Robust Analysis using RoMulOC

for the Longitudinal Control of a Civil Aircraft

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Introduction

- Test robust analysis tools on aerospace industrial application
  - LMIs for parameter-dependent Lyapunov functions results
  - Two type of results based on two different uncertain models
  - Stability and performances (pole location, H_∞, H_2, impulse-to-peak)
- RoMulOC
  - Tests performed using the RoMulOC toolbox
  - LMIs in YALMIP format, solved using SeDuMi and SDPT3
  - Indications on the numerical performances of the toolbox
- Aircraft motion in the vertical plane (longitudinal)
  - LTI uncertain modeling of the non-linear aircraft and the control
  - Models that cover the flight envelope

Outline

1. Uncertain modeling
2. LMIs for parameter-dependent Lyapunov functions results
3. RoMulOC toolbox
4. Numerical results
5. Conclusions

- Aircraft motion in the vertical plane (longitudinal)

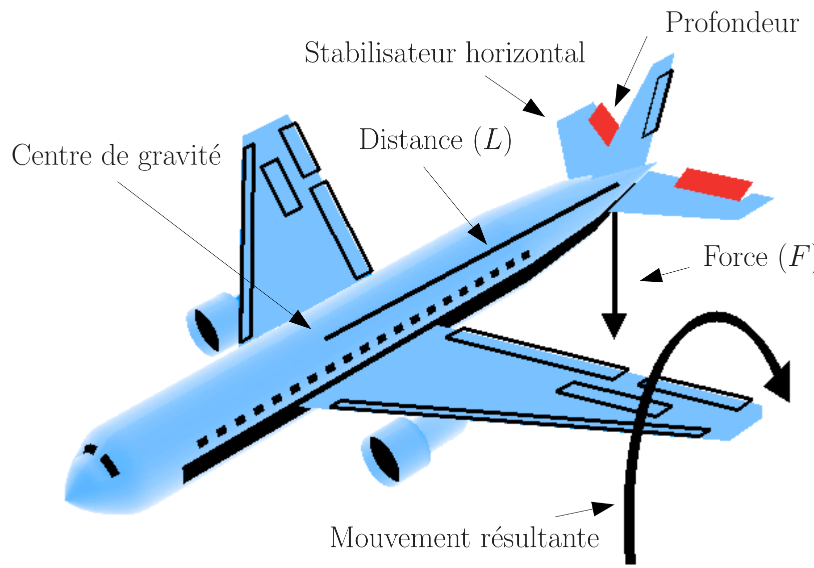
Actuators: elevators

Dynamics: angle of attack + pitch rate

Sensors: modeled as first order

Control: gain scheduled dynamic

Closed-loop system of order 9

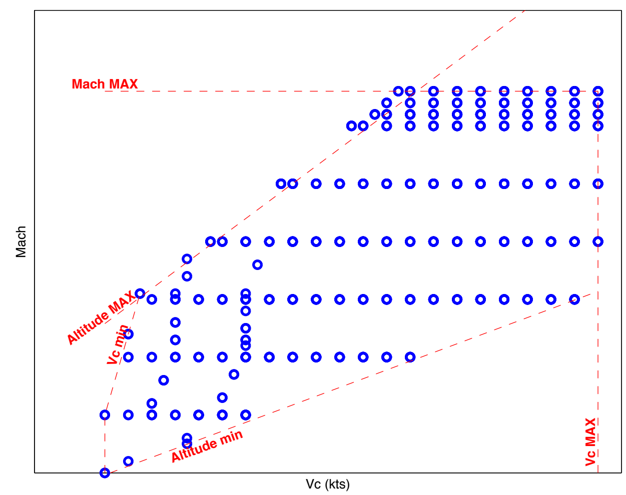


- Non-linear model + controller are linearized at 633 flight configurations

6 parameters:

weight, balance, speed,

Mach nb, altitude, motor thrust.



