Uncertain systems and robust control LMI methods

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Relaxation Approaches for Control of Uncertain Complex Systems:

Methodologies and Tools

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- Objectives of this presentation:
  - Recall some existing results in robust control
  - Demonstrate how to test these with RoMulOC toolbox

    `http://projects.laas.fr/OLOCEP/romuloc/`

- Underlying point of view:
  - Few techniques
  - Many results
  - Depend on modeling choices

- Two classes of uncertain systems : polytopic & LFT
  - Airplane example
  - DEMETER satellite example
  - Prospectives: descriptor uncertain modeling
- Robust analysis
  - Stability and performances - the well-posedness point of view
  - System augmentation approach for sequences of SOS-like relaxations
  - "Slack variable" results and "quadratic stability" as a special case
- State-feedback design: multi-performance
  - Based on dual system
  - Almost LMI results in "slack variable" approach

- Aircraft example

- Complicated non-linear model - linearized around operation point (V_o, ...)
- 9 uncertain parameters:

(Not precisely known parameters such as inertia etc.

& uncertainties on operating point)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & L_p & L_\beta & L_r \\ g/(V_o + v) & 0 & Y_\beta & -1 \\ N_{\dot{\beta}} g/(V_o + v) & N_p & N_\beta + N_{\dot{\beta}} Y_\beta & N_r - N_{\dot{\beta}} \end{bmatrix} x + Bu$$

- Uncertainties given in intervals: $\underline{L_p} \leq L_p \leq \overline{L_p}$, $\underline{L_\beta} \leq L_\beta \leq \overline{L_\beta}$ ...

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & L_p & L_\beta & L_r \\ g/(V_o+v) & 0 & Y_\beta & -1 \\ N_{\dot\beta} g/(V_o+v) & N_p & N_\beta + N_{\dot\beta} Y_\beta & N_r - N_{\dot\beta} \end{bmatrix} x + Bu$$

- Affine-dependent representation with bounds given by interval analysis

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \alpha_{22} & \alpha_{23} & \alpha_{23} \\ \alpha_{31} & 0 & \alpha_{33} & -1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} x + Bu$$

$$\underline{L_p} \leq \alpha_{22} \leq \overline{L_p} \quad \ldots \quad \underline{N_{\dot\beta}} g/(V_o + \overline{v}) \leq \alpha_{41} \leq \overline{N_{\dot\beta}} g/(V_o + \underline{v}) \ldots$$

- Includes the original uncertain model but coupling between coefficients is lost
- Conservative: if a property is proved for polytopic model, it also holds for original one

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \alpha_{22} & \alpha_{23} & \alpha_{23} \\ \alpha_{31} & 0 & \alpha_{33} & -1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} x + Bu$$

$$\underline{\alpha}_{ij} \leq \alpha_{ij} \leq \overline{\alpha}_{ij}$$

- This is an **interval** model: all coefficients are independent and in intervals
- Sub-class of **parallelotopic** models (centered at A_0 with deviations along axes A_1, A_2 etc.)

$$A(\beta) = A_0 + \beta_1 A_1 + \beta_2 A_2 + \ldots \quad , \quad \beta_i \in [-1 \;\; 1]$$

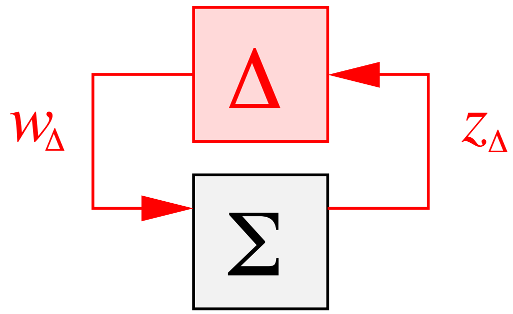
- Sub class of **polytopic** models described as convex hull of vertices (in ex. $\overline{v} = 2^9 = 512$!)

$$A(\xi) = \sum_{v=1}^{\overline{v}} \xi_v A^{[v]} \quad : \quad \sum_{v=1}^{\overline{v}} \xi_v = 1 \quad , \quad \xi_v \geq 0$$

▲ **`Demo in RoMulOC`**

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & L_p & L_\beta & L_r \\ g/(V_o+v) & 0 & Y_\beta & -1 \\ N_{\dot{\beta}}g/(V_o+v) & N_p & N_\beta + N_{\dot{\beta}}Y_\beta & N_r - N_{\dot{\beta}} \end{bmatrix} x + Bu$$

- Linear-Fractional Transformation (LFT): make it linear in the uncertainties via feedback



$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & L_p & L_\beta & L_r \\ g/(V_o + v) & 0 & Y_\beta & -1 \\ N_{\dot{\beta}} g/(V_o + v) & N_p & N_\beta + N_{\dot{\beta}} Y_\beta & N_r - N_{\dot{\beta}} \end{bmatrix} x + Bu$$

- Handling the $1/(V_o + v)$ terms.

$$\begin{aligned} z_1 = 1/(V_o + v) x_1 \; & \Leftrightarrow \; V_o z_1 + v z_1 = x_1 \\ & \Leftrightarrow \begin{cases} w_1 = v z_1 \\ V_o z_1 + w_1 = x_1 \end{cases} \\ & \Leftrightarrow \begin{cases} w_1 = v z_1 \\ z_1 = 1/V_o x_1 - 1/V_o w_1 \end{cases} \end{aligned}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & L_p & L_\beta & L_r \\ g/(V_o+v) & 0 & Y_\beta & -1 \\ N_{\dot\beta}g/(V_o+v) & N_p & N_\beta + N_{\dot\beta}Y_\beta & N_r - N_{\dot\beta} \end{bmatrix} x + Bu$$

