LMI results for robust control design of observer-based controllers, the discrete-time case with polytopic uncertainties

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- Curiosity at the origin of this work:
- Many existing LMI results for robust state-feedback design
- The Luenberger type observer problem is “dual" to state-feedback, but... few LMI results
- Many results for robust output filtering issue, but applicable only to stable plants

- Issues raised by the study of robust observers:

$$\begin{array}{l} x_{k+1} = A(\theta)x_k + Bu_k \\ y_k = Cx_k \end{array} \quad , \quad \begin{array}{l} \hat{x}_{k+1} = A_o\hat{x}_k + Bu_k + L(C\hat{x}_k - y_k) \\ e_k = x_k - \hat{x}_k \end{array}$$

- What model A_o for the observer ?
  - Usual answer is to decompose *a priori* $A(\theta) = A_o + \Delta(\theta)$
  - For example in NL observers, A_o models integrators in series
- One cannot expect $e_{k_2>k_1} = 0$ even if $e_{k_1} = 0$
- Separation principle for the design of state-feedback and observer gains?

$$\begin{pmatrix} x_{k+1} \\ e_{k+1} \end{pmatrix} = \begin{bmatrix} A(\theta) + BK & -BK \\ A(\theta) - A_o & A_o + LC \end{bmatrix} \begin{pmatrix} x_k \\ e_k \end{pmatrix}$$

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- S-variable approach to state-feedback design, and analysis of the closed-loop
- Joint model and gain observer design with minimization of influence on state-feedback
- Robust analysis of the state-feedback + observer feedback loop
- Numerical example
- Conclusions

- Discrete-time systems with polytopic uncertainties

$$A(\theta) = \sum_{v=1}^{\bar{v}} \theta_v A^{[v]} \quad : \quad \theta_{v=1\dots\bar{v}} \geq 0\,, \quad \sum_{v=1}^{\bar{v}} \theta_v = 1$$

- Example of existing LMI result for state-feedback design **(SFdesign)**

$$\begin{array}{l} \exists P_1^{[v]} \succ 0 \\ \exists F_1 \\ \exists \hat{K} \end{array} : \begin{bmatrix} P_1^{[v]} & 0 & 0 \\ 0 & B_w B_w^T - P_1^{[v]} & 0 \\ 0 & 0 & -\mu_\infty^2 I \end{bmatrix} \prec \left\{ \begin{bmatrix} F_1 \\ -(A^{[v]} F_1 + B\hat{K}) \\ -C_z^{[v]} F_1 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \end{bmatrix} \right\}^{\mathcal{S}}.$$

- If **(SFdesign)** hold for all vertices then $K = \hat{K} F_1^{-1}$ robustly stabilizes the plant and

$$\begin{aligned} x_{k+1} &= (A(\theta) + BK)x_k + B_w w_k \\ z_k &= C_z(\theta) x_k \end{aligned}$$

has H_∞ norm robustly less than μ_∞, i.e. $\|z\|_2 \leq \mu_\infty \|w\|_2$.

- Robust analysis LMI result for the plant with state-feedback **(SFanalysis)**

$$\begin{array}{l} \exists P_3^{[v]} \succ 0 \\ \exists G_3 \\ \exists Q \succ 0 \end{array} : \begin{bmatrix} P_3^{[v]} & 0 & 0 \\ 0 & Q - P_3^{[v]} & 0 \\ 0 & 0 & -I \end{bmatrix} \prec \left\{ G_3 \begin{bmatrix} I & -(A^{[v]} + BK) & B \end{bmatrix} \right\}^{\mathcal{S}} .$$

- If K is solution to **(SFdesign)** then **(SFanalysis)** is feasible
- If **(SFanalysis)** hold for all vertices then the following plant is robustly stable and

$$x_{k+1} = (A(\theta) + BK)x_k - BKe_k$$
$$g_k = Q^{1/2} x_k$$

has H_∞ norm robustly less than 1, i.e. $\|Q^{-1/2}x\|_2 \leq \|Ke_k\|_2$.

- Virtual output g_k models the excursions of the state due to perturbations on the control.
- Maximizing Q gives indications on the maximal excusions.

LMI result for robust observer design **(Odesign)**

$$\exists P_{42}^{[v]} \succ 0,\ \exists P_{4p}^{[v]} \succeq K^T K,\ \exists F_4,\ \exists \hat{A}_o,\ \exists \hat{L}\ :$$

$$\begin{bmatrix} P_{42}^{[v]} & 0 & 0 \\ 0 & K^TK - P_{42}^{[v]} & 0 \\ 0 & 0 & -\gamma_2^2 Q \end{bmatrix} \prec \left\{ \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_4 & -(\hat{A}_o + \hat{L}C) & \hat{A}_o - F_4 A^{[v]} \end{bmatrix} \right\}^{\mathcal{S}}$$

$$\begin{bmatrix} P_{4p}^{[v]} & 0 & 0 \\ 0 & -P_{4p}^{[v]} & 0 \\ 0 & 0 & -\gamma_p^2 Q \end{bmatrix} \prec \left\{ \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_4 & -(\hat{A}_o + \hat{L}C) & \hat{A}_o - F_4 A^{[v]} \end{bmatrix} \right\}^{\mathcal{S}}$$

