S-variables for the positivity check

of matrix polynomials with matrix indeterminates

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Introduction

- Many mathematical tools in robust control for building LMI conditions

- Lyapunov, S-procedure, KYP, DG-scaling, IQC, Quadratic Separation,
  Finsler lemma, S-variables, Positivstellensatz, SOS, Polyá...

  - Developed separately for specific uncertainties and view points
  - Conservative SDP relaxations
  - Hierarchies of relaxations with decreasing conservatism

- This presentation : Attempt to establish links between these tools

  - Positivity of matrix polynomials with matrix indeterminates
  - Continuation of work of S-variable approach [Ebihara]
  - Strongly inspired by Quadratic Separation [Iwasaki]
  - Connexions to be done with Generalized Frequency Variables [Hara]
  - Technicalities linking SOS and S-variables [Sato]

Motivation - Lyapunov

- Stability of a linear system $\dot{x} = Ax$

- All eigenvalues of A have negative real part
- $sI - A$ is non singular for all $s \in \overline{\mathbb{C}}_+$
- $I - As^{-1}$ is non singular for all $s^{-1} \in \overline{\mathbb{C}}_+$

- $\exists \epsilon \; : \; (I - As^{-1})^*(I - As^{-1}) \succeq \epsilon I \succ 0$ for all $s^{-1} + s^{-*} \geq 0$
  - Matrix valued polynomial inequality constrained by a polynomial inequality
  - Indeterminate is complex-valued $s^{-1} \in \mathbb{C}$

- $\exists P \succeq 0, \exists \epsilon > 0$ such that $A^* P + PA \preceq -\epsilon I$
  - **Equivalent** LMI formulation
  - P is a Lagrange-like multiplier such that $P(s^{-1} + s^{-*}) \succeq 0$

- Stability of a linear system $\dot{x} = Ax$ with $A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\}$

$$CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\} = \left\{ A = \sum_{v=1}^{\bar{v}} \xi_v A^{[v]} \ : \ \xi_v \geq 0, \ \ \sum_{v=1}^{\bar{v}} \xi_v = 1 \right\}$$

- $\lambda(A)$ have negative real part $\forall A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\}$
- $I - As^{-1}$ is non singular $\forall s^{-1} \in \overline{\mathbb{C}}_+, \forall A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\}$
- $\begin{bmatrix} I & -s^{-1}I \\ -A & I \end{bmatrix}$ is non singular $\forall s^{-1} \in \overline{\mathbb{C}}_+, \forall A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\}$
- $\exists \epsilon \ : \ \begin{bmatrix} I & -s^{-1}I \\ -A & I \end{bmatrix}^* \begin{bmatrix} I & -s^{-1}I \\ -A & I \end{bmatrix} \succeq \epsilon I \succ 0$ for all $\begin{matrix} s^{-1} + s^{-*} \geq 0 \\ A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\} \end{matrix}$

- Matrix polynomial inequality with indeterminates constrained by polynomial inequalities & polytopes

- Indeterminates are in independent rows and columns

Motivation - Lyapunov & S-variables

- Stability of a linear system $\dot{x} = Ax$ with $A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\}$

- $\exists \epsilon$: $\begin{bmatrix} I & -s^{-1} \\ -A & I \end{bmatrix}^* \begin{bmatrix} I & -s^{-1} \\ -A & I \end{bmatrix} \succeq \epsilon I \succ 0$ for all $\begin{array}{l} s^{-1} + s^{-*} \geq 0 \\ A \in CO\{A^{[1]}, \ldots, A^{[\bar{v}]}\} \end{array}$

  - Matrix polynomial inequality with indeterminates constrained by polynomial inequalities & polytopes

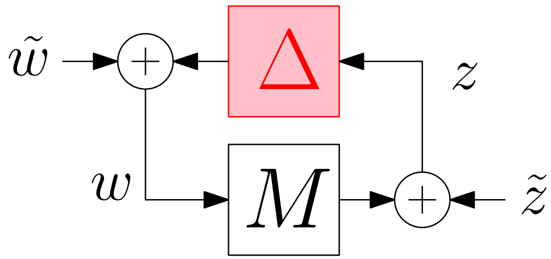
- $\exists S \; : \; \forall v = 1 \ldots \bar{v}, \; \exists P^{[v]} \succeq 0$ such that

$$\begin{bmatrix} \epsilon I & P^{[v]} \\ P^{[v]} & \epsilon I \end{bmatrix} \preceq S \begin{bmatrix} A^{[v]} & -I \end{bmatrix} + \left(S \begin{bmatrix} A^{[v]} & -I \end{bmatrix}\right)^*$$

