Adaptive Control Experiments for LAAS "Helicopter" Benchmark

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***Abstract*— The problem of pitch angle control the Quanser/LAAS "Helicopter" benchmark laboratory setup is studied. Based on the passification design method and "Implicit Reference Model" approach, two adaptive control laws for are designed and experimentally tested. The MATLAB/Simulink and WinCon software environment is used for adaptive control laws implementation and the real-world experiments. The experimental results demonstrate high closed loop system performance and robustness of the suggested control laws with respect to parametric uncertainties and unmodeled plant dynamics.**

Key words: adaptive control, helicopter benchmark, autopilot design, experimental results

I. INTRODUCTION

Recently various computer-controlled equipment units have been described in the literature: cart-pendulum [1], [2], Furuta pendulum [3], pendubot [4], magnetic levitation systems [5], etc. The usage of such an equipment is threefold. First, it may be used for demonstration, attracting newcomers to the control systems area. Secondly, it is used for education, allowing students to enhance their skills in control systems design. Finally, such units are useful for research since they may serve as testbeds for testing new control algorithms under real world constraints. One of impressive laboratory setups is a laboratory-scale bench-top three degrees of freedom (DOF) helicopter produced by Quanser Consulting Inc., [6]. This setup was modified under demand of LAAS to form the "LAAS Helicopter Benchmark" [7], allowing to test 3D flight control algorithms under time-varying conditions. Control problems for helicopter laboratory units attracted an increasing interest recently [8], [9], [10], [11], [12], [13]

In our paper the results of design and testing of adaptive control algorithms for "LAAS Helicopter Benchmark" are presented. Using adaptive control for laboratory equipment has an advantage that the requirements to the controlled system model are significantly reduced and control design is simplified compared with traditional approaches. However, achieving a better performance of the adaptive control system compared to the traditional ones in the real-world applications is a matter of continuing interest and research activities [10]. In this research we attack the problem using adaptive control method based on passification, see [14], [15], [16] for "LAAS Helicopter Benchmark" pitch angle control.

In Section II the description of "LAAS Helicopter Benchmark" is presented. Passification based adaptive control is outlined in Section III. Two adaptive control laws of "Helicopter" pitch angle are described in Section IV. Experimental results are presented in Section V.

II. QUANSER/LAAS "HELICOPTER" BENCHMARK

The 3DOF helicopter setup is manufactured by *Quanser Consulting Inc.*, [6], [8]. Its improved version is used in the MAC Group of LAAS as a benchmark for implementation and testing the robust control laws. The "Helicopter" setup consists of a base on which a long arm is mounted. The arm carries the helicopter body on one end and a counterweight on the other end. The arm can tilt on an elevation axis as well as swivel on a vertical (travel) axis. Quadrature optical encoders mounted on these axes measure the elevation and travel of the arm. The helicopter body is mounted at the end of the arm. The helicopter body is free to pitch about the pitch axis. The pitch angle is measured via a third encoder. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to the motors. The force generated by the propellers causes the helicopter body to lift off the ground. All electrical signals to and from the arm are transmitted via a sliping with eight contacts. The system is also equipped with a motorized lead screw that can drive a mass along the main arm in order to impose known controllable disturbances (the so-called *Active Disturbance Option*, ADS).

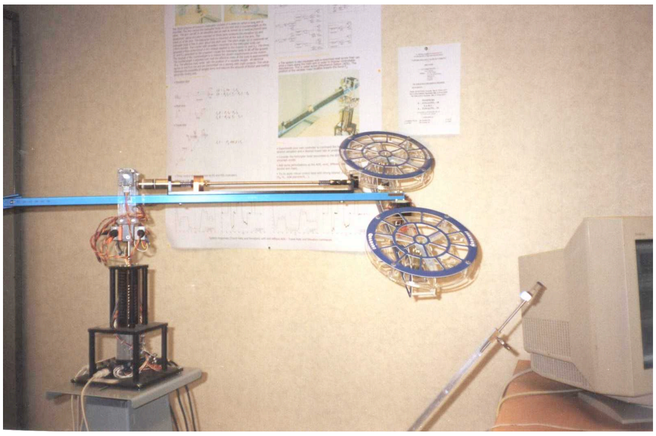

Fig. 1. LAAS-CNRS 'Helicopter' benchmark.