➊ Uncertain modeling

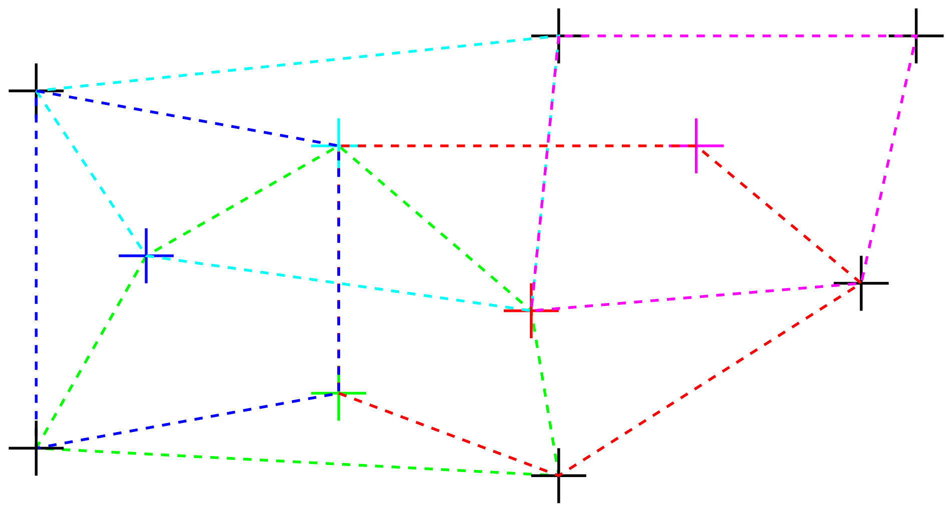
- Analysis of each 633 LTI models

  gives small information on robustness for the total flight envelope

- LFT model can be build to have a parameter-dependent LTI representation

  of the whole flight envelope: uncertainty blocs of size 150!

- Adopted strategy: build uncertain models valid around each flight configuration
- Union of local uncertain models covers the flight envelope



- Robust analysis gives upper bounds on performances achievable locally

- Adopted strategy: build uncertain models valid around each flight configuration
- For a given flight configuration θ_i

  algorithm gives its neighbors in parametric space $\theta_{j\in N(i)}$.
- Heuristic algorithm combines

  Euclidian distance in the 6D space θ + search along parametric directions.
- Tuned to provide 8 to 12 neighbors with a mean value of 11.19.
- Uncertain model around θ_i is defined as the convex hull of models at $\theta_{j\in N(i)}$

$$\begin{pmatrix} \dot{x} \\ z \end{pmatrix} = \begin{bmatrix} A_i(\zeta) & B_i(\zeta) \\ C_i(\zeta) & A_i(\zeta) \end{bmatrix} \begin{pmatrix} x \\ w \end{pmatrix} = \sum_{j\in N(i)} \zeta_j \begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} \begin{pmatrix} x \\ w \end{pmatrix}$$

$$: \quad \sum \zeta_j = 1 \ , \ \ \zeta_j \geq 0$$

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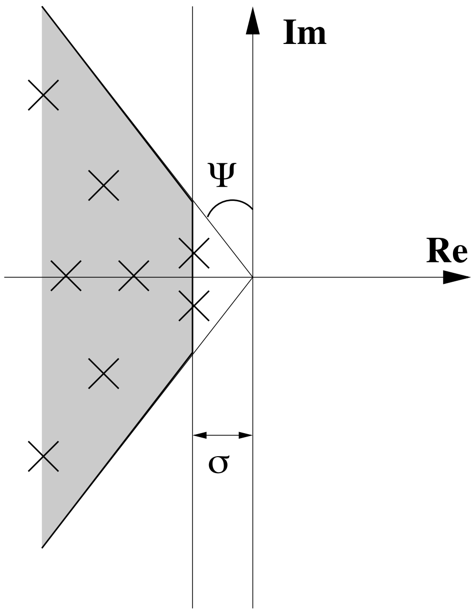
- Each uncertain model is also converted in LFT form

$$\begin{pmatrix} \dot{x} \\ z_\Delta \\ z \end{pmatrix} = \begin{bmatrix} A_i & B_{\Delta i} & B_i \\ C_{\Delta i} & \mathbf{0} & D_{\Delta w i} \\ C_i & D_{z\Delta i} & D_{\Delta i} \end{bmatrix} \begin{pmatrix} x \\ w_\Delta \\ w \end{pmatrix} \ , \ \ w_\Delta = \sum_{j\in N(i)} \zeta_j \Delta^{[j]} z_\Delta$$

$$: \quad \sum \zeta_j = 1 \ , \ \ \zeta_j \geq 0$$

■ Performances to be tested

- Stability
- Pole location



- H_2 norm - measure of control effort due to noise

(w additive noise on measurements, $z = u$ control signal)

- H_∞ norm - stability margin w.r.t. dynamic uncertainty

(w additive signal on control u, $z = y$ measurements)

- Impulse-to-peak - control peak to initial conditions

(w impulse on state vector, $z = u$ control signal)

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■ 2 results for polytopic models

● 'Quadratic stability' - $V(x) = x^T P x$ independent of uncertain parameters

$$A^{[j]T} P + P A^{[j]} < \mathbf{0} \ , \quad P > \mathbf{0}$$

● Polytopic PDLF - $V(x) = x^T \left(\sum \zeta_j P^{[j]} \right) x$

'Slack variable' approach [SCL 00]

$$\begin{bmatrix} \mathbf{0} & P^{[j]} \\ P^{[j]} & \mathbf{0} \end{bmatrix} < F \begin{bmatrix} A^{[j]} & -\mathbf{1} \end{bmatrix} + \begin{bmatrix} A^{[j]T} \\ -\mathbf{1} \end{bmatrix} F^T \ , \quad P^{[j]} > \mathbf{0}$$

