General polynomial parameter-dependent Lyapunov functions for polytopic uncertain systems

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Abstract

New LMI conditions are given for robust H_2 analysis of polytopic systems based on polynomial parameter-dependent Lyapunov functions. Results are derived using a "slack variables" approach. Numerical results illustrate the characteristics of the proposed results in terms of conservatism reduction and numerical complexity.
Keywords: Polynomial parameter-dependent Lyapunov functions, polytopic systems, robustness, stability, H_2.

1 Introduction

Robust stability and performance analysis of polytopic systems via parameter-dependent Lyapunov functions and LMI-based methods have been extensively studied in the past five years leading to four classes of results:

"Slack variables" method.
Initiated by results presented by de Oliveira and Geromel in [4, 5] for disctrete-time systems and then extended to continuous-time systems and robust pole location in [12], this methodology is based on a convexification of polynomial parameter-denendent LMIs by the introduction of additional variables using the elimination lemma backwards. The present paper adopts this same framework to solve robust analysis problems by means of polynomial parameter-dependent Lyapunov functions (PPDLF). The augmentation of the degree of the PPDLF is shown to reduce the conservatism of the derived LMI conditons. The results are extensions of previous contributions in [7, 8] and in [14]. As it will be demonstrated, this technique is quite effective on examples but has two major drawbacks: numerical complexity due to the large number of additional "slack" variables and no proof of convergence towards exact robustness results as the degree of the PPDLF grows.

"Positive polynomials with positive coefficients" method.
This technique that first appeared in [13] and that was recently improved in [11] handles the positivity of polynomial parameter-dependent LMIs over the set of positive uncertain parameters by testing that all matrix coefficients are positive. In the case of affine PDLFs, a sequence of such conditions involving exponentially many LMI constraints is proved to have asymptotic convergence properties. In [10] this technique has been combined with the "slack variables" method, therefore suffering from both the large amount of decision variables and constraints. Impressive results have nevertheless been demonstrated for random systems of small dimensions. At our knowledge, no results based on this method handles general PPDLFs.

”D-scalling” method.
This approach, initiated in papers [6, 9], tends to transform the original problem of stability of polytopic systems into a more general modeling framework where the uncertain parameters enter the system as an exogenous feedback connection. This reformulation allows to apply KYP-lemma, D-scaling, DG-scaling, full block S-procedure or quadratic separation results which are yet another approach to convert polynomial parameter-dependent LMIs into parameter-independent LMIs with additional variables. This methodology was generalized for any PPDLF in [1, 15] and convergence towards exact robustness results is obtained when the degree of the PPDLF grows.

”Sum-of-squares” method.
This more recent approach, [2, 3], considers a general form of PPDLFs and shows that for polytopic systems these polynomial parameter-dependent functions may always be chosen homogeneous. The main result consists in writing conditions for positive definite polynomial parameter-depend matrices to be sum-of-squares leading to sufficient parameter-independent LMI conditions for the original parameter-dependent problem. Results have no proof of convergence towards exact robustness results as the degree of the PPDLF grows. As for the ”slack variables” approach, numerous additional variables are introduced for describing the degrees of freedom of Square Matricial Representation (SMR) of homogeneous forms.

For all these methods the main objectives are: to give implementable LMI formulations for the most general PPDLFs; to evaluate the numerical complexity that in general grows exponentially with the degree of the PPDLF; to prove convergence properties towards ”exact” results. In the present paper the aim is to answer to the first aspect within the ”slack variables” framework. Comparisons with other cited results is not included in this draft but is expected for the final paper.