▲ Handling the $1/(V_o+v)$ terms, continued

$$w_1 = v z_1$$

$$\begin{pmatrix} \dot{x} \\ z_1 \end{pmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & L_p & L_\beta & L_r & 0 \\ g/V_o & 0 & Y_\beta & -1 & -g/V_o \\ N_{\dot\beta}g/V_o & N_p & N_\beta + N_{\dot\beta}Y_\beta & N_r - N_{\dot\beta} & -N_{\dot\beta}g/V_o \\ \hline 1/V_o & 0 & 0 & 0 & -1/V_o \end{array}\right] \begin{pmatrix} x \\ w_1 \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} \dot{x} \\ z_1 \end{pmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & L_p & L_\beta & L_r & 0 \\ g/V_o & 0 & Y_\beta & -1 & -g/V_o \\ N_{\dot\beta} g/V_o & N_p & N_\beta + N_{\dot\beta} Y_\beta & N_r - N_{\dot\beta} & -N_{\dot\beta} g/V_o \\ \hline 1/V_o & 0 & 0 & 0 & -1/V_o \end{array}\right] \begin{pmatrix} x \\ w_1 \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}$$

▲ Handling the $N_{\dot\beta}$ term:

$$w_1 = v z_1 \quad , \quad w_2 = N_{\dot\beta} z_2$$

$$\begin{pmatrix} \dot{x} \\ z_1 \\ z_2 \end{pmatrix} = \left[\begin{array}{cccc|cc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & L_p & L_\beta & L_r & 0 & 0 \\ g/V_o & 0 & Y_\beta & -1 & -g/V_o & 0 \\ 0 & N_p & N_\beta & N_r & 0 & 1 \\ \hline 1/V_o & 0 & 0 & 0 & -1/V_o & 0 \\ g/V_o & 0 & Y_\beta & -1 & -g/V_o & 0 \end{array}\right] \begin{pmatrix} x \\ w_1 \\ w_2 \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

- In the end $w = \Delta z$ with $\Delta = \text{diag}(v, N_{\dot\beta}, Y_\beta, L_p, L_\beta, L_r, N_p, N_\beta, N_r)$ and

$$\begin{pmatrix} \dot{x} \\ z \end{pmatrix} = \left[\begin{array}{cccc|ccccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ g/V_o & 0 & 0 & -1 & -g/V_o & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1/V_o & 0 & 0 & 0 & -1/V_o & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g/V_o & 0 & 0 & -1 & -g/V_o & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{pmatrix} x \\ w \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

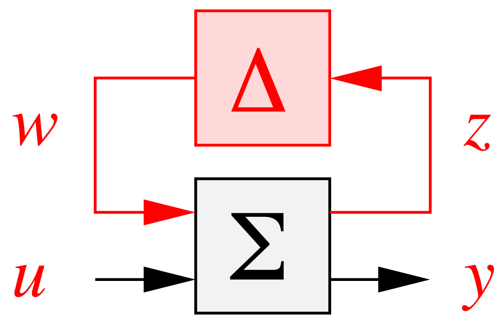
- LFT model is a feedback-loop of a purely uncertain matrix with purely certain system
- Can always be obtained if uncertainty enters rationally in the model
- Issue: having an LFT of minimal size (size of Δ)

- Manipulation LFT made easy using the star-product

$$\Delta \star \left[\begin{array}{c|c} M_d & M_c \\ \hline M_b & M_a \end{array}\right] = M_a + M_b \Delta (\mathbf{I} - M_d \Delta)^{-1} M_c$$

- Corresponds to the following loop:

$$w = \Delta z \quad \star \quad \left\{ \begin{array}{l} z = M_d w + M_c u \\ y = M_b w + M_a u \end{array} \right.$$



- Always assumed to be well-posed: $(\mathbf{I} - M_d \Delta)$ non singular for all uncertainties

- Elementary operations on LFTs

$$\Delta_1 \star \left[\begin{array}{c|c} M_d & M_c \\ \hline M_b & M_a \end{array}\right] + \Delta_2 \star \left[\begin{array}{c|c} N_d & N_c \\ \hline N_b & N_a \end{array}\right] = \left[\begin{array}{cc} \Delta_1 & \mathbf{0} \\ \mathbf{0} & \Delta_2 \end{array}\right] \star \left[\begin{array}{cc|c} M_d & \mathbf{0} & M_c \\ \mathbf{0} & N_d & N_c \\ \hline M_b & N_b & M_a + N_a \end{array}\right]$$

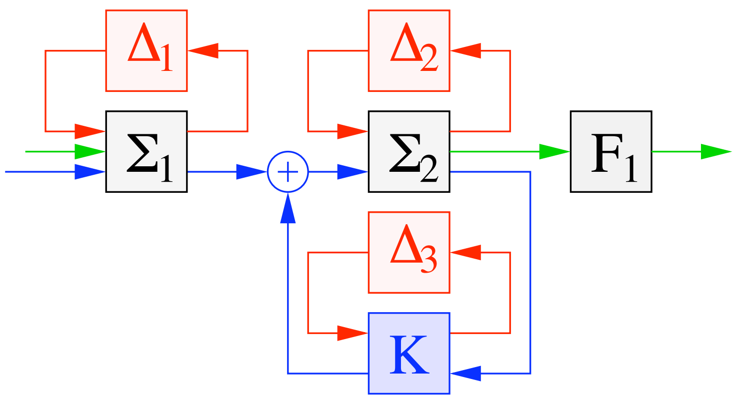
$$\Delta_1 \star \left[\begin{array}{c|c} M_d & M_c \\ \hline M_b & M_a \end{array}\right] \cdot \Delta_2 \star \left[\begin{array}{c|c} N_d & N_c \\ \hline N_b & N_a \end{array}\right] = \left[\begin{array}{cc} \Delta_1 & \mathbf{0} \\ \mathbf{0} & \Delta_2 \end{array}\right] \star \left[\begin{array}{cc|c} M_d & M_c N_b & M_c N_a \\ \mathbf{0} & N_d & N_c \\ \hline M_b & M_a N_b & M_a N_a \end{array}\right]$$

$$\left(\Delta \star \left[\begin{array}{c|c} M_d & M_c \\ \hline M_b & M_a \end{array}\right]\right)^{-1} = \Delta \star \left[\begin{array}{c|c} M_d - M_c M_a^{-1} M_b & -M_c M_a^{-1} \\ \hline M_a^{-1} M_b & M_a^{-1} \end{array}\right]$$

- Coded in Matlab's Robust Control toolbox & in LFRToolbox

- **`Demo in Robust Control toolbox & RoMulOC`**

- Allows also to manipulated complex control schemes



- DEMETER: a satellite of the MYRIAD family developed by CNES

- All MYRIAD microsatellites share common platform (including the control components), the load is different (and the gains of the control law are tuned accordingly).
- On DEMETER the scientific load includes four long appendices that study the ionospheric disturbances (`smsc.cnes.fr/DEMETER`).

