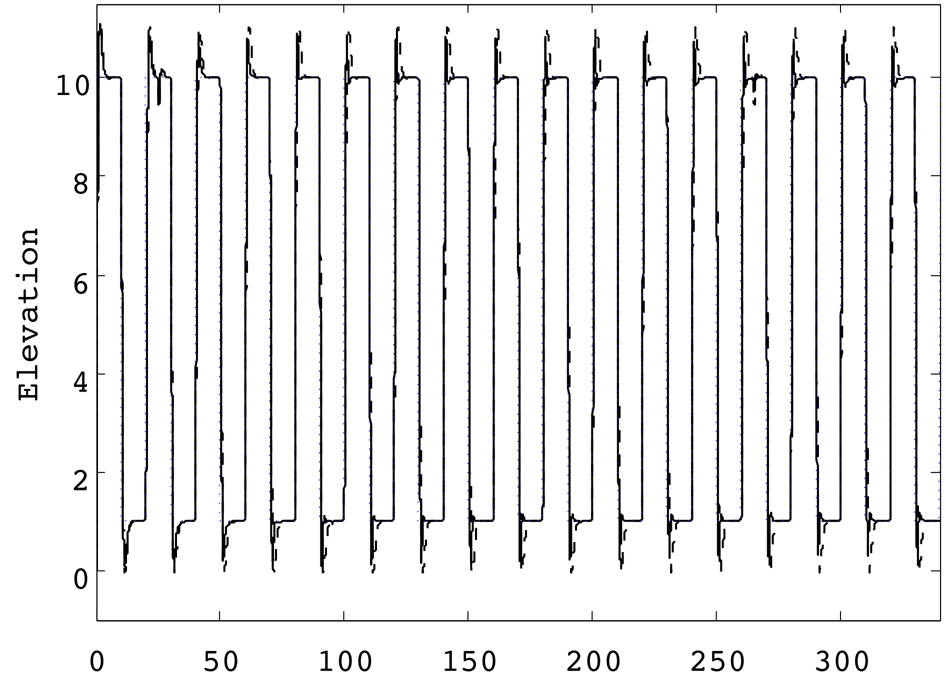


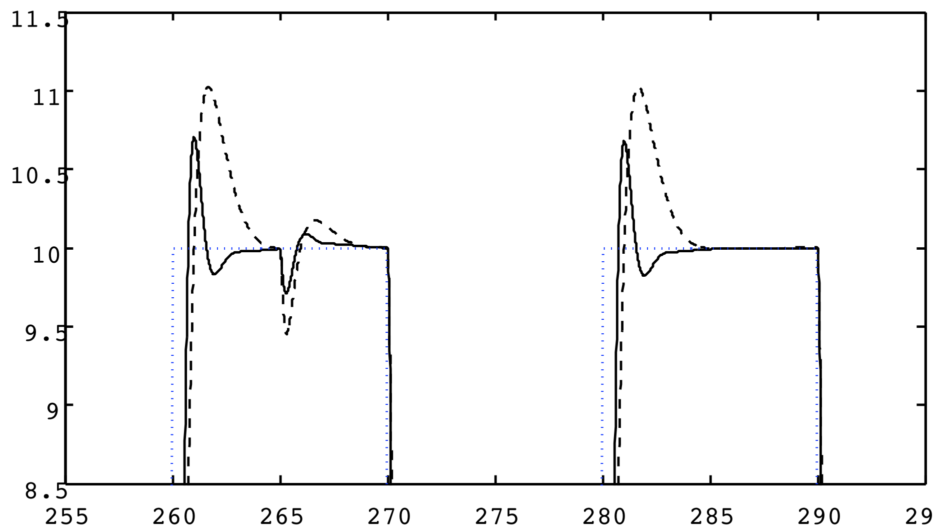
- All states measured: elevation ϵ, pitch θ, travel λ and their derivatives
- Two inputs: drag forces due to the propellers
- Non linear model:
  - linearized at operating point: allows to design an initial PID controller (state-feddback problem)
  - non-linearities taken into account through an uncertain linear model