2 Preliminaries

Define two polytopic parameter-dependent matrices as follows

$$A(\zeta) = \sum_{i=1}^{N} \zeta_i A_i \ , \ \ \Pi(\zeta) = \sum_{i=1}^{N} \zeta_i \Pi_i$$

where the ζ is the vector of the N uncertain parameters constrained by

$$\zeta_i \geq 0 \ , \ \ \sum_{i=1}^{N} \zeta_i = 1 \ . \tag{1}$$

For the analysis of polytopic uncertain linear time-invariant (LTI) systems defined by

$$\dot{x}(t) = A(\zeta)x(t)$$

we have suggested in [7] to do the analysis of the artificially augmented system

$$\left\{ \begin{array}{l} \dot{x}(t) = A(\zeta)x(t) \\ A(\zeta)\dot{x}(t) = A^2(\zeta)x(t) \\ \vdots \\ A^r(\zeta)\dot{x}(t) = A^{r+1}(\zeta)x(t) \end{array} \right. \tag{2}$$

and proved that it amounts to attesting robust stability of the original system using a PPDLF of the type

$$x^T \begin{bmatrix} 1 \\ A(\zeta) \\ \vdots \\ A^r(\zeta) \end{bmatrix}^T \Pi(\zeta) \begin{bmatrix} 1 \\ A(\zeta) \\ \vdots \\ A^r(\zeta) \end{bmatrix} x \ .$$

In [8], this results has been extended by using a PPDLF of the type

$$x^T \begin{bmatrix} 1 \\ M(\zeta) \\ \vdots \\ M^r(\zeta) \end{bmatrix}^T \Pi(\zeta) \begin{bmatrix} 1 \\ M(\zeta) \\ \vdots \\ M^r(\zeta) \end{bmatrix} x$$

where $M(\zeta)$ is an arbitrarily chosen polytopic parameter-dependent matrix. The obtained results amount to the robust analysis of the artificially augmented system

$$\left\{ \begin{array}{l} \dot{x}(t) = A(\zeta)x(t) \\ M(\zeta)\dot{x}(t) = M(\zeta)A(\zeta)x(t) \\ \vdots \\ M^r(\zeta)\dot{x}(t) = M^r(\zeta)A(\zeta)x(t) \ . \end{array} \right. \quad (3)$$

A "good" choice for $M(\zeta)$ is non trivial. The last formulation includes the former one but has the same disadvantage of considering special parameter-dependent polynomial Lyapunov functions of degree $2r+1$. Therefore, conditions for more general types of PPDLFs of fixed degree are needed to extend these results. To do so, one can introduce the following redundant equations:

$$\zeta_i^{j_i} \dot{x}(t) = \zeta_i^{j_i} A x(t) \ .$$

This approach leads to write PPDLFs of the type

$$x^T (\zeta^{[r]} \otimes 1_n)^T \Pi(\zeta) (\zeta^{[r]} \otimes 1_n) x \quad (4)$$

where $\zeta^{[r]} \otimes 1_n$ is a block column matrix composed of $\zeta_i^{j_i} 1_n$ elements of degrees constrained by $j_i \leq r_i$. For example, in case of three parameters $N = 3$, the choice $r = (2\ 1\ 0)$ corresponds to $\zeta^{[r]} = \begin{pmatrix} 1 & \zeta_1 & \zeta_1^2 & \zeta_2 \end{pmatrix}^T$.

Applied with the same methodology as proposed in [8] and on the same second example, we get the numerical results of Table 1. γ_r stands for the minimal attainable guaranteed H_2 cost when applying the methods for a chosen vector r. Only some results among all tested vectors r are reported, including the best ones. For each choice of r, the number of decision variables depends only on $\sum r_i$. For $\sum r_i = 0$, 1, 2, 3, 4 and 5 the number of variables are respectively 52, 217, 499, 898, 1414 and 2047. Note that if $\tilde{r} \leq r$ (element-wize) then the guaranteed costs can be proved to be ordered as $\gamma_{\tilde{r}} \geq \gamma_r$. This is indeed the case for the results of Table 1.