  - **Conservative** LMI formulation
  - $P(A) = \sum_{v=1}^{\bar{v}} \xi_v P^{[v]}$, parameter-dependent, s.t. $P(A)(s^{-1} + s^{-*}) \succeq 0$
  - S-variable copes with the polytopic uncertainty

■ Well-posedness of $\Delta \star M$

- $\Delta = \begin{bmatrix} \delta_1 I_{r_1} & & 0 \\ & \ddots & \\ 0 & & \Delta_{\bar{k}} \end{bmatrix}$
  - independent uncertainties
  - scalar repeated or matrix valued
  - real or complex
  - norm-bounded by 1 : $|\delta_k| \leq 1$ or $\|\Delta_k\| \leq 1$
- $\star$: feedback-loop



- M : Complex valued matrix
- Well-posedness : internal (w, z) bounded for all bounded disturbances $(\tilde{w}, \tilde{z})$

Motivation - DG-scalings

- Well-posedness of $\Delta \star M$

- $\Delta = \begin{bmatrix} \delta_1 I_{r_1} & & 0 \\ & \ddots & \\ 0 & & \Delta_{\bar{k}} \end{bmatrix}$

  - independent uncertainties
  - scalar repeated or matrix valued
  - real or complex
  - norm-bounded by 1 : $|\delta_k| \leq 1$ or $\|\Delta_k\| \leq 1$

$$\delta_k \in \mathbb{C}, \quad |\delta_k| \leq 1 \quad \Leftrightarrow \quad 1 \geq \delta_k{}^* \delta_k$$

$$\Delta_k \in \mathbb{C}^{m_{1k}, m_{2k}}, \quad \|\Delta_k\| \leq 1 \quad \Leftrightarrow \quad I \succeq \Delta_k{}^* \Delta_k$$

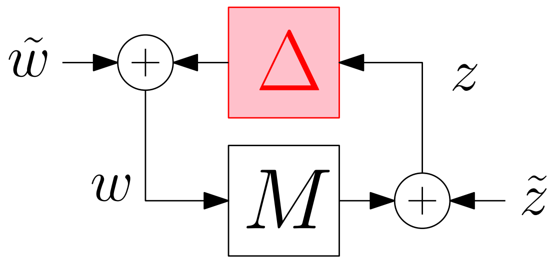
$$\delta_k \in \mathbb{R}, |\delta_k| \leq 1 \quad \Leftrightarrow \quad -j\delta_k{}^* + j\delta_k = 0, \quad 1 \geq \delta_k{}^* \delta_k$$

$$\delta_k \in \mathbb{R}, |\delta_k| \leq 1 \quad \Leftrightarrow \quad \delta_k \in CO\{-1, 1\}$$

- Indeterminates constrained by polynomial inequalities/equalities & polytopes
- Uncertainties are repeated $I_{r_k} \otimes \Delta_k$ (generalization of $\delta_1 I_{r_1}$ to matrices)

Motivation - DG-scalings

- Well-posedness of $\Delta \star M$
- $\Delta = ...$
- $\star$: feedback-loop



- Well-posedness : internal (w, z) bounded for all bounded disturbances $(\tilde{w}, \tilde{z})$

$$\begin{bmatrix} I_{m_1} & \Delta \\ M & I_{m_2} \end{bmatrix} \begin{pmatrix} w \\ z \end{pmatrix} = \begin{pmatrix} \tilde{w} \\ \tilde{z} \end{pmatrix}$$

- $\exists \epsilon > 0$ such that for all admissible Δ

$$(I_{m_2} - M\Delta)^*(I_{m_2} - M\Delta) \succeq \epsilon I_{m_2}$$

- Matrix polynomial inequality with indeterminates constrained by polynomial inequalities/equalities & polytopes
- Indeterminates are in independent rows and columns (Δ block-diagonal)