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III. Implicit Reference Model Concept for Adaptive Control

Theoretical background of passification-based design of adaptive control systems with Implicit Reference Model (IRM) may be found in [14], [15], [16], [17], [18]. Some essentials of the method are outlined below.

Consider a linear time invariant (LTI) system (A,B,C) described in the state-space form as

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$ is a state vector, $u = u(t) \in \mathbb{R}^m$ is a control vector, $y = y(t) \in \mathbb{R}^m$ is a measurable output vector, A, B, C are constant real matrices of sizes $n \times n$, $n \times m$, $m \times n$ respectively.

Definition: System (1) is called *minimum phase* if the polynomial

$$\varphi_0(s) = \det \begin{bmatrix} sI_n - A & -B \\ C & 0 \end{bmatrix} \tag{2}$$

is Hurwitz; *strictly minimum phase* (SMP), if it is minimum phase and the matrix CB is nonsingular ($\det CB \neq 0$); *hyper minimum phase* (HMP), if it is minimum phase and the matrix CB is symmetric and positive definite.

Theorem 1 (*Feedback Kalman-Yakubovich Lemma*, [14], [17], [18], [19]): Let $\operatorname{rank} B = m$ (i.e. the matrix B is of full rank) and let G be a symmetric negative semidefinite $n \times n$-matrix. Then the following statements are equivalent:

(A1) There exist a positive definite $n \times n$-matrix H and an $n \times m$ matrix K such that the relations

$$H(A+BKC) + (A+BKC)^T H < G, \quad HB = C^T \tag{3}$$

hold.

(B1) The system (1) is HMP.

Note, that if the condition (B1) is satisfied then a matrix K in (3) can be found in the form $K = -\alpha C$ where α is a sufficiently large positive real number.

Theorem 1 gives conditions of solvability of matrix inequalities related to classical Kalman-Yakubovich Lemma (KYL) with a feedback matrix [16], [19] and has various applications in control theory since the 1970s. It provides also solvability of the system passification by means of static output feedback. In this paper Theorem 1 is applied to design of adaptive control systems with Implicit Reference Model [14], [15]. Let the LTI SISO system (1) be described in the input-output form as follows:

$$A(p)y(t) = B(p)u(t), \ t \geq 0, \tag{4}$$

where u, y are the scalar variables, $A(p) = p^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0$, $B(p) = b_m p^m + b_{m-1}p^{m-1} + \cdots + b_1 p + b_0$ are polynomials with *a priori* unknown plant model parameters a_i, b_j $(i = 0,1,\ldots,n-1, \ j = 1,2,\ldots,m)$, $p \equiv \mathrm{d}/\mathrm{d}t$ denotes the time derivative. The plant (4) *relative degree* $k = n - m > 0$. Consider the control problem of tracking the reference (command) signal $r(t)$. For the disturbance free case the control goal is $\lim\limits_{t\to\infty} \big(r(t) - y(t)\big) = 0$. To solve the posed problem introduce the secondary goal (the *adaptation goal*) prescribing the desired tracking error behavior:

$$|\sigma(t)| \leq \Delta \quad \text{when} \quad t \geq t_*, \tag{5}$$

where $\sigma(t) = G(p)y(t) - D(p)r(t)$ is the *adaptation error* signal, $G(p) = p^l + g_{l-1}p^{l-1} + \cdots + g_1 p + g_0$, $D(p) = d_s p^s + d_{s-1}p^{s-1} + \cdots + d_1 p + d_0$ are given polynomials specifying the desired properties of the closed-loop system. $G(\lambda)$ is assumed to be stable (Hurwitz) polynomial. Note that the signal $\sigma(t)$ may be treated as equation error for the equation