Fine pointing towards earth is required.

- CNES: French national space center - governmental
- Accepted to provide data about DEMETER to the scientific community, [PA06]
- It is purely a benchmark: no possible implementation (no more on orbit)

- LAAS studies for the attitude control of DEMETER
- Uncertain modeling at small depointing errors
- Mixed H_2/H_∞ reduced order control design (small depointing) [ADGH11]
- Robustness analysis of the uncertain LTI model in closed-loop (small depointing) [PBG$^+$10]
- Periodic control law design using reaction wheels and magneto torquers (medium to large depointing) [TAP$^+$11]
- Design of an adaptive control law replacing a commuting control (small to medium depointing) [PDPM11]

$$\begin{bmatrix} J(\Delta) & \hat{J}(\Delta) \\ \hat{J}^T(\Delta) & \mathbf{I}_8 \end{bmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\eta} \end{pmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ C(\Delta) & K(\Delta) \end{bmatrix} \begin{pmatrix} \dot{\eta} \\ \eta \end{pmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} u$$

- $\theta \in \mathbb{R}^3$ is the attitude of the satellite close to an orientation defined as $\theta = 0$
- $\eta = \begin{pmatrix} {\eta_1}^T & \cdots & {\eta_4}^T \end{pmatrix}^T \in \mathbb{R}^8$ are the state of the flexible modes
- 4: number of appendices on DEMETER
- each flexible mode is described by 2 states: bending & torsion
- Flexible modes in higher frequencies are neglected (including solar panel)
- All parameters are uncertain (cannot be measured on earth nor on orbit)
- 14 uncertainties enter the model in polynomial (2nd order) manner

■ Natural frequency and damping uncertainties

$$\ddot{\eta} + \overbrace{2\Omega Z}^{C(\Delta)} \dot{\eta} + \overbrace{\Omega^2}^{K(\Delta)} \eta = -\hat{J}^T(\Delta)\ddot{\theta}$$

$$\left[\begin{array}{cc} C(\Delta) & K(\Delta) \end{array}\right] = \left[\begin{array}{cc} 2\Omega Z & \Omega^2 \end{array}\right] = \Omega \left[\begin{array}{cc} 2Z & \Omega \end{array}\right]$$

$$\Omega = \left[\begin{array}{ccc} \omega_1 \mathbf{I}_2 & & \\ & \ddots & \\ & & \omega_4 \mathbf{I}_2 \end{array}\right], \quad Z = \left[\begin{array}{ccc} \zeta_1 \mathbf{I}_2 & & \\ & \ddots & \\ & & \zeta_4 \mathbf{I}_2 \end{array}\right]$$

- Flexible modes of the four identical masts
  - $\omega_i \in [0.2 \cdot 2\pi \ , \ 0.6 \cdot 2\pi] \ \forall i$
  - $\zeta_i \in [5 \cdot 10^{-4} \ , \ 5 \cdot 10^{-3}] \ \forall i$
- 2 states per mast: bending and torsion

■ Natural frequency and damping uncertainties

$$\begin{bmatrix} C(\Delta) & K(\Delta) \end{bmatrix} = \Omega \begin{bmatrix} 2Z & \Omega \end{bmatrix}$$

● LFT modeling with $\Omega = \Omega_a + \Omega_b \Delta_\Omega$ and $Z = Z_a + Z_b \Delta_Z$
such that Δ_Ω and Δ_Z diagonal composed of $|\Delta_{\omega_i}| \leq 1$ and $|\Delta_{\zeta_i}| \leq 1$.

$$\Omega = \Delta_\Omega \star \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \Omega_b & \Omega_a \end{bmatrix} \ , \ Z = \Delta_Z \star \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ Z_b & Z_a \end{bmatrix}$$

$$\begin{bmatrix} 2Z & \Omega \end{bmatrix} = \begin{bmatrix} \Delta_Z & \mathbf{0} \\ \mathbf{0} & \Delta_\Omega \end{bmatrix} \star \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \begin{bmatrix} 2Z_b & \Omega_b \end{bmatrix} & \begin{bmatrix} 2Z_a & \Omega_a \end{bmatrix} \end{bmatrix}$$

$$\Omega \begin{bmatrix} 2Z & \Omega \end{bmatrix} = \begin{bmatrix} \Delta_\Omega & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Delta_Z & 0 \\ \mathbf{0} & \mathbf{0} & \Delta_\Omega \end{bmatrix} \star \left[\begin{array}{cc|c} \mathbf{0} & \begin{bmatrix} 2Z_b & \Omega_b \end{bmatrix} & \begin{bmatrix} 2Z_a & \Omega_a \end{bmatrix} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \hline \Omega_b & \begin{bmatrix} 2\Omega_a Z_b & \Omega_a \Omega_b \end{bmatrix} & \begin{bmatrix} 2\Omega_a Z_a & \Omega_a^2 \end{bmatrix} \end{array}\right]$$