Table 1: Numerical results for example 2 in [8] using a PPDLF of the type (4)

r	γ_r	r	γ_r	r	γ_r
(0 0 0)	8.31	(1 1 1)	4.14		
		(2 1 0)	3.73	(2 1 1)	3.73
(1 0 0)	4.83	(1 2 0)	4.09		
(0 1 0)	5.29	(1 0 2)	4.10	(2 2 1)	3.68
(0 0 1)	4.90	(3 0 0)	3.98	(3 1 1)	3.54
		(0 3 0)	5.14	(1 1 3)	4.08
(1 1 0)	4.14	(0 0 3)	4.80	(3 0 2)	3.51
(2 0 0)	4.17				
(0 2 0)	5.14				
(0 0 2)	4.83				

The numerical results of Table 1 are to be compared with those of [8]. Choosing $M(\zeta) = A(\zeta)$ we had obtained for $r = 1$, 2 and 3 respectively $\gamma = 4.7339$, 4.2177 and 3.8307. Then the choice of $M(\zeta) = \zeta_1 1_n$

was tested and gave for $r = 1$ and 2 better results (see $r = (1\ 0\ 0)$ and $r = (2\ 0\ 0)$ in Table 1). These results are much less conservative than the quadratic stability framework (parameter-independent Lyapunov function) that gives the upper bound of 18.1490. Nevertheless they seem still quite conservative compared to a lower bound of 1.3208 obtained by fine griding search over the set of uncertainties.

A clear conclusion of these numerical tests is that the "slack variables" framework may be applied to solve robustness problems with PPDLFs, but results depend on the choice of the monomials involved in the PPDLF. More precisely, in (4) the parameter-dependent Lyapunov matrix $P(\zeta)$ such that $V(x,\zeta) = x^T P(\zeta) x$ is of the type

$$P(\zeta) \;= (\zeta^{\{r\}} \otimes 1_n)^T \Pi(\zeta)(\zeta^{\{r\}} \otimes 1_n) \;= \sum_{i_1,i_2,i_3} \zeta_{i_1}\zeta_{i_2}^{j_{i_2}}\zeta_{i_3}^{j_{i_3}} P_{i_1,i_2,j_{i_2},i_3,j_{i_3}}$$

and whatever the maximal degrees chosen for each parameter, the monomials composed of more than three parameters are left aside. For that reason the present paper focuses on a more general result where the PPDLFs involve all possible monomials up to a certain degree. The proposed PPDLFs are of the type

$$V_q(x,\zeta) = x^T P_q(\zeta) x = x^T (\sum \alpha_j(\zeta)\Pi_j(\zeta))x \tag{5}$$

where $\Pi_j(\zeta)$ are affine parameter-dependent symmetric matrices and $\alpha_j(\zeta) = \zeta_1^{j_1}\zeta_2^{j_2}\cdots\zeta_N^{j_N}$ are all the monomials up to a degree q ($\sum j_i \leq q$).

3 Notations

$\mathsf{R}^{m\times n}$ is the set of m-by-n real matrices. A^T is the transpose of the matrix A. $\mathsf{1}$ and $\mathsf{0}$ are respectively the identity and the zero matrices of appropriate dimensions. For Hermitian matrices, $A > (\geq) B$ if and only if $A - B$ is positive (semi) definite.

Let a vector of integers $j = \begin{pmatrix} j_1 & \dots & j_N \end{pmatrix}$. Define the notation $\Sigma j = \sum_{i=1}^N j_i$. For all such j vectors, $\alpha_j(\zeta) = \zeta_1^{j_1}\zeta_2^{j_2}\cdots\zeta_N^{j_N}$ is a monomial of degree Σj of the variables ζ_i, $i \in \{1 \dots N\}$. The number of monomials of degree going from 0 to q is given by

$$p_{q,N} = (N+q)!/(N!q!)$$

Define $\beta_{k,q}(\zeta)$ the vector of all monomials of degrees k to q and let $\beta_k(\zeta) = \beta_{k,k}(\zeta)$. Trivially $\beta_0(\zeta) = 1$ and $\beta_1(\zeta) = (\zeta_1 \dots \zeta_N)^T$. Define recursively the following linear parameter-dependent matrices:

$$M_k^0(\zeta) = \begin{bmatrix} \zeta_k \\ \zeta_{k+1} \\ \vdots \\ \zeta_N \end{bmatrix} \;,\; M_k^{j+1}(\zeta) = \begin{bmatrix} M_k^j(\zeta) & \mathsf{0} & \cdots & \mathsf{0} \\ \mathsf{0} & M_{k+1}^j(\zeta) & & \\ \vdots & & \ddots & \\ \mathsf{0} & & & M_N^j(\zeta) \end{bmatrix}$$

and let the affine parameter-dependent matrices

$$\begin{bmatrix} \Upsilon_q(\zeta) & \Psi_q(\zeta) \end{bmatrix} = \left[\begin{array}{c|ccccc} M_1^0(\zeta) & -\mathsf{1} & \mathsf{0} & \cdots & & \mathsf{0} \\ \mathsf{0} & M_1^1(\zeta) & -\mathsf{1} & & & \\ \vdots & & & \ddots & \ddots & \\ \mathsf{0} & & & & M_1^{q-1}(\zeta) & -\mathsf{1} \end{array}\right],$$

$$\Phi_{A,q}(\zeta) = \begin{bmatrix} A(\zeta) & -\mathsf{1}_n & \mathsf{0} \\ \mathsf{0} & \Upsilon_q(\zeta)\otimes \mathsf{1}_n & \Psi_q(\zeta)\otimes \mathsf{1}_n \end{bmatrix},$$

$$\Phi_{B,q}(\zeta) = \begin{bmatrix} \mathrm{vec}(B(\zeta)) & -\mathsf{1}_m \otimes \begin{bmatrix} \mathsf{1}_n & \mathsf{0} \end{bmatrix} \\ \mathsf{0} & \mathsf{1}_m \otimes \begin{bmatrix} \Upsilon_q(\zeta)\otimes \mathsf{1}_n & \Psi_q(\zeta)\otimes \mathsf{1}_n \end{bmatrix} \end{bmatrix}$$

where $\mathrm{vec}(B(\zeta))$ is the vector composed of the m columns of $B(\zeta)$ stacked together.

These matrices are such that $\beta_1(\zeta) = M_1^0(\zeta)$ and recursively $\beta_k(\zeta) = M_1^{k-1}(\zeta)\beta_{k-1}$. Therefore, the vector $\beta_{0,q}(\zeta)$ containing all $p_{q,N}$ monomials $\alpha_j(\zeta)$ of degrees 0 to q can be described by

$$\left[\begin{array}{cc} \Upsilon_q(\zeta) & \Psi_q(\zeta) \end{array} \right] \beta_{0,q}(\zeta) = 0 \; .$$

Moreover, one gets that

$$\Phi_{A,q}(\zeta) \underbrace{\left(\begin{array}{c} x \\ (\beta_{0,q}(\zeta) \otimes 1_n) A(\zeta) x \end{array} \right)}_{z_{A,q}(\zeta)} = 0 \tag{6}$$

and

$$\Phi_{B,q}(\zeta) \underbrace{\left(\begin{array}{c} 1 \\ (1_m \otimes (\beta_{0,q}(\zeta) \otimes 1_n)) \mathrm{vec}(B(\zeta)) \end{array} \right)}_{z_{B,q}(\zeta)} = 0 \; . \tag{7}$$

4 Stability and guaranteed H_2 performance via PPDLF and LMIs

Theorem 1 *Let an integer q and assume that for one admissible value of the uncertainty $A(\zeta_0)$ is stable. If there exist an affine parameter-dependent row block matrix $\Pi(\zeta) = \left[\begin{array}{ccc} \Pi_1(\zeta) & \cdots & \Pi_{p_{q,N}}(\zeta) \end{array} \right]$ composed of symmetric elements:*

$$\Pi_j(\zeta) = \sum_{i=1}^{N} \zeta_i \Pi_{ji} \;\; , \;\; \Pi_{ji} = \Pi_{ji}^T \in \mathsf{R}^{n \times n} \tag{8}$$

and a matrix F such that the following LMIs are fulfilled for all N vertices ($\zeta_i = 1$, $\zeta_{j \neq i} = 0$):

$$\left[\begin{array}{cc} 0 & \Pi(\zeta) \\ \Pi^T(\zeta) & 0 \end{array} \right] + F\Phi_{A,q}(\zeta) + \Phi_{A,q}^T(\zeta) F^T < 0 \tag{9}$$

then robust stability of $\dot{x} = A(\zeta)x$ is attested by the PPDLF (5).