- For all K and Q the LMI problem **(Odesign)** is feasible.
- **(Odesign)** provides $A_o = F_4^{-1}\hat{A}_o$, $L = F_4^{-1}\hat{L}$ s.t. the error dynamics are stable

$$e_{k+1} = (A_o + LC)e_k + (A(\theta) - A_o)x_k$$

and guarantees the following robust properties:

$$\|Ke\|_2 \leq \gamma_2 \|Q^{-1}x\|_2 \quad , \quad \|Ke\|_p = \max_k \|Ke_k\| \leq \gamma_p \|Q^{-1}x\|_2$$

- Small gain theorem guarantees closed-loop robust stability if $\gamma_2 < 1$.
- Minimizing γ_2 improves stability,

  but tends to give high gain observers with large peak responses.
- Minimizing a linear combination of γ_2 and γ_p gives a trade-off between the two effects

■ Robust analysis LMI result for the closed-loop plant **(Oanalysis)**

$$\exists P_6^{[v]} \succ 0,\ \exists G_6\ :$$

$$\begin{bmatrix} P_6^{[v]} & 0 & 0 \\ 0 & \begin{bmatrix} C_z^{[v]T} \\ 0 \end{bmatrix} \begin{bmatrix} C_z^{[v]} \\ 0 \end{bmatrix}^T - P_6^{[v]} & 0 \\ 0 & 0 & -{\nu_\infty}^2 I \end{bmatrix} \prec \left\{ G_6 \left[\begin{array}{cc|cc|c} I & 0 & -A^{[v]} & -BK & -B_w \\ 0 & I & LC & -A_o - BK - LC & 0 \end{array} \right] \right\}^{\mathcal{S}}.$$

- If **(Oanalysis)** hold for all vertices then the state-fedback + observer loop robustly stabilizes the plant and

$$x_{k+1} = A(\theta)x_k + Bu_k + B_w w_k, \qquad \hat{x}_{k+1} = (A_o + BK + LC)\hat{x}_k - Ly_k$$
$$y_k = Cx_k, \quad z_k = C_z(\theta)x_k \qquad u_k = K\hat{x}_k$$

has H_∞ norm robustly less than ν_∞, i.e. $\|z\|_2 \leq \nu_\infty \|\hat{w}\|_2$.

- One can expect $\mu_\infty \leq \nu_\infty$, i.e. that the observer-based control degrades the performance compared to the ideal state-feedback.

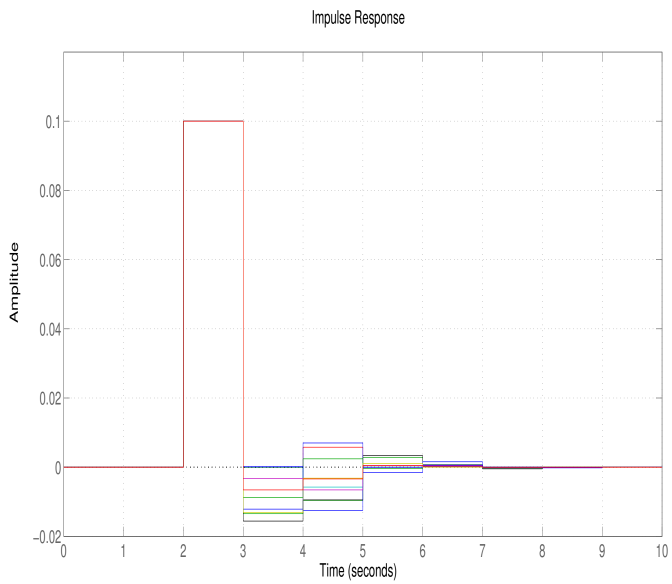
$$x_{k+1} = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} w_k, \quad \begin{array}{l} y_k = \begin{bmatrix} 0 & 1 \end{bmatrix} x_k \\ z_k = \begin{bmatrix} 0 & 1 \end{bmatrix} x_k \end{array}$$

- $a \in [0.9,\ 1.1]$ and $b \in [0.9,\ 1.1]$, i.e. $\bar{v} = 4$ vertices. None of the vertices are stable.
- **(SFdesign)** with $\mu_\infty = 1$ gives $K = \begin{bmatrix} -1.0633 & -1.0324 \end{bmatrix}$.
- **(SFanalysis)** with $\max$ Tr(Q) gives $Q = \begin{bmatrix} 0.1239 & 0.0527 \\ 0.0527 & 0.5730 \end{bmatrix}$.
- **(Odesign)** with $\min \gamma_2 + \gamma_p$ gives $\gamma_2 = 0.8797$, $\gamma_p = 0.8575$ and