- Inertia of the satellite and contributions of the flexible modes

$$\begin{bmatrix} J(\Delta) & \hat{J}(\Delta) \\ \hat{J}^T(\Delta) & \mathbf{I} \end{bmatrix} = \begin{bmatrix} J_1 + {J_1}^T + J_2^T \tilde{J}^2 J_2 & J_2^T \tilde{J} J_3 \\ J_3^T \tilde{J} J_2 & \mathbf{I} \end{bmatrix}$$

- cross inertia $J_1 = \begin{bmatrix} 0 & J_{12} & J_{13} \\ 0 & 0 & J_{23} \\ 0 & 0 & 0 \end{bmatrix}$

- Coupling between axes is unknown but limited:

- $J_{12} \in [-x.xx\ ,\ y.yy]$, $J_{13} \in [-x.xx\ ,\ y.yy]$, $J_{23} \in [-x.xx\ ,\ y.yy]$

- LFT modeling $J_1 = \begin{bmatrix} 0 & J_{12} & J_{13} \\ 0 & 0 & J_{23} \\ 0 & 0 & 0 \end{bmatrix} = \Delta_{J_1} \star \begin{bmatrix} \mathbf{0} & J_{1c} \\ J_{1b} & J_{1a} \end{bmatrix}$ with

$$J_{1a} = \begin{bmatrix} 0 & J_{12c} & J_{13c} \\ 0 & 0 & J_{23c} \\ 0 & 0 & 0 \end{bmatrix} \quad J_{1b} = \begin{bmatrix} J_{12b} & J_{13b} & 0 \\ 0 & 0 & J_{23b} \\ 0 & 0 & 0 \end{bmatrix} \quad \Delta_{J_1} = \begin{bmatrix} \Delta_{J_{12}} & 0 & 0 \\ 0 & \Delta_{J_{13}} & 0 \\ 0 & 0 & \Delta_{J_{23}} \end{bmatrix} \quad J_{1c} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- and the normalized uncertainties are such that $|\Delta_{J_{ij}}| \leq 1$.

$$J_1 + {J_1}^T = \begin{bmatrix} \Delta_{J_1} & \mathbf{0} \\ \mathbf{0} & \Delta_{J_1} \end{bmatrix} \star \left[\begin{array}{cc|c} \mathbf{0} & \mathbf{0} & J_{1c} \\ \mathbf{0} & \mathbf{0} & J_{1b}^T \\ \hline J_{1b} & J_{1c}^T & J_{1a} + J_{1a}^T \end{array}\right]$$

$$\begin{bmatrix} J_1 + {J_1}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \Delta_{J_1} & 0 \\ 0 & \Delta_{J_1} \end{bmatrix} \star \left[\begin{array}{cc|cc} \mathbf{0} & \mathbf{0} & J_{1c} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & J_{1b}^T & \mathbf{0} \\ \hline J_{1b} & J_{1c}^T & J_{1a} + J_{1a}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right]$$

- Inertia of the satellite and contributions of the flexible modes

$$\begin{bmatrix} J(\Delta) & \hat{J}(\Delta) \\ \hat{J}^T(\Delta) & \mathbf{I} \end{bmatrix} = \begin{bmatrix} J_1 + {J_1}^T + J_2^T \tilde{J}^2 J_2 & J_2^T \tilde{J} J_3 \\ J_3^T \tilde{J} J_2 & \mathbf{I} \end{bmatrix}$$

- square root of uncertainties on direct inertia $\tilde{J} = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix}$
  - Inertia of each axis is positive unknown and bounded.
  - Coupling with flexible modes depend of these.

▲ $J_{11} \in [xx.x\ ,\ yy.y]$, $J_{22} \in [xx.x\ ,\ yy.y]$, $J_{33} \in [xx.x\ ,\ yy.y]$

$$\begin{bmatrix} J_2^T \tilde{J}^2 J_2 & J_2^T \tilde{J} J_3 \\ J_3^T \tilde{J} J_2 & J_3^T J_3 \end{bmatrix} = \begin{bmatrix} J_2^T \tilde{J} \\ J_3^T \end{bmatrix} \begin{bmatrix} \tilde{J} J_2 & J_3 \end{bmatrix}$$

- LFT modeling with $\tilde{J} = \mathbf{I} + \Delta_{\tilde{J}} \tilde{J}_c$ where $|\Delta_{\tilde{J}_{ii}}| \leq 1$:

$$\begin{bmatrix} J_2^T \tilde{J} \\ J_3^T \end{bmatrix} = \Delta_{\tilde{J}} \star \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline J_2^T \tilde{J}_c^T & J_2^T \\ \mathbf{0} & J_3^T \end{array}\right], \quad \begin{bmatrix} \tilde{J} J_2 & J_3 \end{bmatrix} = \Delta_{\tilde{J}} \star \left[\begin{array}{c|cc} \mathbf{0} & \tilde{J}_c J_2 & \mathbf{0} \\ \hline \mathbf{I} & J_2 & J_3 \end{array}\right]$$

$$\begin{bmatrix} J_2^T \tilde{J}^2 J_2 & J_2^T \tilde{J} J_3 \\ J_3^T \tilde{J} J_2 & J_3^T J_3 \end{bmatrix} = \begin{bmatrix} \Delta_{\tilde{J}} & \mathbf{0} \\ \mathbf{0} & \Delta_{\tilde{J}} \end{bmatrix} \star \left[\begin{array}{cc|cc} \mathbf{0} & \mathbf{I} & J_2 & J_3 \\ \mathbf{0} & \mathbf{0} & \tilde{J}_c J_2 & \mathbf{0} \\ \hline J_2^T \tilde{J}_c^T & J_2^T & J_2^T J_2 & J_2^T J_3 \\ \mathbf{0} & J_3^T & J_3^T J_2 & J_3^T J_3 \end{array}\right]$$