- 1 result for LFT models

- Quadratic PDLF - $V(x) = x^T \begin{bmatrix} \mathbf{1} & \Delta^T \end{bmatrix} \hat{P} \begin{bmatrix} \mathbf{1} \\ \Delta \end{bmatrix} x, \quad \Delta = \sum \zeta_j \Delta^{[j]}$

'Quadratic separation' approach [Iwasaki 01]

$$\mathcal{L}(\hat{P}, \Theta) < \mathbf{0} \ , \quad \begin{bmatrix} \mathbf{1} & \Delta^{[j]T} \end{bmatrix} \Theta \begin{bmatrix} \mathbf{1} \\ \Delta^{[j]} \end{bmatrix} \leq \mathbf{0} \ , \quad \hat{P} > \mathbf{0}$$

- Results of all three methods are extended to deal with the performance criteria (pole location, H_2, H_∞ and impulse-to-peak)

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❸ RoMulOC toolbox

- Robust Multi-Objective Control toolbox
- Freely distributed at `www.laas.fr/OLOCEP/romuloc`
- Includes uncertain modeling features

```
>> usys_h2
Uncertain model : polytope 11 vertices
               n=9    mw=2    mu=1
 n=9    dx  =   A*x +  Bw*w +  Bu*u
pz=1     z  =  Cz*x + Dzw*w
py=2     y  =  Cy*x         + Dyu*u
continuous time ( dx: derivative )
```

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```
>> usys_hinf
Uncertain model : LFT
-------- WITH --------
               n=9     md=6     mw=1     mu=1
 n=9     dx  =   A*x +  Bd*wd +  Bw*w +  Bu*u
pd=7     zd  =  Cd*x          + Ddw*w + Ddu*u
pz=3      z  =  Cz*x + Dzd*wd + Dzw*w
py=2      y  =  Cy*x
continuous time ( dx : derivative operator )
-------- AND  --------
 wd = #1 * zd
index   size    constraint                       name
#1      6x7     polytope 11 vertices    real
```

- Robust Multi-Objective Control toolbox
- Freely distributed at `www.laas.fr/OLOCEP/romuloc`
- LMI formulas pre-coded - easy to generate

```
quiz = ctrpb('a',LyapType)+ h2(usys_h2)
```

`LyapType` defines the method to be applied

`h2` or `stability`, `dstability`, `hinfty`, `i2p`: performance to test

`quiz` contains the LMI constraints and variables in YALMIP format

- Solve the LMI problem with any solver

```
result = solvesdp(quiz, sdpsettings(...))
```

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Table 1: LMI sizes and times for stability tests

	No. of vars	No. of rows	Mean time
quad-poly	45	110	0.25s
PDLF-poly	676	215	0.93s
PDLF-LFT	456	221	1.08s

Table 2: Results for settling time criterion

	$\sigma_{\%}$	Mean time per LMIs	Mean nb iter
quad-poly	15.27%	0.35s	7.29
PDLF-poly	2.38%	1.35s	1.95
PDLF-LFT	2.38%	1.45s	1.96

- Robust upper bound on σ optimized by bisection (iterative LMI algorithm)
- $\sigma_{\%}$: Gap between robust upper bound and worst case on vertices

Table 3: Results for damping criterion

	$\psi_{\%}$	Mean time per LMIs	Mean nb iter
quad-poly	11.40%	0.46s	6.45
PDLF-poly	1.44%	1.76s	1.25
PDLF-LFT	1.62%	1.52s	1.75

Table 4: Damping criterion for two particular flight points

		$\psi^*(i)$		
i	$\psi_m(i)$	quad-poly	PDLF-poly	PDLF-LFT
15	0.7286	0.5408	0.7213	0.6650
517	0.4978	0.4200	0.4735	0.4766

Table 5: Results for robust $\mathcal{H}_\infty$ cost

	$\gamma_\infty\%$	Mean time	Less conservative
quad-poly	39.64%	0.55s	
PDLF-poly	0.19%	2.38s	52
PDLF-LFT	0.26%	9.04s	2

Table 6: Results for robust impulse-to-peak criterion

	$\gamma_{i2p}\%$	Mean time	Less conservative
quad-poly	43.59%	0.81s	
PDLF-poly	27.98%	2.66s	500
PDLF-LFT	30.16%	6.39s	0

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❺ Conclusions

- Parameter-dependent Lyapunov type results tested on an industrial application
  - Overall test over 633 points takes 3 hours on a PC

    (negligible compared to Monte Carlo tests on high order non-linear model)
  - May be used at the control design phase to pre-validate (or not) a control law
  - Gives information on robust stability and performances

    Can be used to retune LPV controllers in regions of the flight domain.
- PDLF results show very low conservatism
  - PDLF-Poly always better than PDLF-LFT (can it be proved?)
  - No severe numerical problem - Validates the coding of LMIs in RoMulOC
  - `www.laas.fr/OLOCEP/romuloc`