Robust adaptive PID control design applied

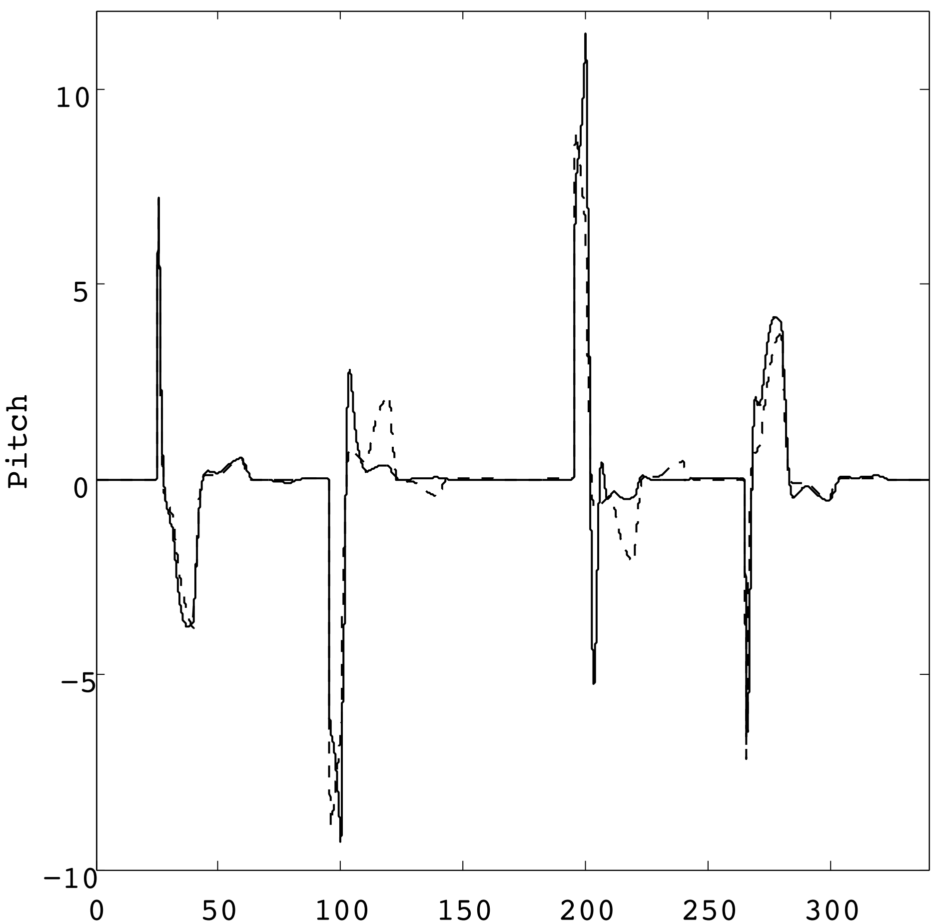
- Simulation results: travel and elevation (dotted: PID - solid: Adaptive)

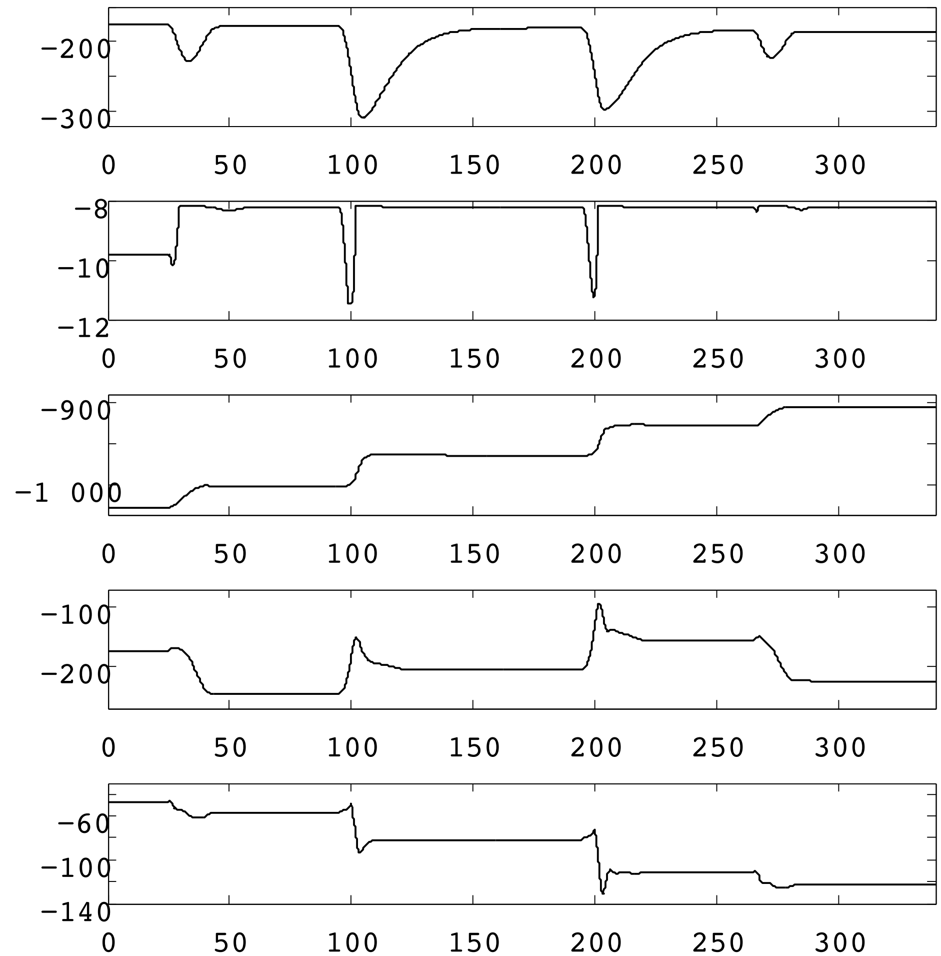






Simulation results: pitch and Adaptive PID gains





Conclusions

- LMI-based method that guarantees robust stability of Adaptive control
  - Applies to any stabilizable LTI MIMO system
  - Allows to keep some structure such as PID
  - Adaptive gains are bounded and remain close to initial given values

- Prospectives
  - More structured control (decentralized etc.)
  - Guaranteed robustness for time-varying uncertainties
  - Take advantage of flexibilities on G for engineering issues (saturations...)
  - Apply to the actual helicopter benchmark