- LFT model of the inertia matrix

$$\begin{bmatrix} J(\Delta) & \hat{J}(\Delta) \\ \hat{J}^T(\Delta) & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \Delta_{J_1} & 0 \\ 0 & \Delta_{J_1} \end{bmatrix} \star \left[\begin{array}{cc|cc} \mathbf{0} & \mathbf{0} & J_{1c} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & J_{1b}^T & \mathbf{0} \\ \hline J_{1b} & J_{1c}^T & J_{1a} + J_{1a}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right]$$

$$+ \begin{bmatrix} \Delta_{\tilde{J}} & \mathbf{0} \\ \mathbf{0} & \Delta_{\tilde{J}} \end{bmatrix} \star \left[\begin{array}{cc|cc} \mathbf{0} & \mathbf{I} & J_2 & J_3 \\ \mathbf{0} & \mathbf{0} & \tilde{J}_c J_2 & \mathbf{0} \\ \hline J_2^T \tilde{J}_c^T & J_2^T & J_2^T J_2 & J_2^T J_3 \\ \mathbf{0} & J_3^T & J_3^T J_2 & J_3^T J_3 \end{array}\right] + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} - J_3^T J_3 \end{bmatrix}$$

$$= \begin{bmatrix} \Delta_{J_1} & & & \\ & \Delta_{J_1} & & \\ & & \Delta_{\tilde{J}} & \\ & & & \Delta_{\tilde{J}} \end{bmatrix} \star \left[\begin{array}{cccc|cc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & J_{1c} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & J_{1b}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & J_2 & J_3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \tilde{J}_c J_2 & \mathbf{0} \\ \hline J_{1b} & J_{1c}^T & J_2^T \tilde{J}_c^T & J_2^T & J_{1a} + J_{1a}^T + J_2^T J_2 & J_2^T J_3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & J_3^T & J_3^T J_2 & \mathbf{I} \end{array}\right]$$

$$\begin{bmatrix} J(\Delta) & \hat{J}(\Delta) \\ \hat{J}^T(\Delta) & \mathbf{I}_8 \end{bmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\eta} \end{pmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ C(\Delta) & K(\Delta) \end{bmatrix} \begin{pmatrix} \dot{\eta} \\ \eta \end{pmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} u$$

- Descriptor LFT model with $X^T = \begin{pmatrix} \dot{\theta}^T & \dot{\eta}^T & \theta^T & \eta^T \end{pmatrix}$:

$$\text{diag}\begin{bmatrix} \Delta_{J_1} \\ \Delta_{J_1} \\ \Delta_{\tilde{J}} \\ \Delta_{\tilde{J}} \end{bmatrix} \star \begin{bmatrix} E_d & E_c \\ E_b & E_a \end{bmatrix} \dot{X} = \text{diag}\begin{bmatrix} \Delta_\Omega \\ \Delta_Z \\ \Delta_\Omega \end{bmatrix} \star \begin{bmatrix} A_d & A_c \\ A_b & A_a \end{bmatrix} X + Bu$$

- Non-descriptor LFT model (taking the inverse of the left-hand side matrix):

$$\dot{X} = \Delta \star \left[\begin{array}{cc|cc} E_d - E_c E_a^{-1} E_b & -E_c E_a^{-1} A_b & -E_c E_a^{-1} A_a & -E_c E_a^{-1} B \\ \mathbf{0} & A_d & A_c & \mathbf{0} \\ \hline E_a^{-1} E_b & E_a^{-1} A_b & E_a^{-1} A_a & E_a^{-1} B \end{array}\right] \begin{pmatrix} X \\ u \end{pmatrix}$$

with $\Delta = \text{diag}\left[\begin{array}{cccc|ccc} \Delta_{J_1} & \Delta_{J_1} & \Delta_{\tilde{J}} & \Delta_{\tilde{J}} & \Delta_\Omega & \Delta_Z & \Delta_\Omega \end{array}\right]$.

- Benchmark coded with possibility to choose
  - 1, 2 or 3 axis model
  - 0 to 4 appendices (each contributing to 2 flexibles dynamics)
  - Identical or distinct models of appendices
  - With or without models of reaction wheel actuators
- Models with more or less numerical complexity
  - **`Demo in RoMulOC`**

- Prospective for RoMulOC future versions: **descriptor modeling**

- LFT descriptor model of DEMETER - simple, closer to original equations

$$\Delta \star \left[\left[\begin{array}{cc|c} E_d & \mathbf{0} & E_c \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline E_b & \mathbf{0} & E_a \end{array} \right] \dot{X} = \left[\begin{array}{cc|c} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_d & A_c \\ \hline \mathbf{0} & A_b & A_a \end{array} \right] X + Bu \right]$$

- $\Delta \in \mathrm{R}^{36\times 36}$: exogenous LFT signals are $z_\Delta \in \mathrm{R}^{36}$, $w_\Delta \in \mathrm{R}^{36}$

- Descriptor model also allows to model rationally dependent models in polytopic affine form

$$\left[\begin{array}{cc} J_1 + J_1^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \hline \tilde{J} J_2 & J_3 \\ \mathbf{0} & \mathbf{0} \end{array} \right] \begin{pmatrix} \ddot{\theta} \\ \ddot{\eta} \end{pmatrix} + \left[\begin{array}{cc} J_2^T \tilde{J} & \mathbf{0} \\ J_3^T & \Omega \\ \hline -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array} \right] \begin{pmatrix} \pi_J \\ \pi_\eta \end{pmatrix} + \left[\begin{array}{cc} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} \\ 2Z & \Omega \end{array} \right] \begin{pmatrix} \dot{\theta} \\ \eta \end{pmatrix} = \left[\begin{array}{c} \mathbf{I} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{array} \right] u$$

- Includes fictive signals $\pi_J \in \mathrm{R}^3$ and $\pi_\eta \in \mathrm{R}^8$ - only 11 exogenous signals!

