

Robust analysis in a quadratic separation framework and application to Demeter satellite attitude control system

Denis Arzelier, Alberto Bortott, Frédéric Gouaisbaut, Dimitri Peaucelle



LAAS-CNRS

The logo for LAAS-CNRS features the text "LAAS-CNRS" in a bold, blue, sans-serif font. It is framed by a thick magenta horizontal bar above and a thick yellow horizontal bar below.

Christelle Pittet, Catherine Charbonnel



CCT SCA & MOSAR - Toulouse - October 14th, 2009

- Results are part of a joint project involving
CNES, LAAS-CNRS and Thales Alenia Space
- Flexible satellite attitude control benchmark Demeter developed at CNES
- Robust analysis methodology developed at LAAS-CNRS
and coded in a Matlab toolbox : RoMuIOC
`www.laas.fr/OLOCEP/romuIOC`
- Dedicated codes for Demeter and tests realized at LAAS-CNRS
- Further software developments done at Thales Alenia Space
- Large scale tests done at CNES and Thales Alenia Space

- ① Demeter satellite
- ② Integral Quadratic Separation (IQS)
- ③ Heuristic algorithm for optimization of stable domains
- ④ Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
- ⑤ Conclusions

■ M-C-K uncertain model

$$M(\delta_J) \begin{pmatrix} \delta\ddot{\theta} \\ \dot{\eta} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & C_S^-(\delta_\omega, \delta_\xi) \end{bmatrix} \begin{pmatrix} \delta\dot{\theta} \\ \dot{\eta} \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_S^-(\delta_\omega) \end{bmatrix} \begin{pmatrix} \delta\theta \\ \eta \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U$$

▲ $\theta, \dot{\theta}$: satellite orientation (3D)

▲ $\eta, \dot{\eta}$: states of the flexible modes (up to 4 for each axis)

▲ δ_J : 6 scalar uncertainties on the inertia

▲ $\delta_\omega, \delta_\xi$: scalar uncertainties on the frequency and damping of flexible modes

● State space model: $\dot{X} = A(\delta)X + B(\delta)U$

$A(\delta), B(\delta)$ are rational w.r.t. uncertainties.

■ LFT model

$$\begin{aligned}\dot{X} &= AX + B_{\Delta}w_{\Delta} + B_u u \\ z_{\Delta} &= C_{\Delta}X + D_{\Delta\Delta}w_{\Delta} + D_{\Delta u}u \quad , \quad w_{\Delta} = \Delta z_{\Delta} \\ y &= C_y X + D_{y\Delta}w_{\Delta} + D_{yu}u\end{aligned}$$

- ▲ Δ : diagonal matrix with δ_J , δ_{ω} and δ_{ξ} elements
- ▲ Some elements δ are repeated
- ▲ The problem is normalized: $\delta \in [-1 \ 1]$
- Modeling is made possible in the following toolboxes

Control (Matlab©) LFR (J.F. Magni) RoMuLOC (LAAS)

1 Demeter satellite

```
>> Ax = [1]; Fm = 1; model_type = 2;
>> usys = demeter2romuloc(Ax,Fm,model_type)
Uncertain model : LFT
----- WITH -----
                n=4      md=5      mu=1
n=4      dx  =  A*x +  Bd*wd +  Bu*u
pd=5      zd  =  Cd*x + Ddd*wd + Ddu*u
py=1      y   =  Cy*x
continuous time ( dx : derivative operator )
----- AND -----
diagonal structured uncertainty
size: 5x5 | nb blocks: 5 | independent blocks: 3
wd = diag( #1 #1 #2 #2 #3 ) * zd
index      size      constraint
#1         1x1       interval 1 param      real      dJ11
#2         1x1       interval 1 param      real      dW1
#3         1x1       interval 1 param      real      dX1
```

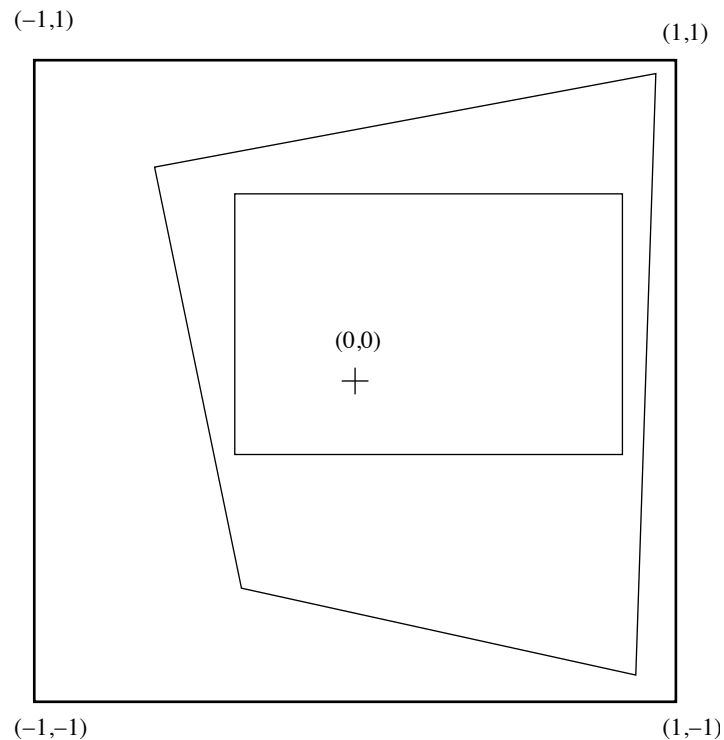
1 Demeter satellite

● RoMulOC allows also polytopic models

size: 5x5 | nb blocks: 1 | independent blocks: 1

```
wd = diag( #4 ) * zd
```

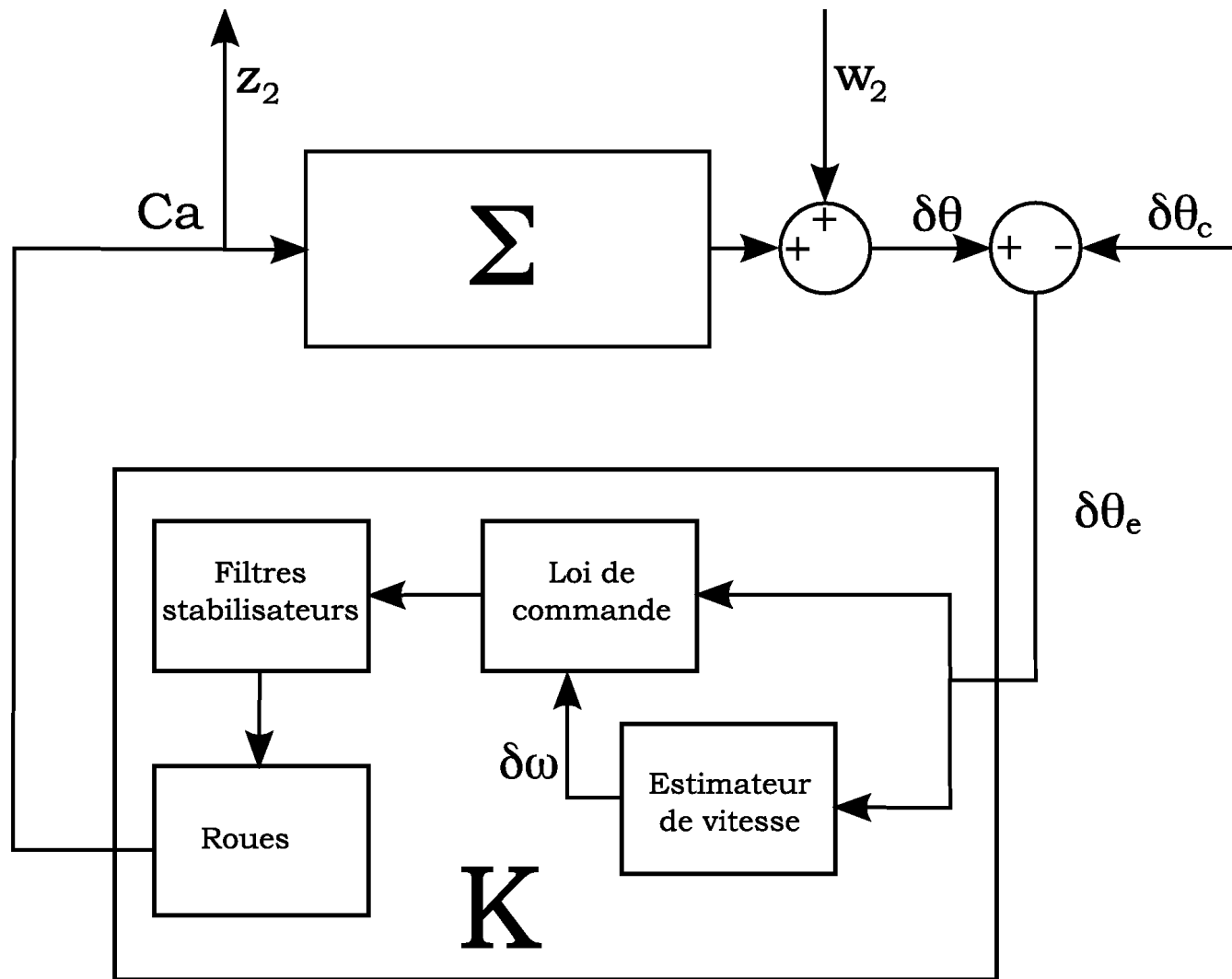
index	size	constraint	name
#4	5x5	polytope 8 vertices	real dJ11, dW1, dX1