Motivation - DG-scalings

- Well-posedness of $\Delta \star M$

- $\Delta = \begin{bmatrix} I_{r_1} \otimes \Delta_1 & & 0 \\ & \ddots & \\ 0 & & I_{r_{\bar{k}}} \otimes \Delta_{\bar{k}} \end{bmatrix}$
  - polynomial inequality on Δ_k
  - polynomial equality on Δ_k
  - polytopic constraint Δ_k

- $\exists \epsilon > 0$ such that for all admissible Δ

$$(I_{m_2} - M\Delta)^*(I_{m_2} - M\Delta) \succeq \epsilon I_{m_2}$$

- **Conservative** LMI condition

$$\exists D_k \succeq 0, G_k \quad : \quad \begin{bmatrix} I & M^* \end{bmatrix} \Theta(D_k, G_k) \begin{bmatrix} I \\ M \end{bmatrix} \succeq \epsilon I$$

- $\Theta(D_k, G_k)$: linear in the decision variables
- D_k, G_k : Lagrange-like multipliers w.r.t inequality and equality constraints

Motivation - Proving positivity under constraints

- Robustness analysis of linear time-invariant systems

- Most problems can be recast as proving positivity of polynomials
  - matrix valued (semi-definite constraints)
  - indeterminates are matrices (or scalars), complex valued
  - constrained by polynomial inequalities, equalities and in polytopes

- Many LMI results in the litterature,
  - in general **conservative** (problems are NP-hard)
  - some results are proved to be less conservative
  - on examples conservatism may vanish
  - duality of SDPs can extract worst case indeterminates (prove conservatism vanishes)
  - Numerical issues : limit size of LMIs using the structure of the data

Introduction

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  Finsler lemma, S-variables, **Positivstellensatz, SOS**, Pólya...
  - Developed separately for specific uncertainties and view points
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Sum-Of-Squares

- [Lasserre], [Parillo], [Scherer], [Chesi] ... vast literature

- Goal : proving positivity of a matrix valued polynomial $F(\delta) \succeq 0$
  - with scalar indeterminates $\delta \in \mathbb{R}^{\bar{k}}$
  - constrained by scalar polynomial inequalities $f_i(\delta) \geq 0$.

- Some key methods for solving the problem using SDPs
  - Positivstellensatz
  - Polynomials modeled as quadratic functions of monomials
  - SDP relaxation
  - Hierarchies
  - Moment problem

- $F(\delta) \succeq 0$ for all $\delta \in \mathbb{R}^{\bar{k}}$ such that $f_i(\delta) \geq 0$

- Positivstellensatz [Putinar] : Polynomial sum-of-squares multipliers for each constraint

$$\exists D_i(\delta) \quad SOS \quad : \quad F_D(\delta) = d_0(\delta)F(\delta) + \sum D_i(\delta)f_i(\delta) \quad SOS$$

- Lossless when taking polynomials $D_i(\delta)$ with sufficiently high order
- Variants of Positivstellensatz are : S-procedure, D-scalings, Lagrange multipliers...

Sum-Of-Squares

- $F(\delta) \succeq 0$ for all $\delta \in \mathbb{R}^{\bar{k}}$ such that $f_i(\delta) \geq 0$

- Positivstellensatz : Polynomial positive multipliers for each constraint

$$\exists D_i(\delta) \quad SOS \quad : \quad F_D(\delta) = d_0(\delta) F(\delta) + \sum D_i(\delta) f_i(\delta) \quad SOS$$

- Express the polynomials as a quadratic functions of monomials

$$\delta^{\{0:p\}*} = \left[\begin{array}{cccccccc} I & \delta_1 I & \delta_2 I & \cdots & \delta_1 \delta_2 I & \cdots & {\delta_1}^{p_1} {\delta_2}^{p_2} I & \cdots \end{array} \right]$$

$$D_i(\delta) = \delta^{\{0:p\}*} Q_i(d_i) \delta^{\{0:p\}}, \qquad F_D(\delta) = \delta^{\{0:p\}*} Q(d) \delta^{\{0:p\}}$$

$Q_i(d_i)$ and $Q(d)$ are linear in decision variables d_i (coef. of polynomials $D_i(\delta)$)