- [CTF02], [TD13] Descriptor models also appropriate for rationally non-linear systems,
- e.g. satellite

$$J\dot{\omega} + \omega^\times J\omega = u \quad , \quad \dot{q} = \frac{1}{2} \left[\begin{array}{cc} -\omega^\times & \omega \\ -\omega^T & 0 \end{array} \right] q$$

- Two classes of uncertain systems : polytopic & LFT
  - Airplane example
  - DEMETER satellite example
  - Prospectives: descriptor uncertain modeling
- Robust analysis
  - Stability and performances - the well-posedness point of view
  - System augmentation approach for sequences of SOS-like relaxations
  - "Slack variable" results and "quadratic stability" as a special case
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- Dynamical system viewed as an LFT

$$\dot{x} = Ax \quad \Leftrightarrow \quad s^{-1}\mathbf{I} \star A \quad \Leftrightarrow \quad \begin{cases} w = s^{-1}z \\ z = Aw \end{cases}$$

- s^{-1}: integrator. It is characterized by $s^{-1} + s^{-*} \geq 0$
  - Property to be understood in the sense of scalar products of signals, with zero initial conditions:

$$< \dot{x}|s^{-1}\dot{x} >_\tau = < \dot{x}|x >_\tau = \int_0^\tau \dot{x}^T(t)x(t)dt = x(\tau)^T x(\tau) \geq 0$$

- Well-posedness of the loop: $(s\mathbf{I} - A)$ non-singular for all $s^{-1} + s^{-*} \geq 0$
  - i.e. all poles of A are in the open left-half of the complex plane

- Stability is equivalent to the well-posedness problem
  - Robust stability of LFT model equivalent to well-posedness of

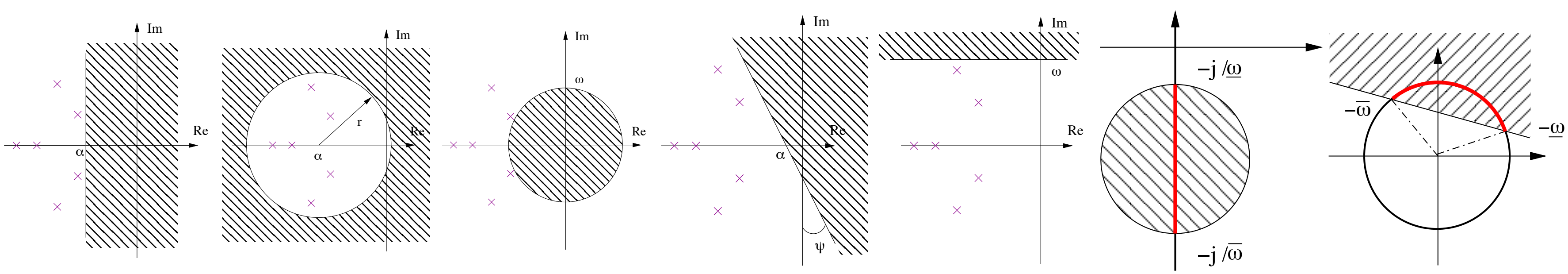
$$\begin{bmatrix} s^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Delta \end{bmatrix} \star \begin{bmatrix} A & B_\Delta \\ C_\Delta & D_{\Delta\Delta} \end{bmatrix} \quad , \quad \begin{matrix} s^{-1} \in \mathbb{C}_+ \\ \Delta \in \mathbf{\Delta} \end{matrix}$$

- Dynamical system viewed as an LFT - discrete-time case

$$x_{k+1} = Ax_k \quad \Leftrightarrow \quad z^{-1}\mathbf{I} \star A \quad \Leftrightarrow \quad \begin{cases} w = z^{-1}z \\ z = Aw \end{cases}$$

- z^{-1}: delay operator. It is characterized by $z^{-*}z^{-1} \leq 1$.
- Well-posedness of the loop: $(z\mathbf{I} - A)$ non-singular for all $z^*z \geq 1$
- i.e. all poles of A are in the open unit disc

- Same procedure applies to pole location & frequency-dependent specifications [IH05, IHF05]



- Input/Output performances can also be written as well-posedness problems
  - For H_∞ (or induced L_2) norm: small gain theorem

$$\|D + C(s\mathbf{I} - A)^{-1}B\|_\infty \leq \gamma \quad \Leftrightarrow \quad \begin{bmatrix} s^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Delta \end{bmatrix} \star \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad , \quad \begin{array}{c} s^{-1} \in \mathbb{C}_+ \\ \|\Delta\| \leq \frac{1}{\gamma} \end{array}$$

  - Similar well-posedness interpretations for H_2 & impulse-to-peak performances [PBG09]

- All robust analysis problems coded in RoMulOC boil down to well-posedness of

$$\nabla \star M \quad \text{or} \quad \nabla \star M(\xi)$$

  - with $\nabla \in \mathbb{W}$ a structured (block-diagonal) uncertain operator
  - and $M(\xi) = \sum \xi_v M^{[v]}$ affine polytopic with respect to parametric uncertainties

- Deriving LMIs for guaranteeing well-posedness of $\nabla \star M \;,\; \nabla \in \mathbb{V}$
  - Done in the framework of **topological separation** [Saf80]
  - **Quadratic separation** in the context of linear systems with parametric uncertainties [IH98, IS01, PAHG07, Pea07]
  - **Integral quadratic separation** in the context of linear systems with TV/NL operators [PBG09, Pea09, PTGSB12]
  - Similar to results from **Integral quadratic constraints** literature [MR97, JM99, SK08]
- Well-posedness **iff** exists a matrix Θ solution to one LMI + one IQC

$$\begin{bmatrix} \mathbf{I} & M^T \end{bmatrix} \Theta \begin{bmatrix} \mathbf{I} \\ M \end{bmatrix} \succ \mathbf{0} \;,\quad \left\langle \begin{bmatrix} \nabla \\ \mathbf{I} \end{bmatrix} \middle| \Theta \begin{bmatrix} \nabla \\ \mathbf{I} \end{bmatrix} \right\rangle \leq 0 \;,\; \forall \nabla \in \mathbb{V}$$

  - IQC can be converted to finite set of LMI constraints $F_{\mathbb{V}}(\Theta) \prec \mathbf{0}$
  - Conservative in general, except some lossless cases [MSF97])
  - Contains generalized KYP-lemma [IH05], S-procedure, DG-scalings etc.