▲ Aim: Guaranteed closed-loop robust stability for the "biggest" polytope

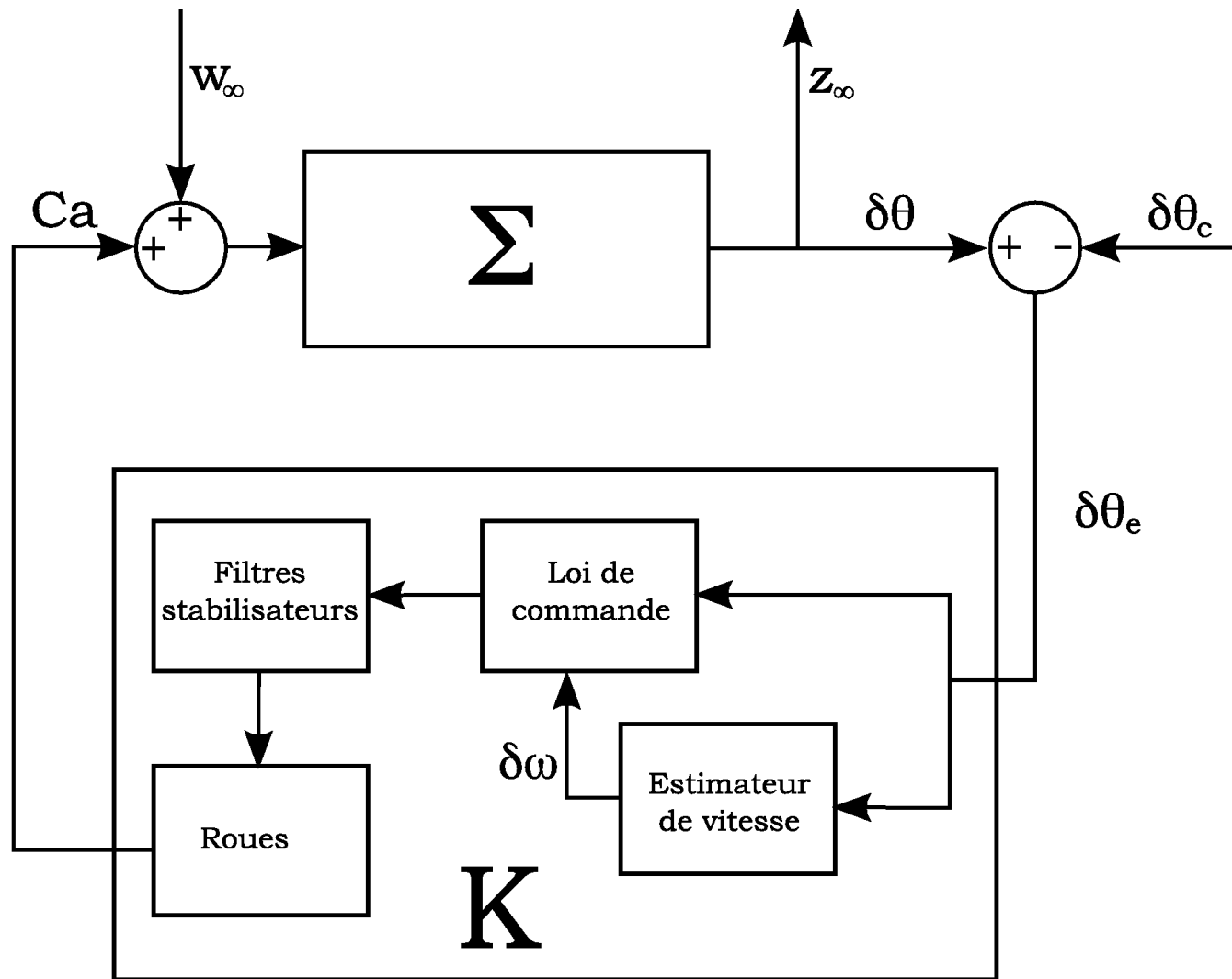
1 Demeter satellite

- ▲ Secondary problem: Robust guaranteed H_2 norm (consumption)



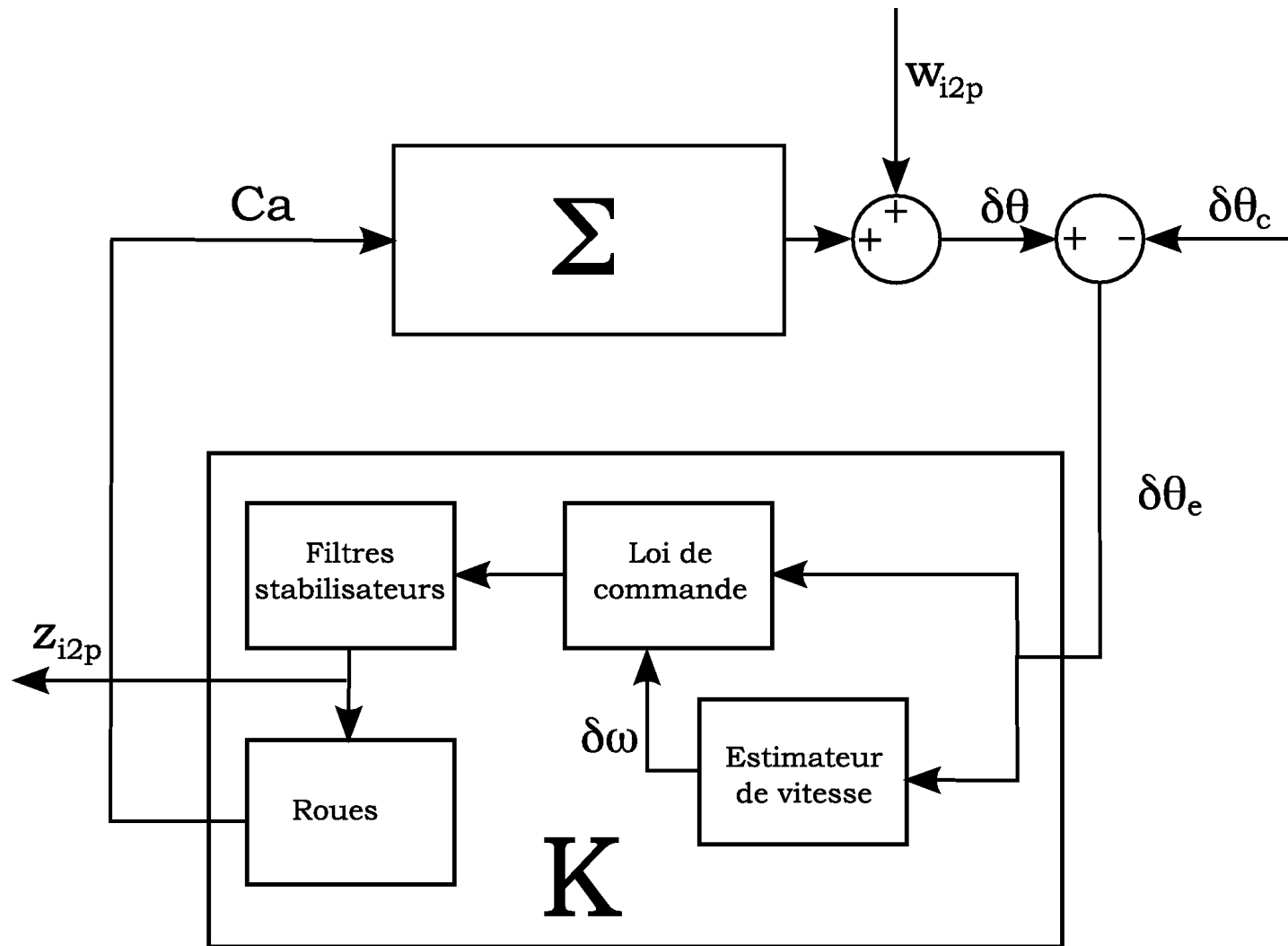
1 Demeter satellite

- ▲ Secondary problem: Robust guaranteed H_∞ norm
(robustness to unmodeled dynamics)

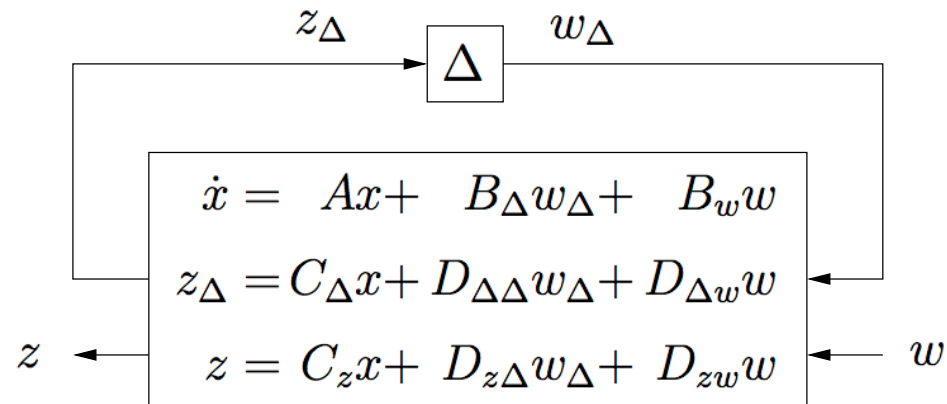


1 Demeter satellite

- ▲ Secondary problem: Robust guaranteed impulse-to-peak performance (control input saturation w.r.t. to initial depointing)



1 Demeter satellite



Uncertain model : closed-loop satellite with performances

----- WITH -----

n=4 md=5 mw=1

$$n=4 \quad dx = A*x + Bd*wd + Bw*w$$

$$pd=5 \quad zd = Cd*x + Ddd*wd + Ddw*w$$

$$pz=1 \quad z = Cz*x$$

continuous time (dx : derivative operator)

----- AND -----

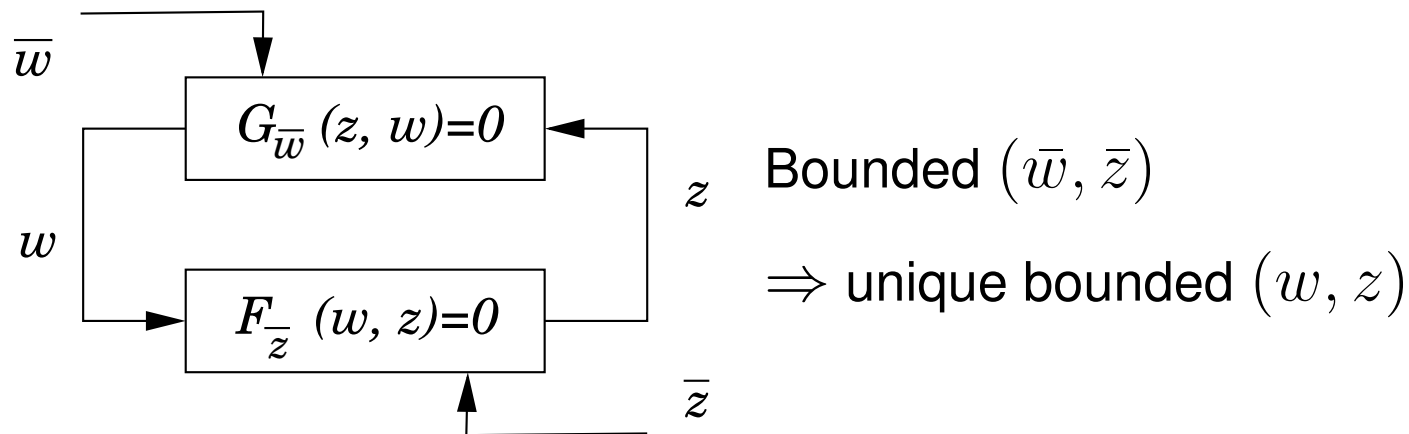
$$wd = \text{diag}(\#4) * zd$$

index	size	constraint	name
#4	5x5	polytope 8 vertices	real dJ11, dW1, dX1

- ① Demeter satellite
- ② Integral Quadratic Separation (IQS)
- ③ Heuristic algorithm for optimization of stable domains
- ④ Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
- ⑤ Conclusions

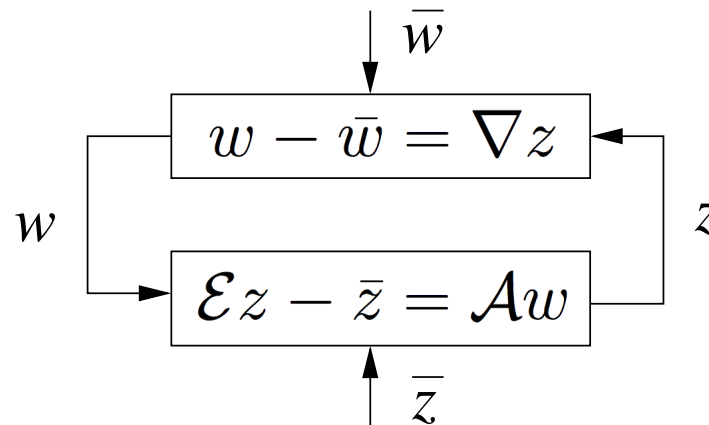
② Integral Quadratic Separation (IQS)

Well-posedness



Considered case

- Linear implicit application: \mathcal{E} and \mathcal{A} are matrices (possibly not square)
- $\nabla \in \mathbb{W}$ is bloc-diagonal. Contains scalar, full-bloc, LTI and LTV uncertainties



- For $\mathcal{E} = 1$ and $\mathcal{A} = H(j\omega)$ one recovers IQC framework

② Integral Quadratic Separation (IQS)

- For dynamic systems $\dot{x} = Ax$: well posedness \equiv internal stability

$$\overbrace{z(t) = Aw(t) + \bar{z}(t)}^F, \quad \overbrace{w(t) = \int_0^t z(\tau) d\tau + \bar{w}(t)}^G$$

$\underbrace{w(t)}_{x(t)} \quad \underbrace{z(\tau)}_{\dot{x}(t)} \quad \underbrace{\bar{w}(t)}_{x(0)}$

▲ \bar{w} contains information on initial conditions

● Well-posedness

$$\Leftrightarrow \forall (\bar{w}, \bar{z}) \in L_2, \exists!(w, z) \in L_2 : \left\| \begin{array}{c} w \\ z \end{array} \right\| \leq \gamma \left\| \begin{array}{c} \bar{w} \\ \bar{z} \end{array} \right\|$$

\Rightarrow for zero initial conditions and zero perturbations:

$w = z = 0$ is the unique solution (equilibrium point).