- Quadratic representations are not unique

$$F_D(\delta) = \delta^{\{0:p\}*} \left(Q(d) + V \right) \delta^{\{0:p\}} \quad \text{whatever } V \text{ s.t. } \quad \delta^{\{0:p\}*} V \delta^{\{0:p\}} = 0 \quad \forall \delta$$

these are linear constraints on V denoted $V \in \mathcal{N}(\delta^{\{0:p\}})$

Sum-Of-Squares

- $F(\delta) \succeq 0$ for all $\delta \in \mathbb{R}^{\bar{k}}$ such that $f_i(\delta) \geq 0$

- Positivstellensatz + monomials (for large enough order p)

$$\exists \delta^{\{0:p\}*} (Q_i(d_i)) \, \delta^{\{0:p\}} \quad SOS \quad : \quad \delta^{\{0:p\}*} (Q(d)) \, \delta^{\{0:p\}} \quad SOS$$

- SDP relaxation (for fixed order p)

$$\exists V_i \in \mathcal{N}(\delta^{\{0:p\}}) \quad : \quad Q_i(d_i) + V_i \succeq 0 \quad , \quad Q(d) + V \succeq 0$$

- LMI : convex and exist efficient solvers (for not too large size problems)
- Conservative
- Parameterization of $\mathcal{N}(\delta^{\{0:p\}})$ is not easy

Sum-Of-Squares

- $F(\delta) \succeq 0$ for all $\delta \in \mathbb{R}^{\bar{k}}$ such that $f_i(\delta) \geq 0$
- Positivstellensatz + monomials + SDP relaxation (for fixed order p)

$$\exists V_i \in \mathcal{N}(\delta^{\{0:p\}}) \quad : \quad Q_i(d_i) + V_i \succeq 0 \quad , \quad Q(d) + V \succeq 0$$

- Hierarchies : increasing p, the order of the polynomials (and scalings)
- ▲ Conservatism decreases (rapidly)
- ▲ Under mild assumption conservatism vanishes (for finite and low orders)
- ▼ Numerical burden increases rapidly (need to exploit the structure)

- Dual of the LMIs is the relaxation of generalized moment problem

Motivation - Proving positivity under constraints

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- indeterminates are matrices (or scalars), complex valued
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- Many LMI results in the litterature,
- in general **conservative** (problems are NP-hard)
- some results are proved to be less conservative
- on examples conservatism may vanish
- duality of SDPs can extract worst case indeterminates (prove conservatism vanishes)
- Numerical issues : limit size of LMIs using the structure of the data

- Same characteristics in the SOS-Moment framework
- Unification needs to manipulate polynomials with matrix indeterminates

Polynomials with matrix indeterminates

- Goal : proving $F(\Delta) = F(I_{r_1} \otimes \Delta_1, \ldots, I_{r_{\bar{k}}} \otimes \Delta_{\bar{k}}) \succeq 0$
  - under polynomials + polytopic constraints of the type

$$F_{ik}(\Delta_k) \succeq 0, \quad F_{ek}(\Delta_k) = 0, \quad \Delta_k \in CO\{\Delta_k^{[1]}, \ldots, \Delta_k^{[\bar{v}_k]}\}$$

Polynomials with matrix indeterminates

- Monomials with matrix indeterminates $\Delta_k \in \mathbb{C}^{m_{1k}\times m_{2k}}$

$$\Delta_k{}^{\{0\}} = I_{m_{2k}} \qquad \Delta_k{}^{\{1\}} = \Delta_k, \qquad \Delta_k{}^{\{2\}} = \Delta_k^*\Delta_k,$$

$$\Delta_k{}^{\{3\}} = \Delta_k\Delta_k^*\Delta_k, \quad \Delta_k{}^{\{4\}} = \Delta_k^*\Delta_k\Delta_k^*\Delta_k, \quad \dots$$

- Matrix with monomials from degree 0 to degree p_k :

$$\Delta_k{}^{\{0:p_k\}} = \begin{bmatrix} \Delta_k{}^{\{0\}} \\ \vdots \\ \Delta_k{}^{\{p_k\}} \end{bmatrix} = \begin{bmatrix} I_{m_{2k}} \\ \Delta_k \\ \Delta_k^*\Delta_k \\ \vdots \end{bmatrix}, \quad (I_{r_k}\otimes\Delta_k)^{\{0:p_k\}} = \begin{bmatrix} I_{r_k}\otimes\Delta_k{}^{\{0\}} \\ \vdots \\ I_{r_k}\otimes\Delta_k{}^{\{p_k\}} \end{bmatrix}$$

$$\Delta^{\{0:p\}} = \begin{bmatrix} (I_{r_1}\otimes\Delta_1)^{\{0:p_1\}} & & 0 \\ & \ddots & \\ 0 & & (I_{r_{\bar{k}}}\otimes\Delta_{\bar{k}})^{\{0:p_{\bar{k}}\}} \end{bmatrix}$$