- Well-posedness of $\nabla \star M \quad , \quad \nabla \in \mathbb{W}$ **if** exists a matrix Θ solution to LMIs

$$\begin{bmatrix} \mathbf{I} & M^T \end{bmatrix} \Theta \begin{bmatrix} \mathbf{I} \\ M \end{bmatrix} \succ \mathbf{0} \quad , \quad F_{\mathbb{W}}(\Theta) \prec \mathbf{0}$$

- No need to understand all that to use RoMulOC

- **`Demo in RoMulOC`**

- Result extend to well-posedness of descriptor LFTs: $\left\{ \begin{array}{l} w = \nabla z \\ M_1 z = M_2 w \end{array} \right.$

- Reducing conservatism: the system augmentation approach [EPAH05, PAHG07]
- Original robust stability problem: well-posedness of

$$\begin{cases} x = s^{-1}\dot{x} \\ w = \Delta z \end{cases} \star \begin{cases} \dot{x} = Ax + Bw \\ z = Cx + Dw \end{cases}$$

- First augmentation (assuming $\dot{\Delta} = \mathbf{0}$)

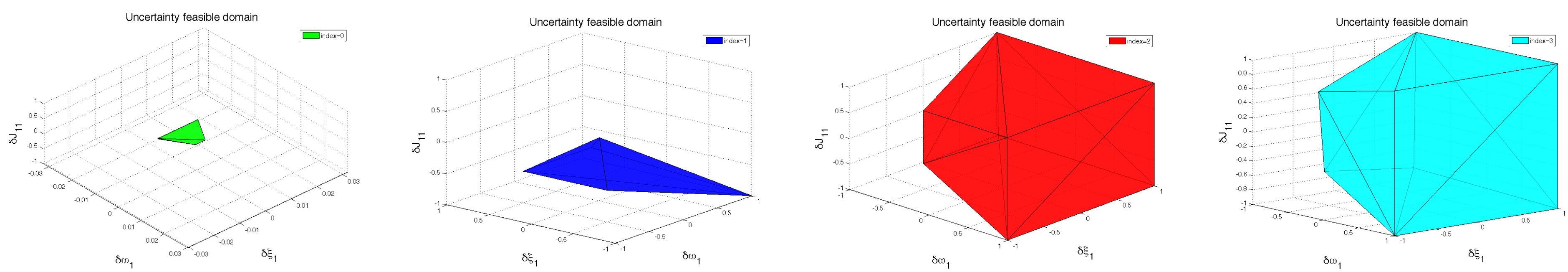
$$\begin{cases} x = s^{-1}\dot{x} \\ w = s^{-1}\dot{w} \\ w = \Delta z \\ \dot{w} = \Delta \dot{z} \end{cases} \star \begin{cases} \dot{x} = Ax + Bw \\ \dot{w} = \dot{w} \\ z = Cx + Dw \\ \dot{z} - C\dot{x} = D\dot{w} \end{cases}$$

- IQS applied to this well-posedness problem gives larger LMIs, proved to be less conservative
- Underlying Lyapunov function is parameter-dependent $V(x, \Delta) =$

$$\begin{pmatrix} x \\ w \end{pmatrix}^T P_1 \begin{pmatrix} x \\ w \end{pmatrix} = x^T \begin{bmatrix} \mathbf{I} \\ \Delta(\mathbf{I} - \Delta D)^{-1} C \end{bmatrix}^T P_1 \begin{bmatrix} \mathbf{I} \\ \Delta(\mathbf{I} - \Delta D)^{-1} C \end{bmatrix} x = x^T P(\Delta) x$$

- First augmentation (assuming $\dot{\Delta} = \mathbf{0}$)
  - **`Demo in RoMulOC`**

- Further augmentations (based on $\dot{\Delta} = \mathbf{0}$, $\ddot{\Delta} = \mathbf{0}$ ...)
  - Provide sequence of less and less conservative LMIs (with increasing dimensions)
  - Method directly related to Lasserre's SOS relaxations [PS09]
  - Method applies also for time-delay systems [GP06], saturations [PTGSB12]...
  - Not yet coded in RoMulOC
  - Tested on DEMETER model [PBG$^+$10]



- Well-posedness in the case of polytopic models $\nabla \star M(\xi)$
  - with $\nabla \in \mathbb{V}$ a structured (block-diagonal) uncertain operator
  - and $M(\xi) = \sum \xi_v M^{[v]}$ affine polytopic with respect to parametric uncertainties

- Applying same method leads to solving for all $\xi \in \left\{ \sum_{v=1}^{\overline{v}} \xi_v = 1 \quad , \quad \xi_v \geq 0 \right\}$

$$\left[\begin{array}{cc} \mathbf{I} & M^T(\xi) \end{array} \right] \Theta(\xi) \left[\begin{array}{c} \mathbf{I} \\ M(\xi) \end{array} \right] \succ \mathbf{0} \quad , \quad F_{\mathbb{V}}(\Theta(\xi)) \prec \mathbf{0}$$

  - Classical **quadratic stability** framework: solve for all vertices $v = 1 \dots \overline{v}$

$$\left[\begin{array}{cc} \mathbf{I} & M^{[v]T} \end{array} \right] \Theta \left[\begin{array}{c} \mathbf{I} \\ M^{[v]} \end{array} \right] \succ \mathbf{0} \ , \ F_{\mathbb{V}}(\Theta) \prec \mathbf{0} \ , \ \Theta_{22} \preceq \mathbf{0}$$