\Rightarrow whatever bounded perturbations the state remains close to equilibrium
(global stability)

2 Integral Quadratic Separation (IQS)

■ Integral Quadratic Separation [Automatica'08, CDC'07, ROCOND'09, ECC'09]

● For the case of linear application with uncertain operator

$$\mathcal{E}z(t) = \mathcal{A}w(t) \quad , \quad w(t) = [\nabla z](t) \quad \nabla \in \mathbb{W}$$

where $\mathcal{E} = \mathcal{E}_1 \mathcal{E}_2$ with \mathcal{E}_1 full column rank,

● Integral Quadratic Separator (IQS) : $\exists \Theta$, matrix, solution of LMI

$$\begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp*} \Theta \begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp} > 0$$

and Integral Quadratic Constraint (IQC) $\forall \nabla \in \mathbb{W}$

$$\int_0^{\infty} \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix}^* \Theta \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix} dt \leq 0$$

- For some given ∇ , \exists LMI conditions for Θ solution to IQC

$$\int_0^{\infty} \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix}^* \Theta \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix} dt \leq 0$$

- ▲ Θ is build out of IQS for elementary blocs of ∇
- ▲ Improved DG -scalings, full-bloc S-procedure, vertex separators...
- ▲ Building Θ and related LMIs is tedious but can be automatized (RoMuloC)
- ▲ It is conservative except in few special cases [Meinsma et al., 1997].

■ Robust analysis in IQS framework:

- 1- Write the robust analysis problem as a well-posedness problem

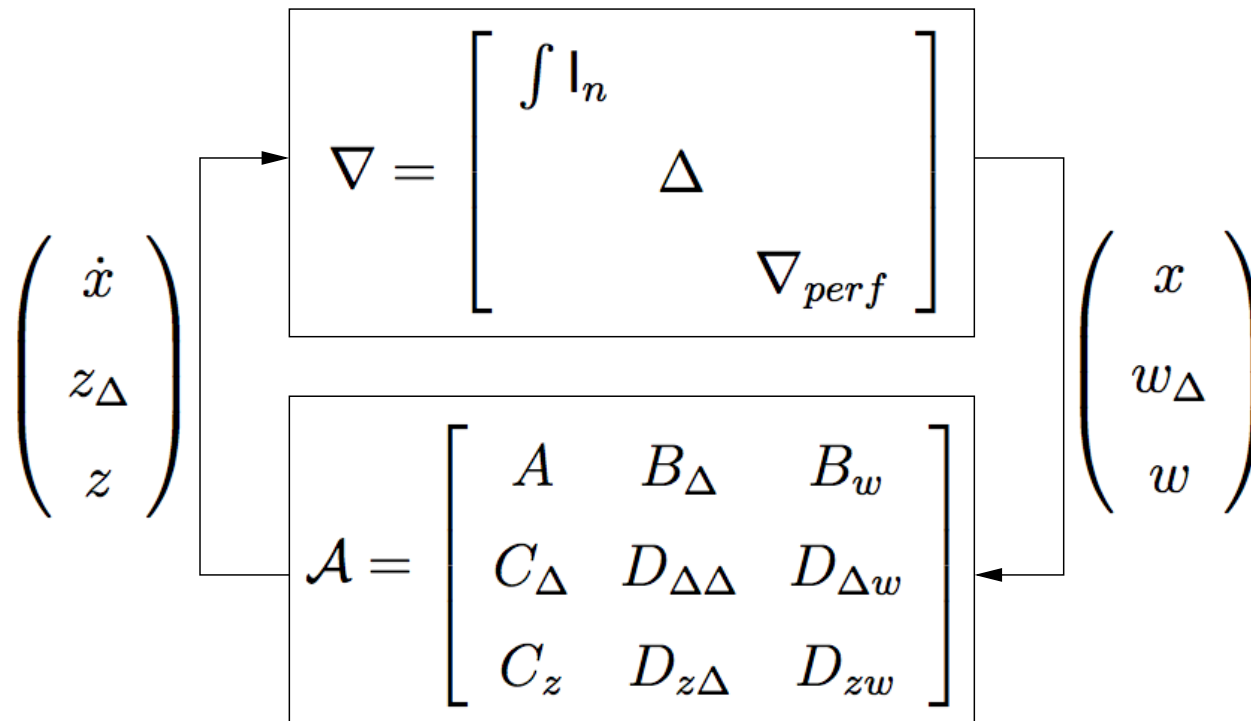
$$\mathcal{E}z = Aw, \quad w = \nabla z$$

- 2- Build Integral Quadratic Separators for each elementary bloc of ∇
- 3- Apply the IQS results to get (conservative) LMIs

② Integral Quadratic Separation (IQS)

■ Demeter analysis problems:

● Well-posedness of



▲ $\int \mathbf{1}_n$ integrator

▲ Δ matrix of uncertainties

▲ ∇_{perf} operator related to performances

② Integral Quadratic Separation (IQS)

■ Other problem modeling produce other LMI conditions

● Dual system: $z_d = \mathcal{A}^T w_d$, $w_d = \nabla^* z_d$

● System augmentation (produces systems in descriptor form)

$$\begin{cases} \dot{x} = Ax + B_{\Delta} w_{\Delta} \\ z_{\Delta} = C_{\Delta} x + D_{\Delta\Delta} w_{\Delta} \end{cases} \Rightarrow \begin{cases} \dot{x} = Ax + B_{\Delta} w_{\Delta} \\ z_{\Delta} = C_{\Delta} x + D_{\Delta\Delta} w_{\Delta} \\ \dot{z}_{\Delta} = C_{\Delta} \dot{x} + D_{\Delta\Delta} \dot{w}_{\Delta} \end{cases}$$

$$w_{\Delta} = \Delta z_{\Delta} \quad \begin{cases} w_{\Delta} = \Delta z_{\Delta} \\ \dot{w}_{\Delta} = \Delta \dot{z}_{\Delta} + \overbrace{\Delta \dot{z}_{\Delta}}^{=0} \end{cases}$$

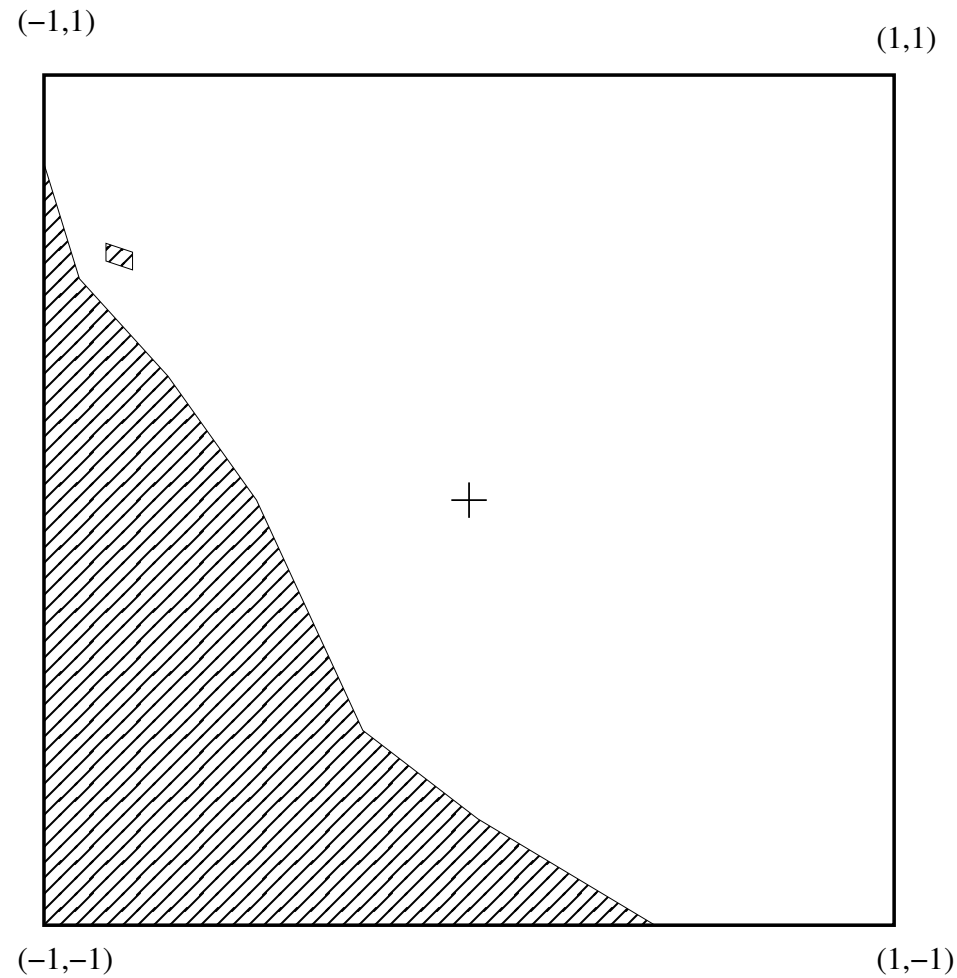
▲ Equivalent to increasing the dependency of the (implicit) Lyapunov function

$$V_0(x) = x^* P x \Rightarrow V_1(x, \Delta) = \begin{pmatrix} x^* & z_{\Delta}^* \end{pmatrix} \hat{P} \begin{pmatrix} x^* & z_{\Delta}^* \end{pmatrix}^*$$

- ① Demeter satellite
- ② Integral Quadratic Separation (IQS)
- ③ Heuristic algorithm for optimization of stable domains
- ④ Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
- ⑤ Conclusions