Polynomials with matrix indeterminates

- With assumption that Δ_k enter in independent columns

$$F(\Delta) = \Delta^{\{0:p\}*}(F_0 + F_1^* F_1)\Delta^{\{0:p\}} \succeq 0$$

under constraints

$$F_{ik}(\Delta_k) = {\Delta_k}^{\{0:p_k\}*}\Phi_{ik}{\Delta_k}^{\{0:p_k\}} \succeq 0, \qquad \Delta_k \in CO\{\Delta_k^{[1]}, \ldots, \Delta_k^{[\bar{v}_k]}\}$$
$$F_{ek}(\Delta_k) = {\Delta_k}^{\{0:p_k\}*}\Phi_{ek}{\Delta_k}^{\{0:p_k\}} = 0,$$

- "Positivstellensatz" : Exist $D_k(\Delta) \succeq 0$, $G_k(\Delta) = G_k^*(\Delta)$ such that

$$\Delta^{\{0:p\}*}\left(F_0 + F_1^* F_1 + \text{diag}\left(\cdots D_k(\Delta) \boxtimes \Phi_{ik} + G_k(\Delta) \boxtimes \Phi_{ek} \cdots\right)\right)\Delta^{\{0:p\}} \succeq 0$$

under polytopic constraints $\Delta_k \in CO\{\Delta_k^{[1]}, \ldots, \Delta_k^{[\bar{v}_k]}\}$ ($\bar{v}_k = 0$ if no constraint)

- Conservative ?
- How to build the SDP relaxation for fixed order p ?

S-variables and SDPs for SOS

- Exists affine $H_k(\Delta_k) = J_0 + J_1(I \otimes \Delta_k)J_2 + J_3(I \otimes {\Delta_k}^*)J_4$
  such that ${\Delta_k}^{\{0:p_k\}}$ spans the null space of $H_k(\Delta_k)$

$$\underbrace{\begin{bmatrix} \Delta_k & -I & 0 & 0 \\ 0 & {\Delta_k}^* & -I & 0 \\ 0 & 0 & \Delta_k & -I \end{bmatrix}}_{M_k(\Delta_k)} \underbrace{\begin{bmatrix} I \\ \Delta_k \\ {\Delta_k}^* \Delta_k \\ \Delta_k {\Delta_k}^* \Delta_k \end{bmatrix}}_{{\Delta_k}^{\{0:p_k\}}} = 0$$

S-variables and SDPs for SOS

- ${\Delta_k}^{\{0:p_k\}}$ null space of $H_k(\Delta_k) = J_0 + J_1(I \otimes \Delta_k)J_2 + J_3(I \otimes {\Delta_k}^*)J_4$
- SDP relaxation for Δ_k in polytope

  assuming $\Psi(X(\Delta))$ is affine of indeterminate-dependent $X(\Delta)$

$$\exists S_k, X^{[v_k]} \ : \ \Psi(X^{[v_k]}) + S_k H_k(\Delta_k^{[v_k]}) + (S_k H_k(\Delta_k^{[v_k]}))^* \succeq 0 \quad \forall v_k = 1 \ldots \bar{v}_k$$

$$\Downarrow$$

$$\exists X(\Delta) \ : \ {\Delta_k}^{\{0:p_k\}*}\Psi(X(\Delta)){\Delta_k}^{\{0:p_k\}} \succeq 0 \quad \forall \Delta_k \in CO\{\Delta_k^{[1]}, \ldots, \Delta_k^{[\bar{v}_k]}\}$$