- ▲ Conservatism due to the choice of Θ unique for all ξ
- ▲ Applies for convex sets $\mathbb{V}$ (not applicable for pole location outside a disc)

$$\left[\begin{array}{cc} \mathbf{I} & M^T(\xi) \end{array}\right] \Theta(\xi) \left[\begin{array}{c} \mathbf{I} \\ M(\xi) \end{array}\right] \succ \mathbf{0} \quad , \quad F_{\blacktriangledown}(\Theta(\xi)) \prec \mathbf{0}$$

- Classical **quadratic stability** framework: solve for all vertices $v = 1 \dots \overline{v}$

$$\left[\begin{array}{cc} \mathbf{I} & M^{[v]T} \end{array}\right] \Theta \left[\begin{array}{c} \mathbf{I} \\ M^{[v]} \end{array}\right] \succ \mathbf{0} \ , \ F_{\blacktriangledown}(\Theta) \prec \mathbf{0} \ , \ \Theta_{22} \preceq \mathbf{0}$$

- The **slack variable** approach [OBG99, OGH99, PABB00, Pea09, PDSV09]

(A book is to come on the topic in 2014 - Y. Ebihara, D. Peaucelle)

$$\Theta^{[v]} \succ G \left[\begin{array}{cc} M^{[v]} & -\mathbf{I} \end{array}\right] + \left[\begin{array}{c} M^{[v]T} \\ -\mathbf{I} \end{array}\right] G^T \ , \ F_{\blacktriangledown}(\Theta^{[v]}) \prec \mathbf{0}$$

- Less conservative and with $\Theta(\xi) = \sum_{v=1}^{\overline{v}} \xi_v \Theta^{[v]}$
- Smartly coded in RoMulOC to avoid unnecessary slack variables
- e.g. if $\Theta^{[v]} = \Theta$ unique and $\Theta_{22} \prec \mathbf{0}$ there is no need for G
- **`Demo in RoMulOC`**

- Two classes of uncertain systems : polytopic & LFT
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■ Most well-posedness have a dual counterpart

$$\nabla \star M \text{ w.p.} \quad \Leftrightarrow \quad \nabla^* \star M^T \text{ w.p.}$$

▲ e.g. poles of $\dot{x} = Ax$ same as poles of $\dot{x}_d = A^T x_d$

▲ e.g. the H_∞ and H_2 norms

$$\|D + C(s\mathbf{I} - A)^{-1}B\|_\infty = \|D^T + B^T(s^*\mathbf{I} - A^T)^{-1}C^T\|_\infty$$

● Interesting feature, even for robust analysis problems

▲ LMIs for $\nabla \star M$ prove stability with Lyapunov matrix $P(\Delta)$ quadratic in Δ

▲ LMIs for $\nabla^* \star M^T$ prove stability with Lyapunov matrix $P^{-1}(\Delta)$ quadratic in Δ

▲ **`Demo in RoMulOC`**

- From analysis to state-feedback problems,
- The stability case

$$\dot{x} = Ax \quad \rightarrow \quad \dot{x} = (A + BK)x$$

- The general well-posedness case

$$\nabla \star M \quad \rightarrow \quad \nabla \star (M_A + M_B \begin{bmatrix} K & \mathbf{0} \end{bmatrix})$$

- Classical methodology to get LMIs:
- Apply analysis conditions to dual model $\nabla^* \star (M_A^T + \begin{bmatrix} K^T \\ \mathbf{0} \end{bmatrix} M_B^T)$ to get a PMI

$$\begin{bmatrix} \mathbf{I} & M_A + M_B \begin{bmatrix} K & \mathbf{0} \end{bmatrix} \end{bmatrix} \Theta \begin{bmatrix} \mathbf{I} \\ M_A^T + \begin{bmatrix} K^T \\ \mathbf{0} \end{bmatrix} M_B^T \end{bmatrix} \succ \mathbf{0}$$

- If $\Theta_{22} \preceq 0$ apply a Schur complement argument to get a BMI
- Apply a linearizing change of variables $XK = Y \quad \Rightarrow \quad F_{QS}(\Theta, Y) \prec 0$

`Demo in RoMulOC`

- The slack variables case
- BMI problem emerging from conditions on the dual model

$$\Theta^{[v]} \succ G \begin{bmatrix} M_A^{[v]T} + \begin{bmatrix} K^T \\ \mathbf{0} \end{bmatrix} M_B^{[v]T} & -\mathbf{I} \end{bmatrix} + (\dots)^T$$

- Exist linearizing change of variables if structuring a priori as $G = \begin{bmatrix} M_G \\ -\mathbf{I} \end{bmatrix} H$

$$\Rightarrow \; F_{SV}(M_G, \Theta^{[v]}, H, Y) \prec 0$$

- $\nabla^* \star M_G$ should be well-posed
- In some case there exists M_G such that (SV) guaranteed to include (QS)

- Some extensions of these results
- For periodic systems, M_G has interesting non causal properties [EPA11]
- Applies to static output feedback design with minor modifications

M_G is then a stabilizing state-feedback, [PA01, AGPP10]

- The multi-performance state-feedback problem, some examples
- Find a unique K that stabilizes three different systems M_1, M_2, M_3
- Find K that locates the closed loop-poles of M_1 in a disc and minimizes H_∞ norm

- In terms of LMI conditions : shaping paradigm [SGC97, ACP06]
- Concatenate each LMI relative to each specification
- Enforce common linearizing variable (X in QS case H in SV case)
- Slack variable allows different Lyapunov matrices for each performance (less conservative)
- Complex to find a priori M_G for each performance

- **`Demo in RoMulOC`**

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Conclusions

- What RoMulOC does:
  - Provides LMI tests for many robust control problems
  - Analysis and state-feedback
  - Performances : stability, pole location, H_∞, H_2, impulse-to-peak
  - LFT and polytopic models
  - Two levels of results with different amount of conservatism

- RoMulOC Prospectives
  - Further relaxations for reduced conservatism
  - Descriptor systems
  - Non-linearities and TV uncertainties
  - Output feedback design

- What is R-RoMulOC ?
  - The same but including randomization features from RACT, [TCD$^+$, TCD13]

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