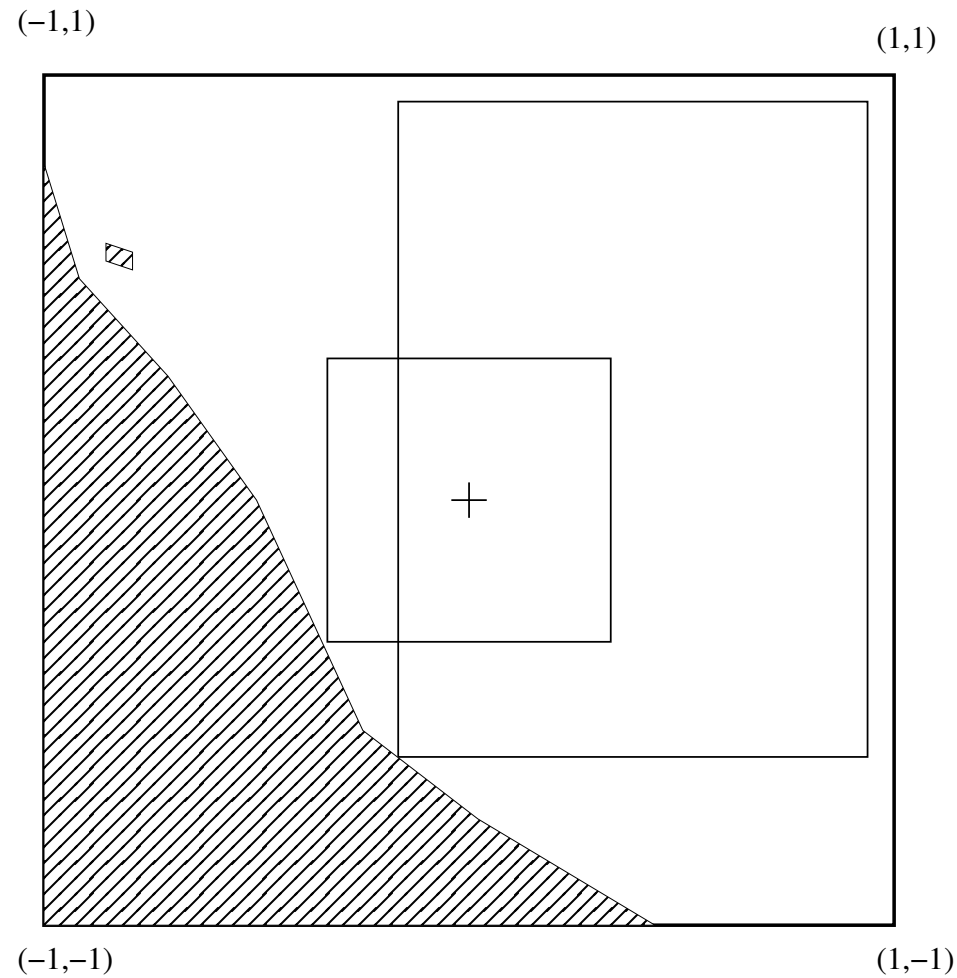
③ Heuristic algorithm for optimization of stable domains

- Values of parameters making the system stable and unstable (unknown)



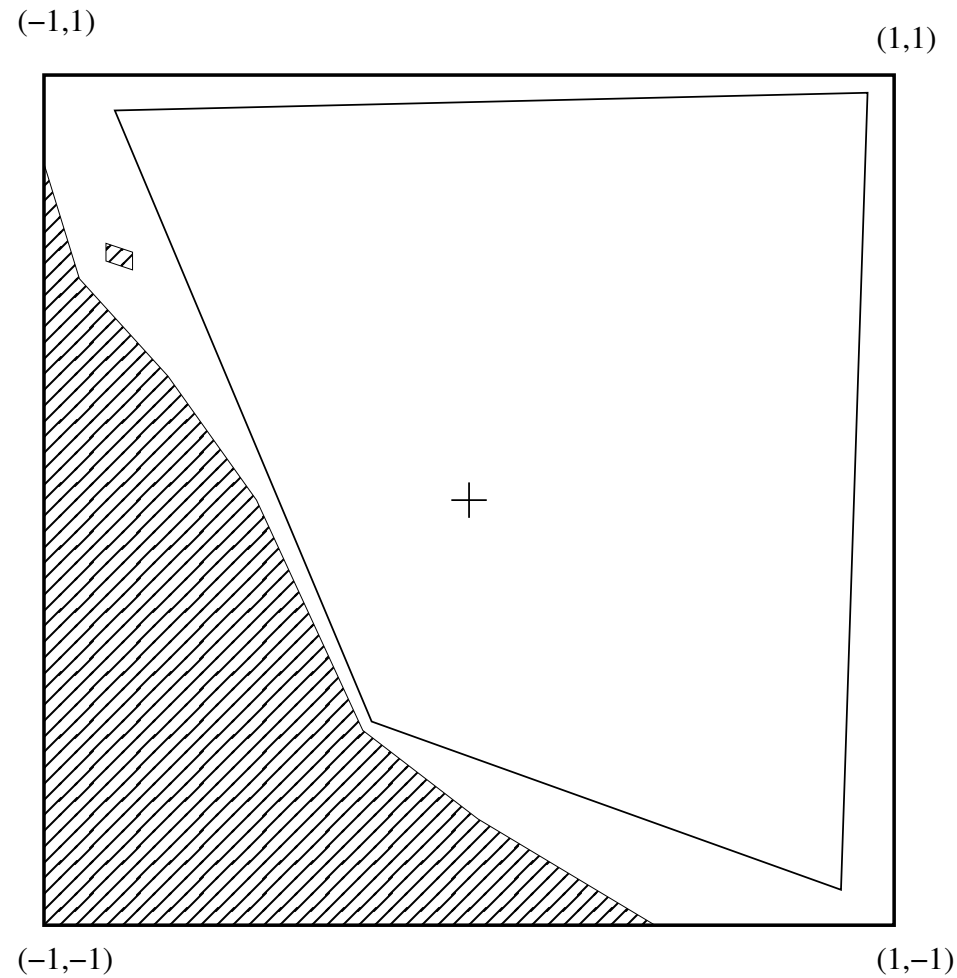
③ Heuristic algorithm for optimization of stable domains

- Biggest squares and rectangles possibly obtained



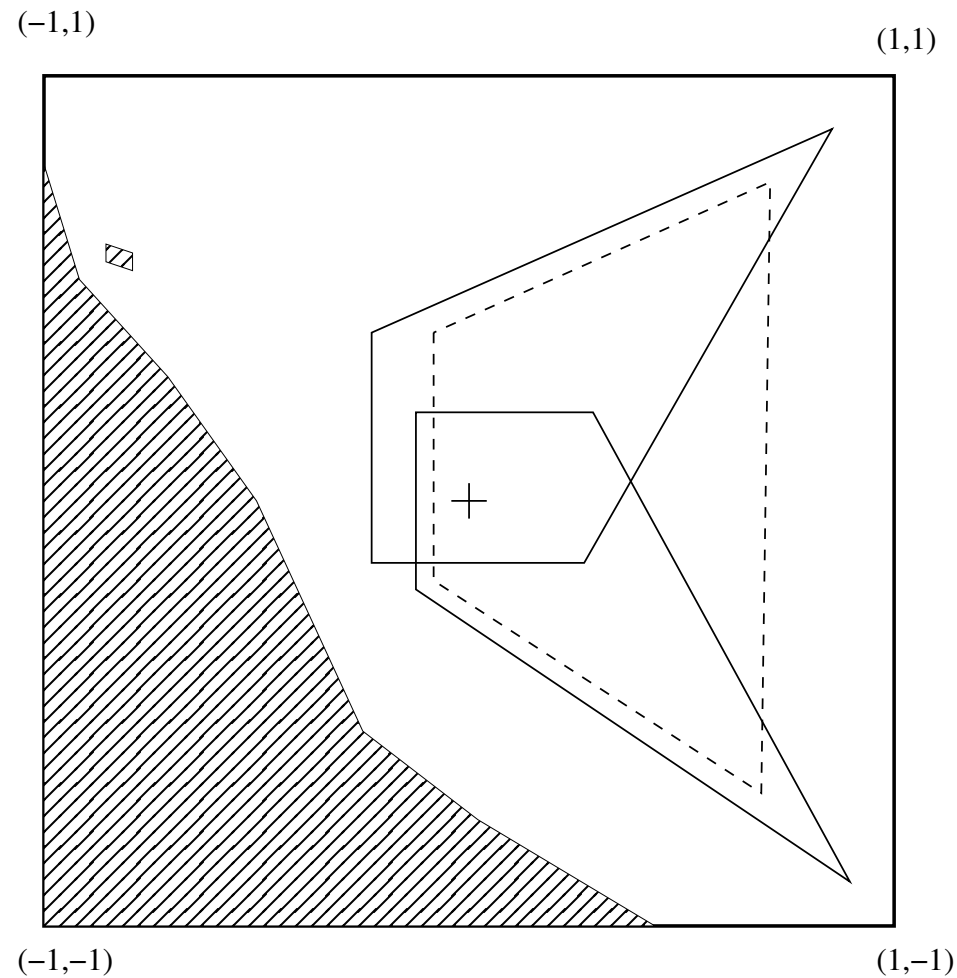
③ Heuristic algorithm for optimization of stable domains

- Biggest polytope possibly obtained (if LMIs are not conservative)



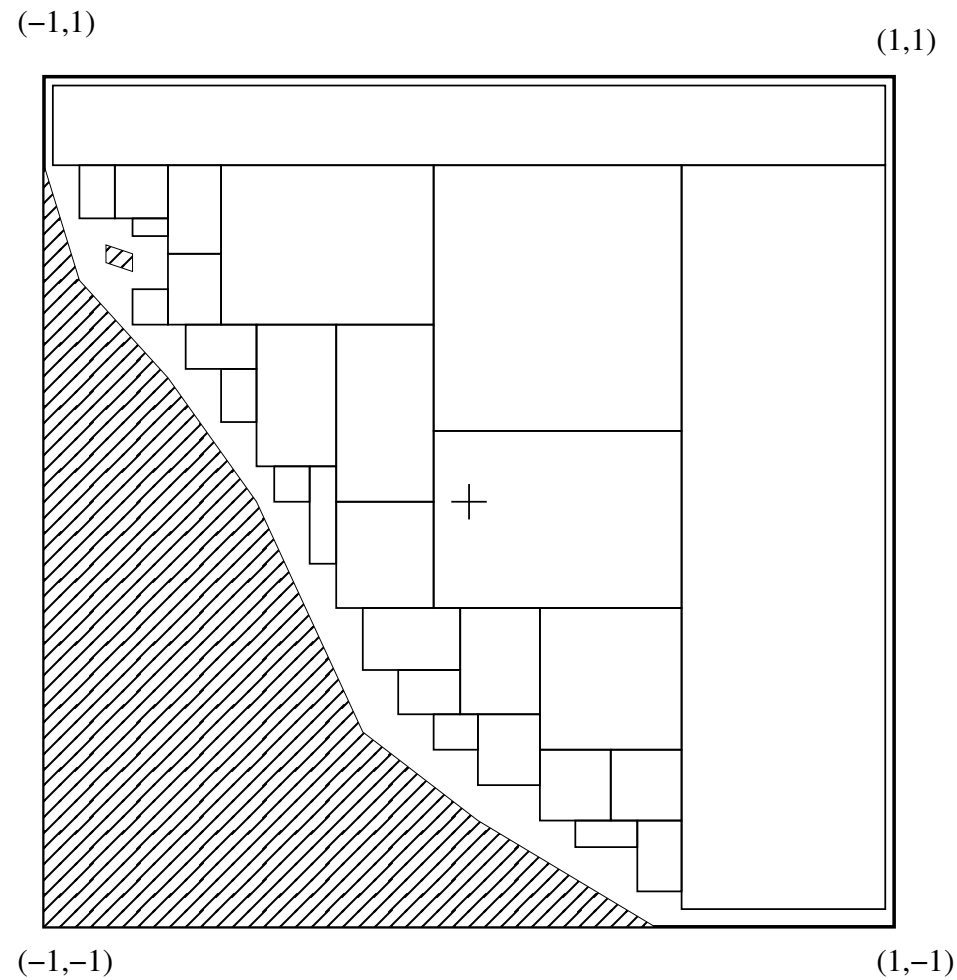
③ Heuristic algorithm for optimization of stable domains