Proof : First note that inequalities (9) are affine with respect to ζ. Therefore, if they are satisfied for all vertices, the inequalities also hold for all uncertainties such that (1). Multiply (9) on both sides by the vector $z_{A,q}(\zeta)$ and its transpose. Due to the conditions (6), one gets that $\dot{V}_q(x,\zeta) < 0$ where $V_q(x,\zeta)$ is the parameter-dependent quadratic Lyapunov function

$$V_q(x,\zeta) = x^T P_q(\zeta) x \;\; , \;\; P_q(\zeta) = \Pi(\zeta)(\beta_{0,q}(\zeta) \otimes 1_n) \; .$$

Note that writing $\Pi(\zeta)$ as a row block matrix of elements $\Pi_j(\zeta)$ on the position corresponding to the monomial $\alpha_j(\zeta)$ gives the Lyapunov function (5) composed of a sum of symmetric matrices $\Pi_j(\zeta)$. Hence $P_q(\zeta)$ is symmetric.

At this stage we have exhibited a quadratic Lyapunov function that is negative along all trajectories of the system and for any admissible uncertain parameter. Moreover, note that $\dot{V}_q(x,\zeta) < 0$ also writes as

$$P_q(\zeta)A(\zeta) + A^T(\zeta)P_q(\zeta) < 0 \; .$$

$A(\zeta_0)$ being stable, this inequality implies that $P_q(\zeta_0)$ is positive definite and moreover, it implies that $P_q(\zeta)$ is non singular whatever the admissible uncertainty. $P_q(\zeta)$ is continuous with respect to the parameters, positive definite for one value in the set and non-singular for all values, therefore it is positive definite for all values. This concludes the proof. ■

The robust stability result is now extended to robust H_2 performance. Let the following uncertain system:

$$\left\{ \begin{array}{l} \dot{x}(t) = A(\zeta)x(t) + B(\zeta)w(t) \\ z(t) = C(\zeta)x(t) \end{array} \right. \tag{10}$$

where $x(t) \in \mathsf{R}^n$ is the state, $w(t) \in \mathsf{R}^m$ is the input perturbation and $z(t) \in \mathsf{R}^p$ is a performance output. The guaranteed H_2 cost problem is to compute an upper bound on the H_2 norms of the transfer $w \mapsto z$ realisations for all admissible values of ζ such that (1). The conservatism of the methods may be evaluated by the gap between the guaranteed upper bound and the worst-case norm.

Theorem 2 *Let an integer q and assume that for one admissible value of the uncertainty $A(\zeta_0)$ is stable. If there exist an affine parameter-dependent matrix $\Pi(\zeta) = \left[\begin{array}{ccc} \Pi_1(\zeta) & \cdots & \Pi_{p_{q,N}}(\zeta) \end{array}\right]$ satisfying (8) and a pair of matrices (F, G) such that the following LMIs are fulfilled for all N vertices ($\zeta_i = 1$, $\zeta_{j\neq i} = 0$):*

$$\begin{array}{c} \left[\begin{array}{cc} C^T(\zeta)C(\zeta) & \Pi(\zeta) \\ \Pi^T(\zeta) & 0 \end{array}\right] + F\Phi_{A,q}(\zeta) + \Phi_{A,q}^T(\zeta)F^T < 0 \\ E^T \left[\begin{array}{cc} 0 & 1_m \otimes \Pi(\zeta) \\ 1_m \otimes \Pi^T(\zeta) & 0 \end{array}\right] E \; -2\gamma^2 FF^T + G\Phi_{B,q}(\zeta) + \Phi_{B,q}^T(\zeta)G^T \leq 0 \end{array} \tag{11}$$