- Proof

- Affine Ψ and M_k and $\Delta_k = \sum_{v_k=1}^{\bar{v}_k} \xi_v \Delta^{[v_k]}$

$$\exists S_k, X^{[v_k]} \; : \; \Psi(X^{[v_k]}) + S_k H_k(\Delta_k^{[v_k]}) + (S_k H_k(\Delta_k^{[v_k]}))^* \succeq 0 \quad \forall v_k = 1 \dots \bar{v}_k$$

$$\Updownarrow$$

$$\exists S_k, \; : \; \Psi(X(\Delta)) + S_k H_k(\Delta_k) + (S_k H_k(\Delta_k))^* \succeq 0 \quad \forall \xi_v \geq 0, \quad \sum_{v=1}^{\bar{v}} \xi_v = 1$$

$$X(\Delta) = \sum_{v_k=1}^{\bar{v}_k} \xi_v X^{[v_k]}$$

- By congruence with the fact that $H_k(\Delta_k) {\Delta_k}^{\{0:p_k\}} = 0$

$$\Downarrow$$

$$\exists X(\Delta) \; : \; {\Delta_k}^{\{0:p_k\}*} \Psi(X(\Delta)) {\Delta_k}^{\{0:p_k\}} \succeq 0 \quad \forall \Delta_k \in CO\{\Delta_k^{[1]}, \dots, \Delta_k^{[\bar{v}_k]}\}$$

- Conservatism comes for the choice of indeterminate-independent S_k

S-variables and SDPs for SOS

- $\Delta_k{}^{\{0:p_k\}}$ null space of $H_k(\Delta_k) = J_0 + J_1(I \otimes \Delta_k)J_2 + J_3(I \otimes \Delta_k{}^*)J_4$

- SDP relaxation for unbounded Δ_k

$$\exists S_k, X \ : \ \Psi(X) + S_k H_k(0) + (S_k H_k(0))^* \succeq 0 \quad \forall v_k = 1 \ldots \bar{v}_k$$

$$S_k = \begin{bmatrix} J_4^T & J_2^T \end{bmatrix} \begin{bmatrix} T_k \otimes I & 0 \\ 0 & T_k \otimes I \end{bmatrix} \begin{bmatrix} J_1^T \\ J_3^T \end{bmatrix}, \quad T_k = -T_k^*$$

$$\Downarrow$$

$$\exists X \ : \ \Delta_k{}^{\{0:p_k\}*} \Psi(X) \Delta_k{}^{\{0:p_k\}} \succeq 0 \quad \forall \Delta_k$$

▲ Structured S-variables provides a parameterization of $\mathcal{N}(\Delta_k{}^{\{0:p_k\}})$

▼ Is $\mathcal{N}(\Delta_k{}^{\{0:p_k\}})$ fully parameterized in this way ?

▲ SOS-Moment SDP relaxations are a special case of S-variable relaxation

- Main result (combining described techniques)

$$\hat{F}_1^{\perp *} \left(\hat{F}_0 - \mathrm{diag} \begin{pmatrix} \vdots \\ \hat{D}_k^{[v_{\mathcal{K}_2}]} \boxtimes \Phi_{ik} + \hat{G}_k^{[v_{\mathcal{K}_2}]} \boxtimes \Psi_{ek} \\ \vdots \end{pmatrix} \right) \hat{F}_1^{\perp}$$
$$+ \sum_{k \in \mathcal{K}_1} \left\{ \hat{F}_1^{\perp *} \hat{S}_k^{[v_{\mathcal{K}_2}]} \hat{H}_k(0) \hat{F}_1^{\perp} \right\}^{\mathcal{H}} + \sum_{k \in \mathcal{K}_2} \left\{ \tilde{S}_k^{[v_{\mathcal{K}_2 \setminus k}]} \hat{H}_k(\Delta_k^{[v_k]}) \hat{F}_1^{\perp} \right\}^{\mathcal{H}} \succeq 0$$

- ▲ $\mathcal{K}_1, \mathcal{K}_2$: uncertainties without and with polytopic constraints respectively
- ▼ Large number of constraints and decision variables (when several polytopes)
- ▼ Are all the decision variables needed ?
- ▼ Can we build hierarchies as in usual SOS methods ?
- ▲ When applied to special cases we get exactly the same LMI conditions

Conclusion

- ▲ Ongoing work to make links between many existing results in Robust Control
- ▲ Used tool is inspired by SOS-Moments relaxations
- ▲ Motivation for dealing with polynomial matrix inequalities of matrix indeterminates
- ▼ Sub-case of all possible polynomial matrix inequalities of matrix indeterminates
- ▼ At this stage does not allow to build non-existing results
- ▼ Numerical experiments to be done