- Due to conservatism some polytopes may give feasible LMIs (full), others not (dotted)



③ Heuristic algorithm for optimization of stable domains

- One solution: pave the feasible set



③ Heuristic algorithm for optimization of stable domains

- Proposed algorithm - Step 1
- Find by bisection a initial hyper-cube of safe parameters

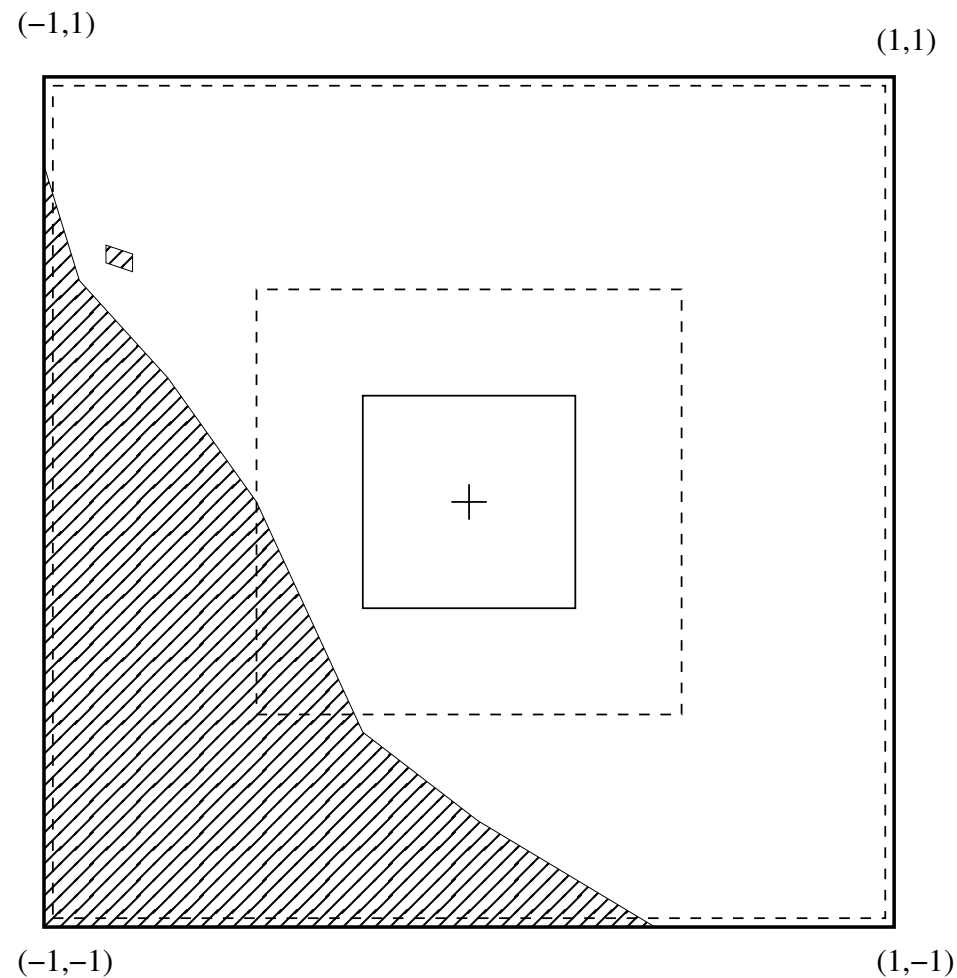
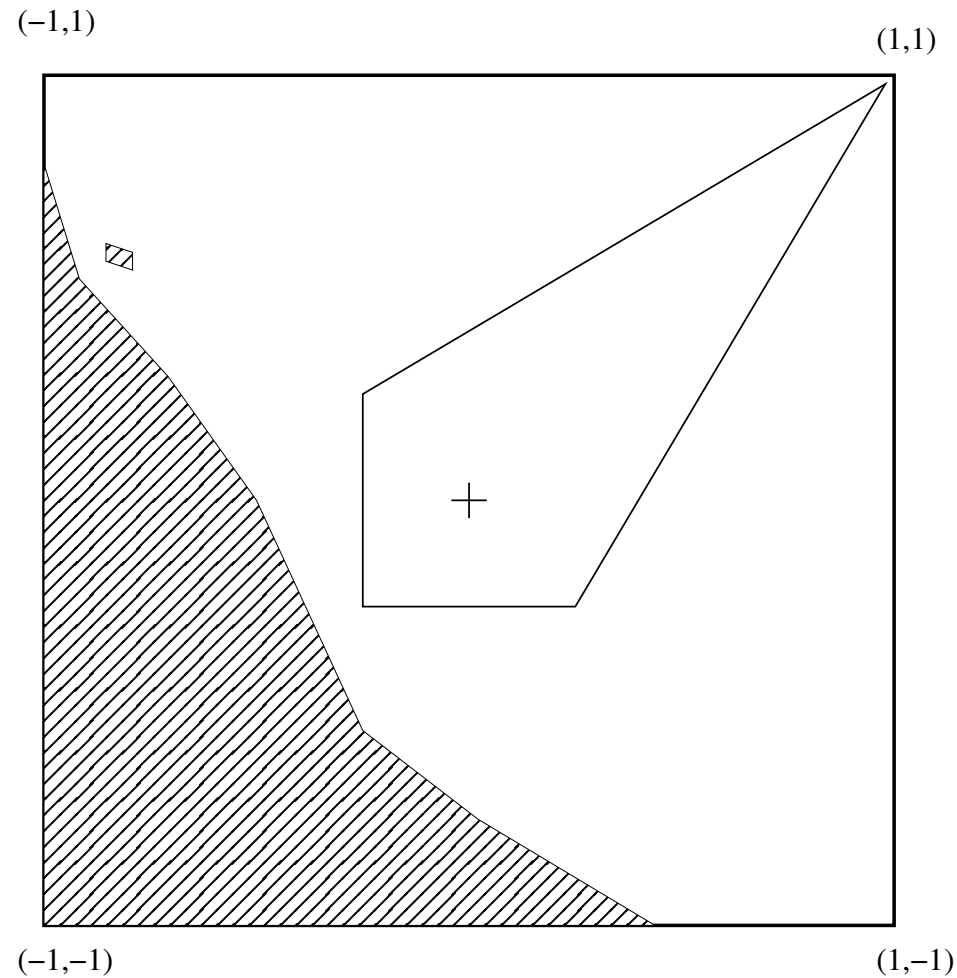


Figure 1: Recherche par bisection d'un carr faisable

③ Heuristic algorithm for optimization of stable domains

■ Proposed algorithm - Step 2

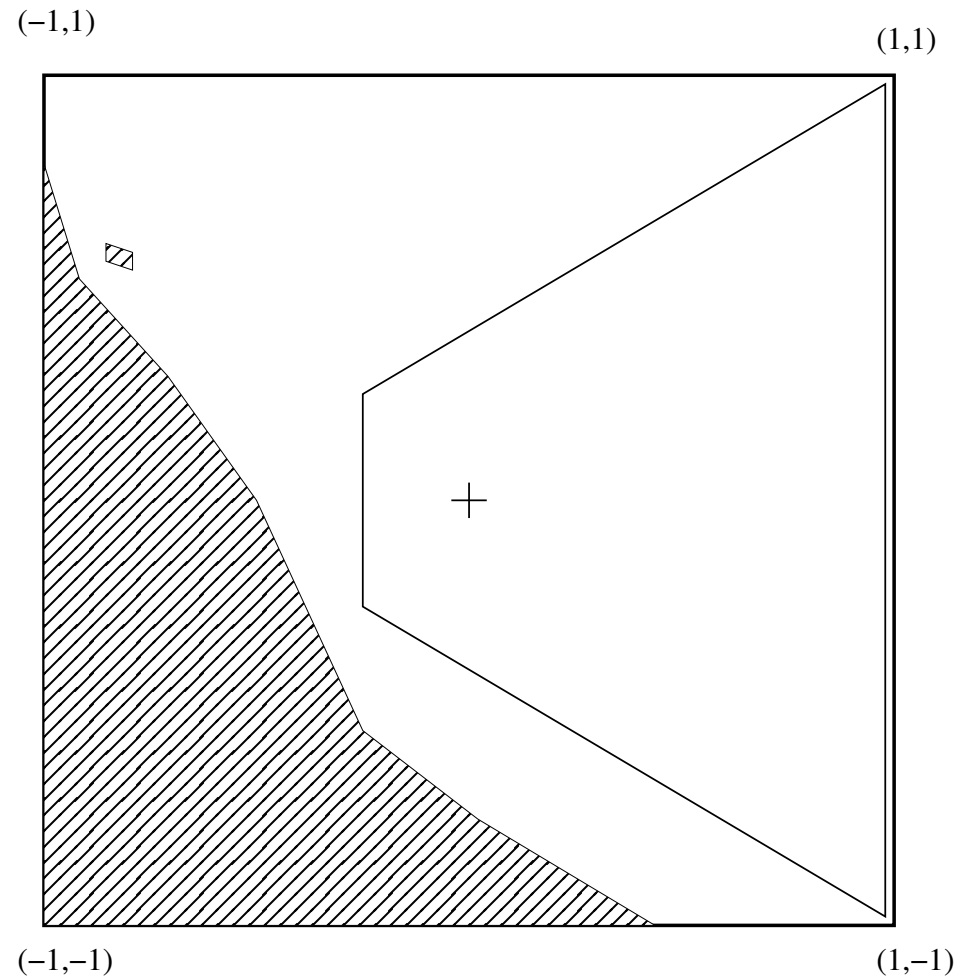
● Pull one after the other the vertices towards the corners



③ Heuristic algorithm for optimization of stable domains

■ Proposed algorithm - Step 2

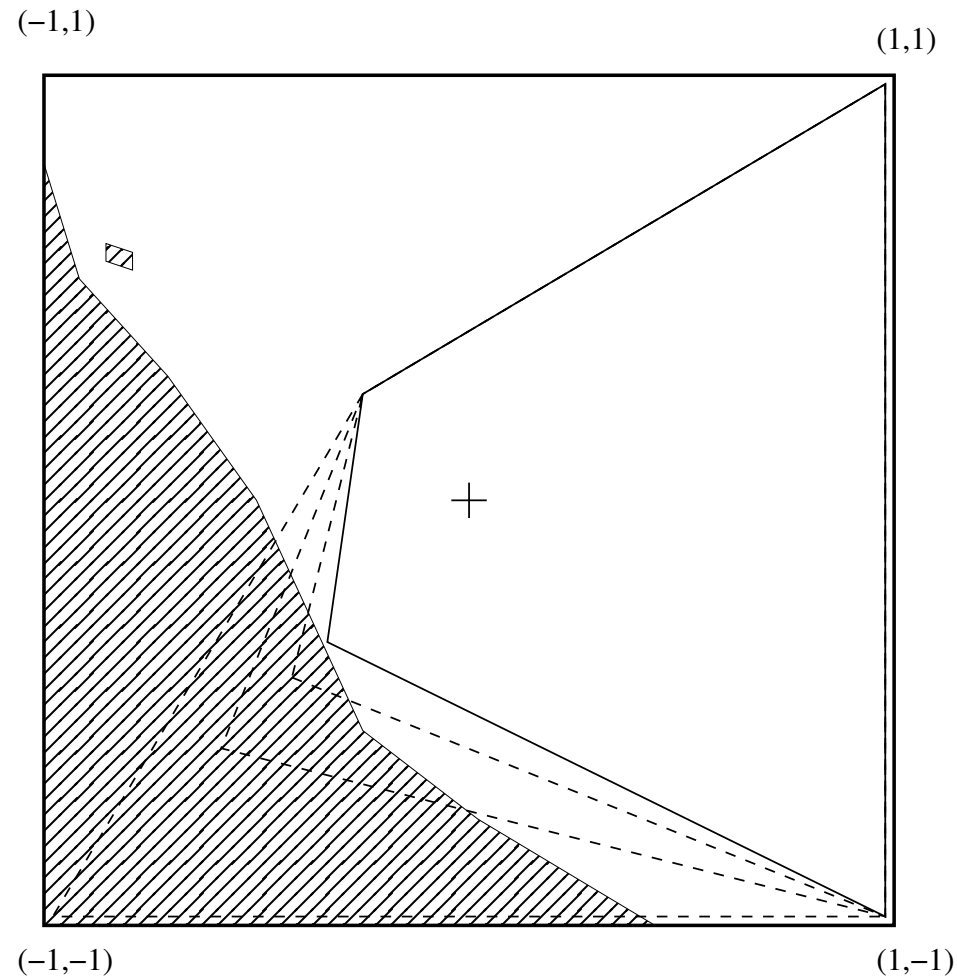
● Pull one after the other the vertices towards the corners