with

$$\begin{array}{l} E = \left[\begin{array}{cc} 0_{mn\times 1} & 1_m \otimes \left[\begin{array}{cc} 1_n & 0_{n\times n(p_{q,N}-1)} \end{array}\right] \\ 0_{mnp_{q,N}\times 1} & 1_{mnp_{q,N}} \end{array}\right] \\ F = \left[\begin{array}{cc} 1 & 0_{1\times mnp_{q,N}} \end{array}\right] , \end{array}$$

then γ is a guaranteed H_2 cost for system (10).

Proof : First note that the inequalities in (11) are convex with respect to ζ. Therefore, their being fulfilled on the vertices implies they are fulfilled for all values inside the polytope. Then, for the same reasons as in the proof of Theorem 1, the first inequality in (11) implies that

$$A^T(\zeta)P_q(\zeta) + P_q(\zeta)A(\zeta) + C^T(\zeta)C(\zeta) < 0$$

and $P_q(\zeta) = P_q^T(\zeta) > 0$. Next, note that for $z_{B,q}(\zeta)$ defined in (6):

$$\begin{array}{c} \Phi_{B,q}(\zeta)z_{B,q}(\zeta) = 0 \ , \ \ Fz_{B,q}(\zeta) = 1 \\ Ez_{B,q}(\zeta) = \left(\begin{array}{c} \text{vec}(B(\zeta)) \\ (1_m \otimes (\beta_{0,q}(\zeta) \otimes 1_n))\text{vec}(B(\zeta)) \end{array}\right) \\ 1_m \otimes P_q(\zeta) = (1_m \otimes \Pi(\zeta))(1_m \otimes (\beta_{0,q}(\zeta) \otimes 1_n)) \ . \end{array}$$

Multiplying second inequality in (11) by $z_{B,q}(\zeta)$ and its transpose implies that

$$\begin{array}{l} -2\gamma^2 + \text{vec}(B(\zeta))^T(1_m \otimes P_q(\zeta))\text{vec}(B(\zeta)) \\ \quad +\text{vec}(B(\zeta))^T(1_m \otimes P_q^T(\zeta))\text{vec}(B(\zeta)) \leq 0 \end{array}$$

which is $Tr(B^T(\zeta)P_q(\zeta)B(\zeta)) \leq \gamma^2$. The obtained parameter-dependent inequalities prove the robust bound on the H_2 norm. ■

Applied to the second example considered in [8], Theorem 2 gives the following results

q	γ_q	nb vars
1	4.18	409
2	2.67	2101

where γ_q are the minimal values fulfilling the LMIs (11) for the two choices of $q = 1$ and 2. These results show that a PPDLF of the type (5) of order 3 is more appropriate than a PPDLF such that (4) of higher degree. Even for LMI conditions of comparable size, the results are significantly improved.

Remark 1 *At the difference of [8] where the constraint $Tr(B^T(\zeta)P(\zeta)B(\zeta)) \leq \gamma^2$ was dealt with by introducing an affine parameter-dependent matrix such that $B^T(\zeta)P(\zeta)B(\zeta) \leq X(\zeta)$ and $Tr(X(\zeta)) \leq \gamma^2$, we have not made any such conservative assumption for obtaining the LMI result of Theorem 2. For the considered example for which there is only one input ($m = 1$) it makes no difference but it may have some advantage for systems with multiple disturbance inputs. This improvement is done at the expense of bigger dimensions of the slack variable G.*

Remark 2 *Define γ_q the minimal value to the LMI constraints (11) for a fixed value q. With a similar method as in [7] one may prove that if $q \leq \tilde{q}$ then $\gamma_{\tilde{q}} \leq \gamma_q$. This means that the conditions are less and less conservative as the degree of the PPDLF grows. But there is no proof at this point of a possible convergence of γ_q towards the exact worst case H_2 cost as q goes to infinity .*

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