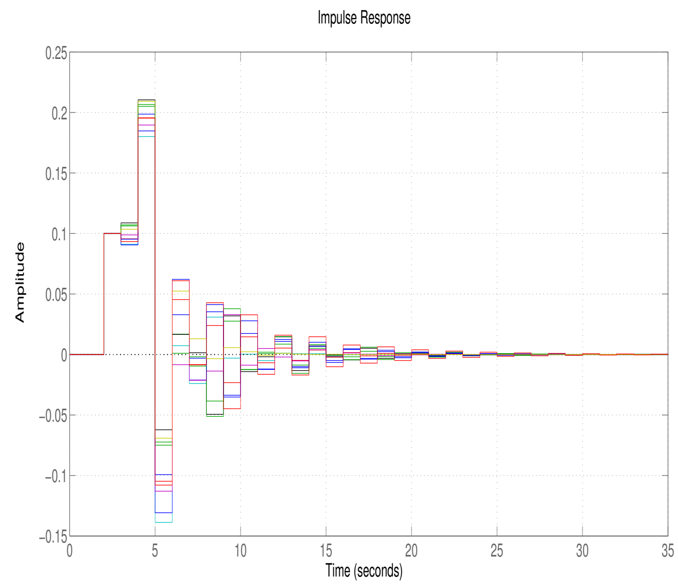
$$A_o = \begin{bmatrix} 0.9946 & 0.9807 \\ 0.9946 & -0.0191 \end{bmatrix}, \quad L = \begin{bmatrix} -2.3637 \\ -1.3565 \end{bmatrix}.$$

- $\gamma_2 < 1$, the closed-loop is robustly stable
- $A_o \neq \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
- **(Oanalysis)** with $\min \nu_\infty$ gives $\nu_\infty = 1.0268$.

- Impulse responses (for several random values of uncertainties)

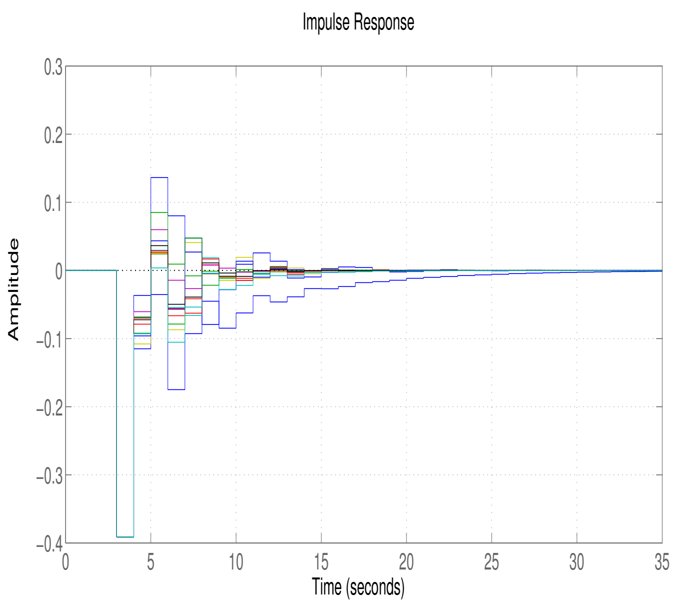


ideal state-feedback

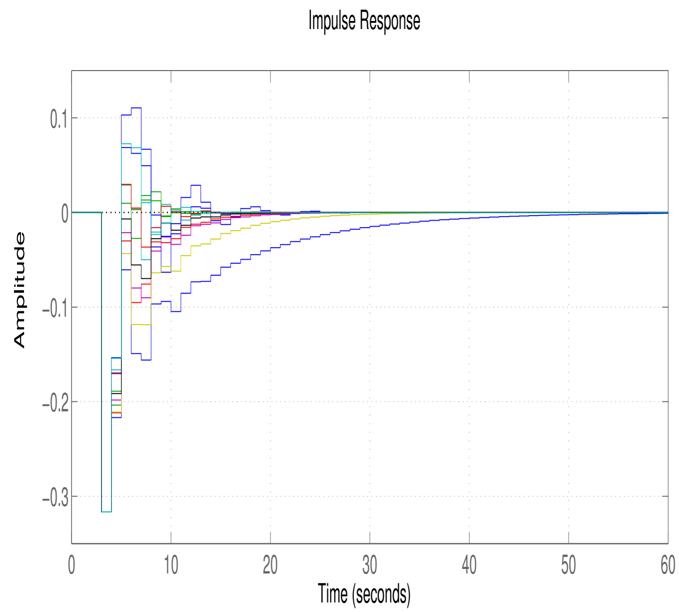


with observer-based control

- Control inputs for two different choices of observers



(Odesign) with $\min \gamma_2 + \gamma_p$



(Odesign) with $\min \gamma_2 + 10^4 \gamma_p$

- Allows to reduce the peak response but with slower convergence

- Robust observer design revisited
- Proposed heuristic based on LMIs only
- Observer optimized not to perturb too strongly the given state-feedback

  Tradeoff between L_2 and peak criteria
- Discussions about S-variable approach LMIs:

  primal (observer case) VS dual (state-feedback)

  analysis (G S-variable) VS design (F structured S-variable)

The S-Variable Approach to LMI-Based Robust Control

Springer, Y. Ebihara, D. Peaucelle, D. Arzelier, 2015