③ Heuristic algorithm for optimization of stable domains

■ Proposed algorithm - Step 2

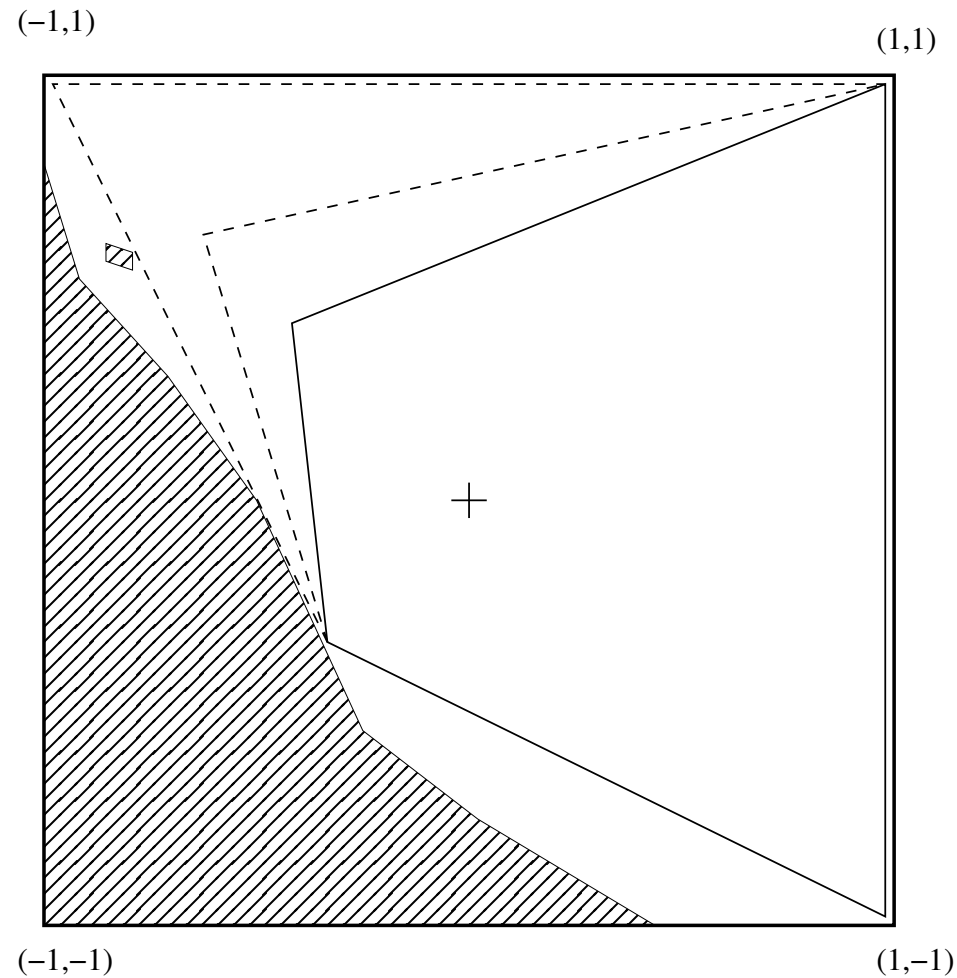
● Pull one after the other the vertices towards the corners



③ Heuristic algorithm for optimization of stable domains

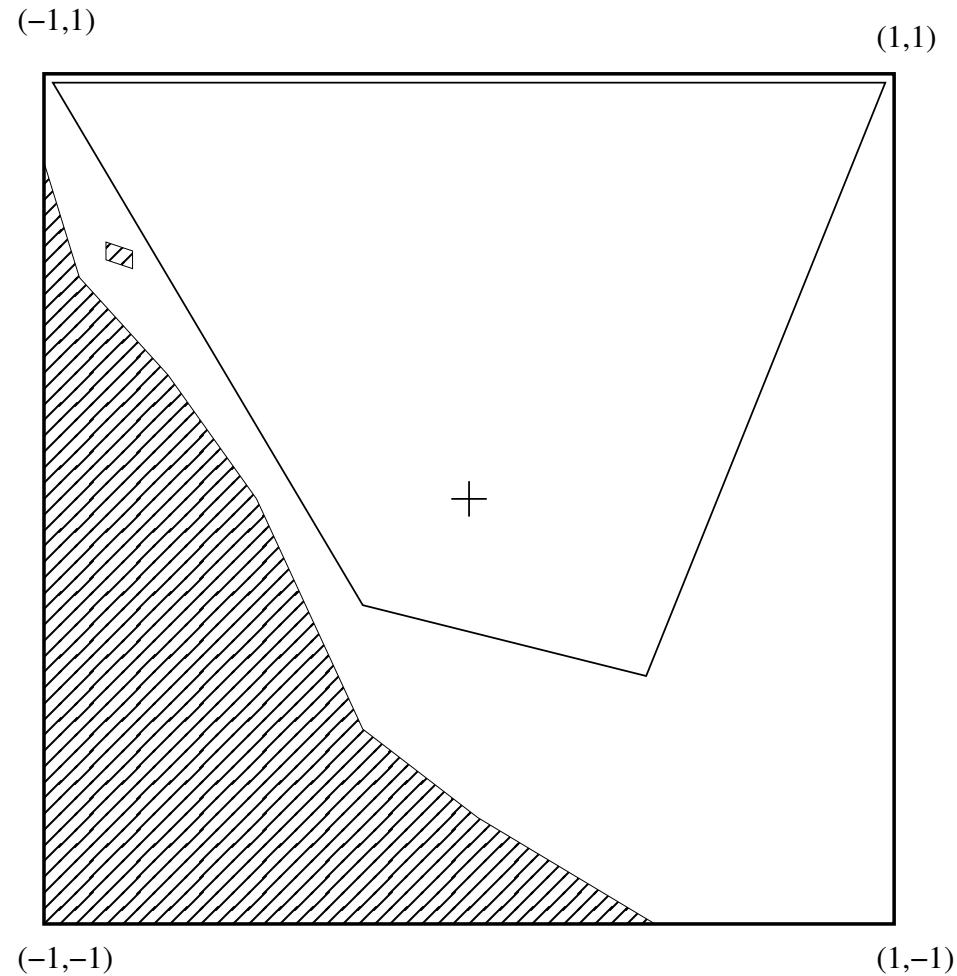
■ Proposed algorithm - Step 2

● Pull one after the other the vertices towards the corners



③ Heuristic algorithm for optimization of stable domains

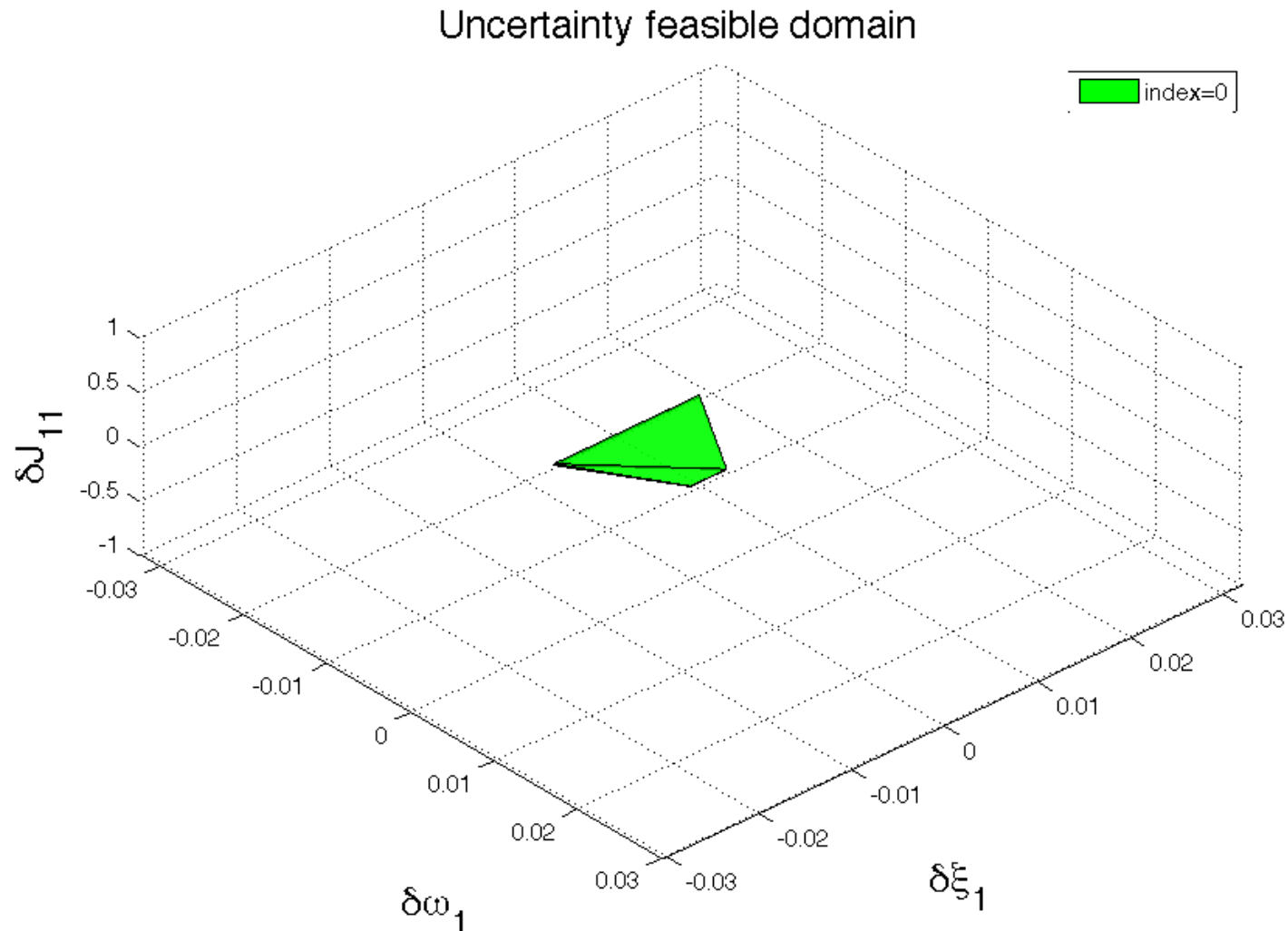
- ▲ Due to the conservatism of the LMIs, the result depends of the ordering
- An other possible solution.



- ① Demeter satellite
- ② Integral Quadratic Separation (IQS)
- ③ Heuristic algorithm for optimization of stable domains
- ④ Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
- ⑤ Conclusions

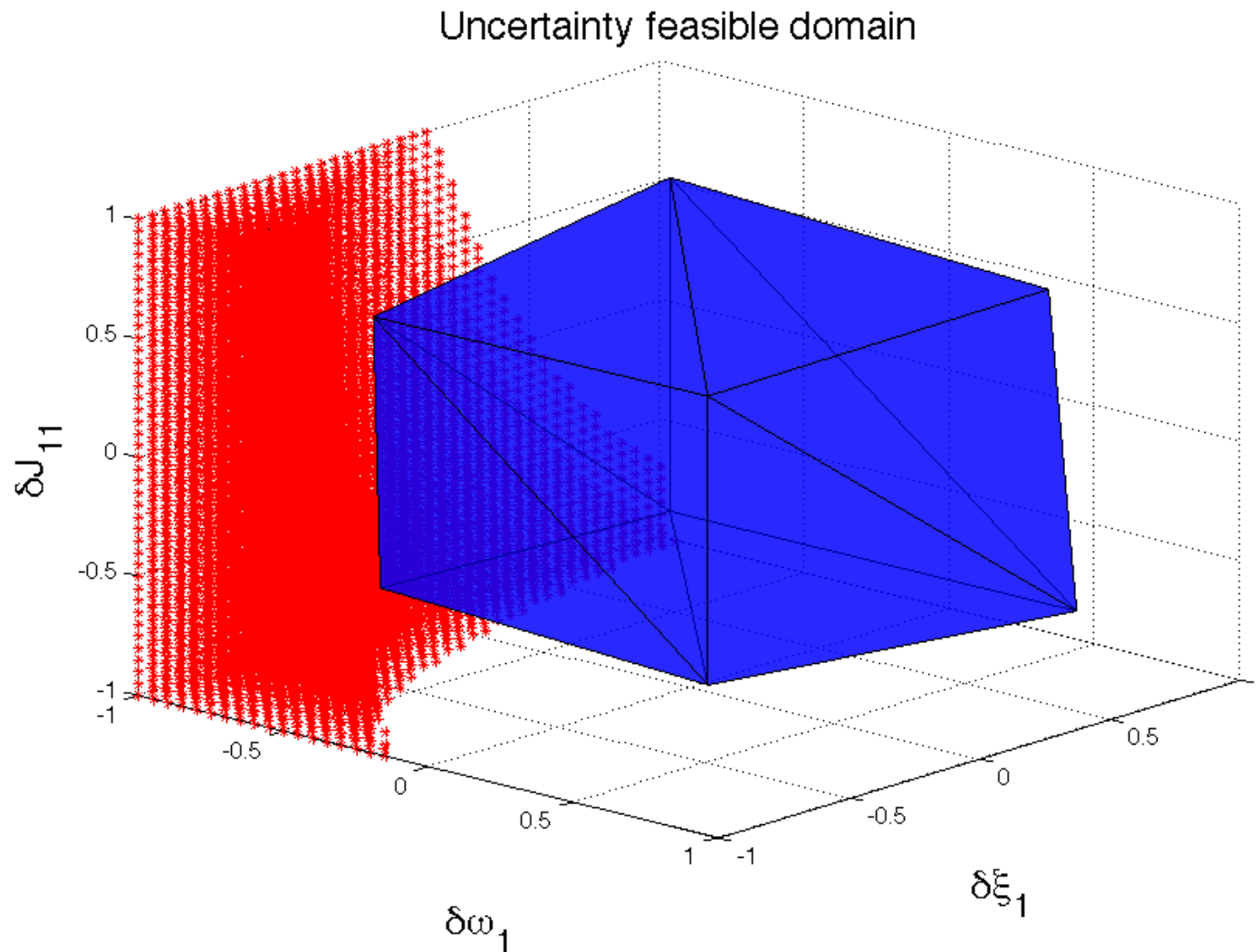
4 Application to Demeter

- X axis, one flexible mode (3 uncertainties)
- IQS result applied to the original modeling
(corresponds to the use of a unique Lyapunov function for all parameters)



4 Application to Demeter

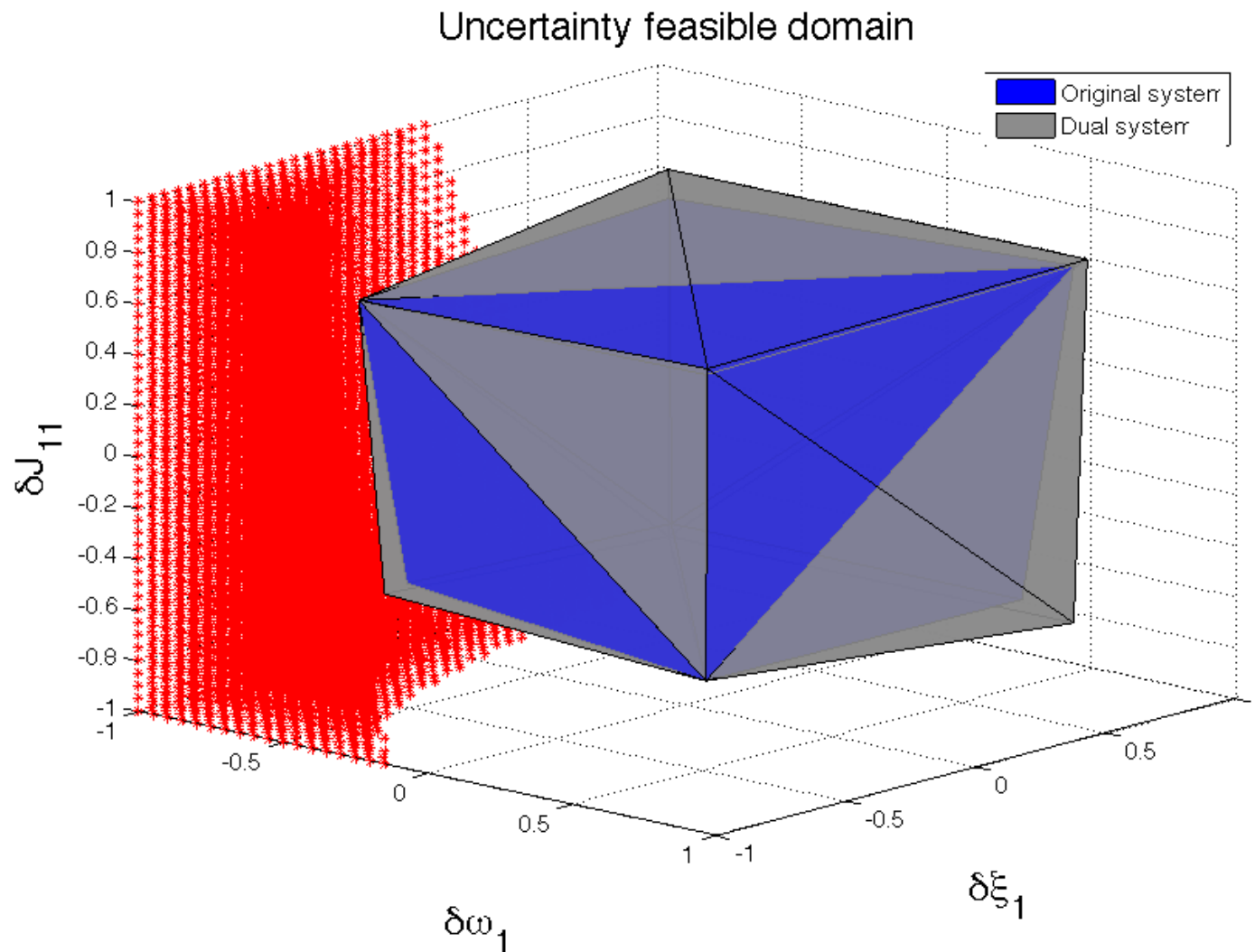
- IQS result applied to the augmented system (added information on $\dot{\Delta} = 0$)
(corresponds to Lyapunov function $V(x) = x^T P(\Delta)x$ with $P(\Delta)$ quadratic)



4 Application to Demeter

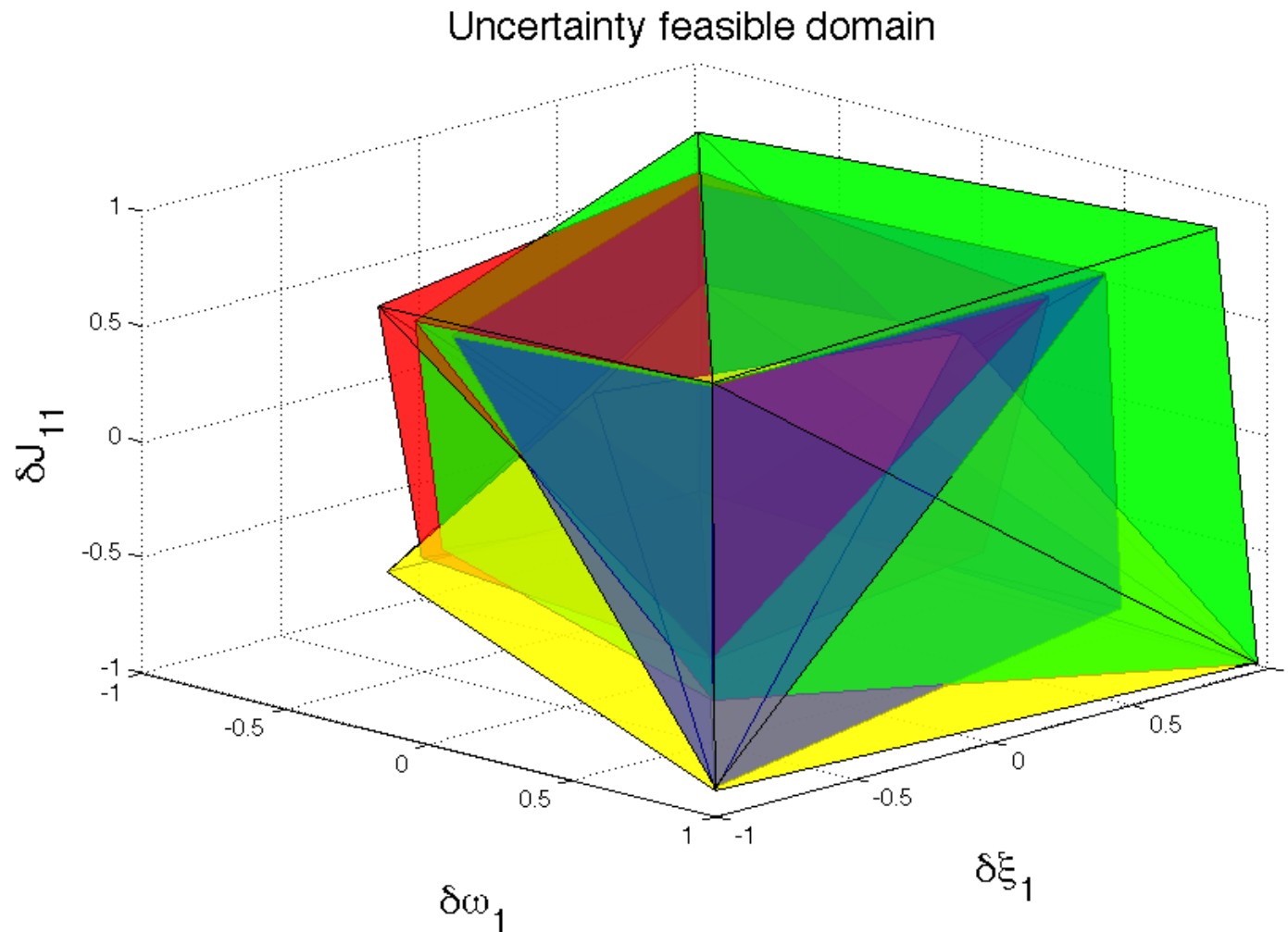
- IQS result applied to the dual of the augmented system

(Lyapunov function $V(x) = x^T P(\Delta)x$ with $X(\Delta) = P^{-1}(\Delta)$ quadratic)



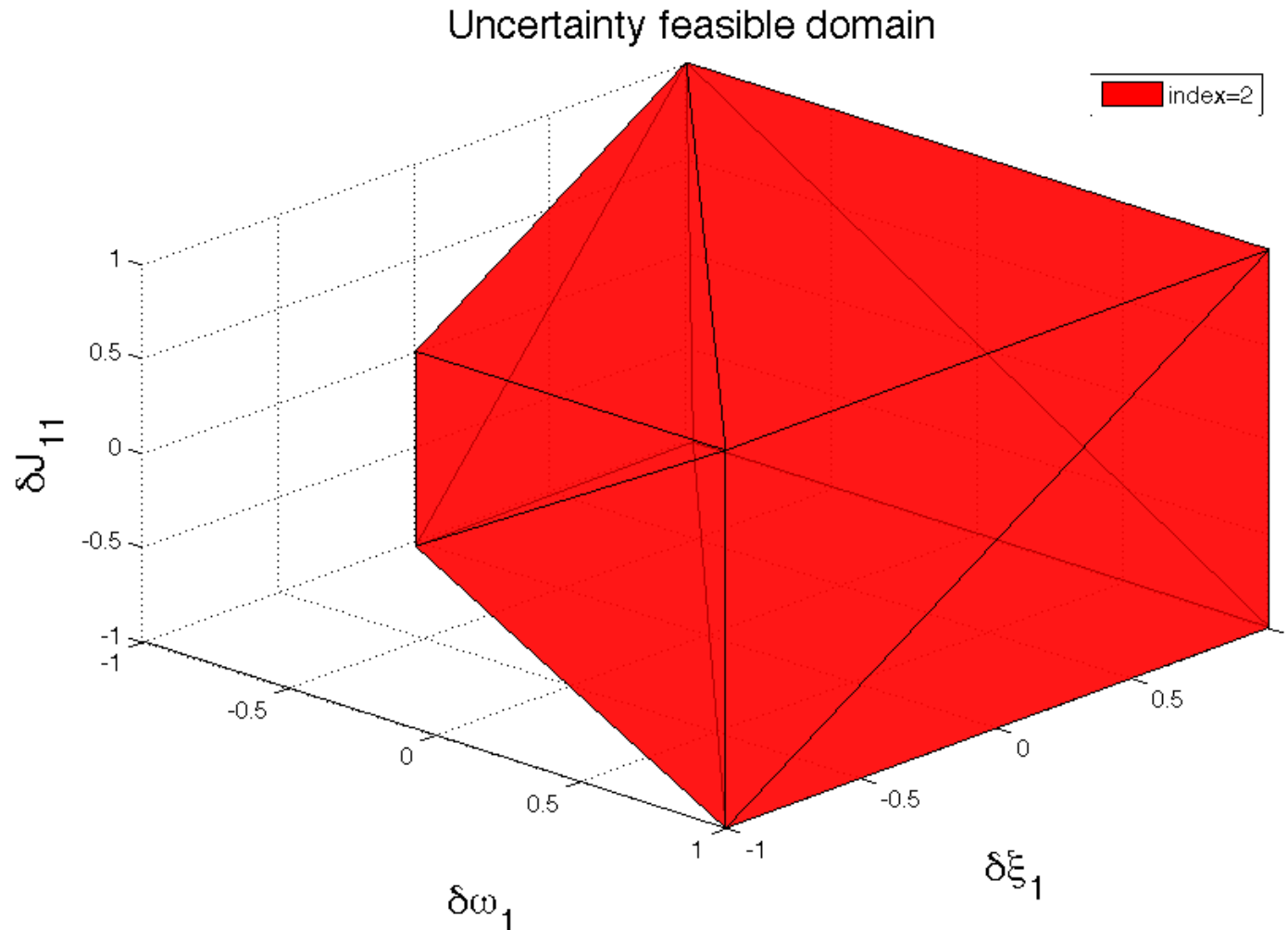
④ Application to Demeter

- IQS result applied to the primal of the augmented system
- ▲ Bisection is replaced by random search in heuristic algorithm (4 tests)



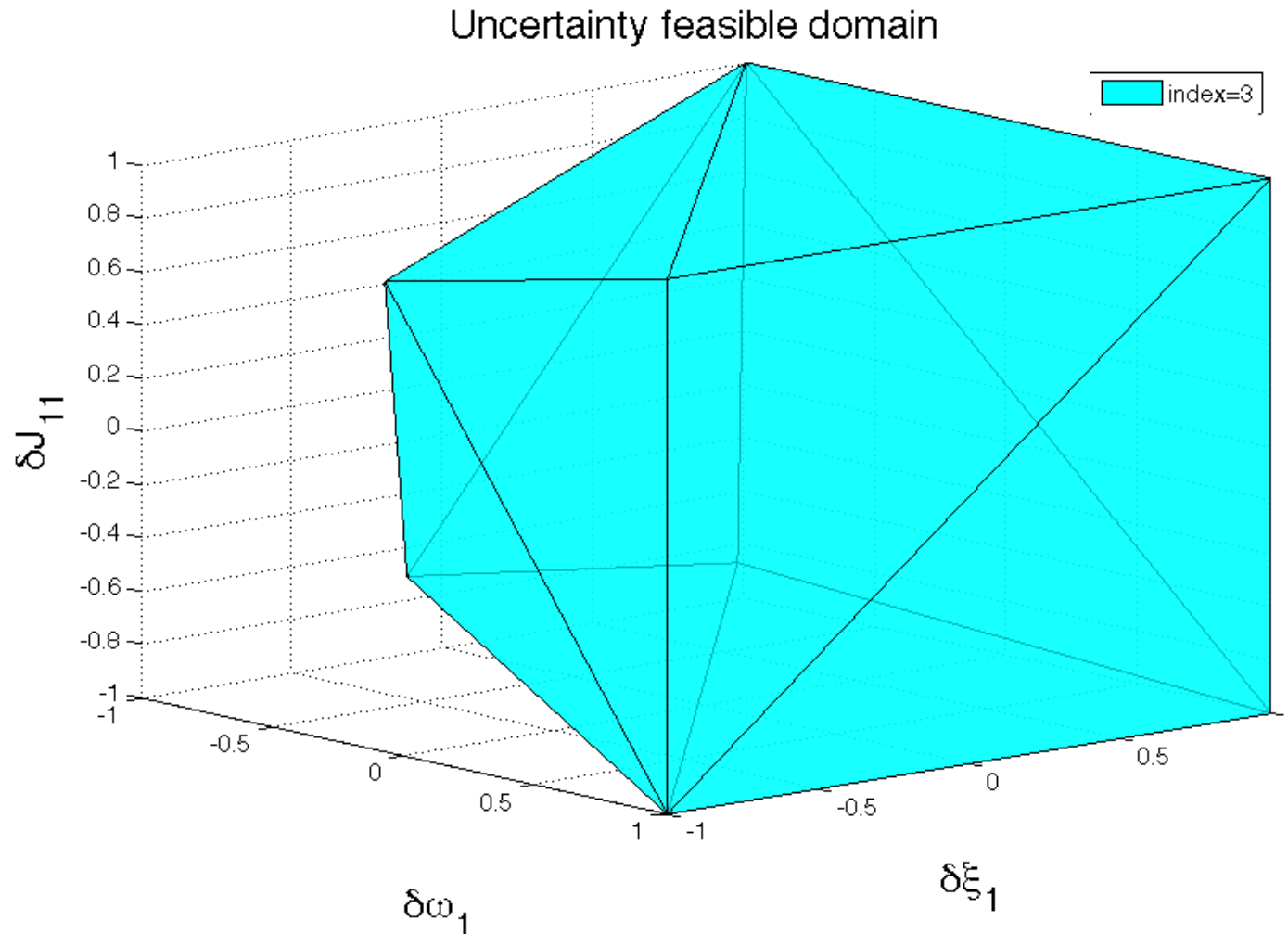
4 Application to Demeter

- IQS result applied to twice augmented system (added information on $\ddot{\Delta} = 0$)
(corresponds to Lyapunov function $V(x) = x^T P(\Delta)x$ with $P(\Delta)$ of order 4)



4 Application to Demeter

- IQS result applied to augmented system 3 times (added information $\Delta^{(3)} = 0$) (corresponds to Lyapunov function $V(x) = x^T P(\Delta)x$ with $P(\Delta)$ of order 6)



4 Application to Demeter

- Vertices obtained for systems augment 2 and 3 times

index=2			index=3		
δJ_{11}	$\delta\omega_1$	$\delta\xi_1$	δJ_{11}	$\delta\omega_1$	$\delta\xi_1$
-0.5156	-0.5156	-0.5156	-0.5312	-0.5312	-0.5312
-0.6562	-0.6562	0.6562	-0.7812	-0.7812	0.7812
-1	1	-1	-1	1	-1
-1	1	1	-1	1	1
0.5156	-0.5156	-0.5156	0.5703	-0.5703	-0.5703
1	-1	1	1	-1	1
1	1	-1	1	1	-1
1	1	1	1	1	1



4 Application to Demeter

- Size of LMIs and computation time

index	nb vars	dim LMI	solve LMI	find polytope
0	216	949×949	0.3s	113s
1	646	2754×2754	3s	746s
2	1614	6185×6185	25s	793s
3	3024	11050×11050	204s	3859s

- ① Demeter satellite
- ② Integral Quadratic Separation (IQS)
- ③ Heuristic algorithm for optimization of stable domains
- ④ Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
- ⑤ Conclusions

5 Conclusions

- IQS framework: Produces new results based on model manipulations
- Adding more equations for higher derivatives of the state:
 - Less conservative LMI conditions
- Same technique works for time varying uncertainties (if known bounds on derivatives)
- Has been applied successfully to time-delay systems [Gouaisbaut]:
 - Gives sequences of LMI conditions with decreasing conservatism
- ▲ Related to SOS representations of positive polynomials [Sato 2009]:
 - Conservatism decreases as the order of the representation is augmented
- No need to manipulate by hand LMIs (Schur complements etc.), polynomials...
- ▲ Does conservatism vanishes? Exactly? Asymptotically? 
- ▲ Is it possible to cope with non-linearities in the same way? 

5 Conclusions

■ Applied currently to attitude control robust analysis:

● Other axis and more flexible modes

● Allows to reduce significantly the validation time: Guaranteed robustness

▲ Tests being done with existing software : RoMulOC

`www.laas.fr/OLOCEP/romuloc`

▲ Contains some analysis (`index = 0, 1`) + some state-feedback features

■ Currently developed software: Romuald

● Dedicated to analysis of descriptor systems

● Fully coded using IQS theory

● Allows systematic system augmentation

```
>> quiz = ctrpb( OrderOfAugmentation ) + h2( usys );
```

```
>> result = solvesdp( quiz )
```

Conclusions