$$G(p)y_*(t) = D(p)r(t), \tag{6}$$

because $\sigma(t) = G(p)\varepsilon(t)$, where $\varepsilon(t) = y(t) - y_*(t)$. Supposing that $\sigma \equiv 0$ one sees that the controlled variable $y(t)$ satisfies equation (6). Hence (6) may be interpreted as the *reference equation*, representing reference model implicitly. In other words, (6) may be called the *Implicit Reference Model* (IRM). Let us take the *control law of the main loop* in the form

$$u(t) = k_r(t)\big(D(p)r(t)\big) + \sum_{i=0}^{l} k_i(t)\big(p^i y(t)\big), \tag{7}$$

where $k_r(t)$, $k_i(t)$ $(i = 0,\ldots,l)$ are tunable parameters. Take the *adaptation algorithm* as follows:

$$\begin{aligned} \dot{k}_r(t) &= \gamma\sigma(t)D(p)r(t) - \alpha\big(k_r(t) - k_r^0\big), \quad k_r^0 = k_r(0), \\ \dot{k}_i(t) &= -\gamma\sigma(t)p^i y(t) - \alpha\big(k_i(t) - k_i^0\big), \quad k_i^0 = k_i(0), \end{aligned} \tag{8}$$

where $\gamma > 0$ is the *adaptation gain*; $\alpha \geq 0$ is the *parametric feedback gain*; k_r^0, k_i^0 are prior estimates of the appropriate values of the tunable parameters, $i = 0,1,\ldots,l$. Theorem 1 provides the following adaptive controller (7), (8):

(C1) $B(p)$ is Hurwitz polynomial,

(C2) $l = k-1$, where $k = n - m$ is the plant model (4) relative degree.

Note that neither degree s of the polynomial $D(p)$ nor its coefficients appear in the above conditions. The degree of $D(p)$ is bounded by the amount of available derivatives of $r(t)$ and is subjected to designer's decision. Note also that a *matching condition* in the form used for the model reference adaptive systems [20] is not needed for IRM adaptive controllers. The order of reference equation (6) is equal to l and can be significantly less then the plant model order n. Moreover, the true plant order need not to be known for adaptive controller design.

The IRM approach can be used for real-time tuning of standard controllers [15]. For instance, consider proportional-integral (PI) control law in the main loop:

$$u(t) = k_P(t)e(t) + k_I(t)\int_0^t e(\tau)\,\mathrm{d}\tau, \tag{9}$$

where $e(t) = r(t) - y(t)$ is the *tracking error*, $k_P(t)$, $k_I(t)$ are tunable controller parameters. Take the second order IRM as follows:

$$T^2 p^2 y(t) + 2\xi T p y(t) + y(t) = r(t), \tag{10}$$

where $p \equiv \mathrm{d}/\mathrm{d}t$. T, ξ are design IRM parameters, describing desirable closed-loop system behavior. After integration and filtering the adaptation error signal σ can be obtained as follows:

$$\sigma(t) = T^2 y(t)\omega_f + (2\xi - T\omega_f)Ty_f(t) - \int_0^t e_f(\tau)\,\mathrm{d}\tau, \quad (11)$$

where $y_f(t)$, $e_f(t)$ are outputs of the low-pass filters with the pass band ω_f driven by the signals $y(t)$ and $e(t)$ correspondingly. The adaptation algorithm (8) in this case takes the form

$$\dot{k}_P(t) = \gamma\sigma(t)e(t) - \alpha\big(k_P(t) - k_P^0\big), \quad k_P(0) = k_P^0$$
$$\dot{k}_I(t) = \gamma\sigma(t)\int_0^t e(\tau)\,\mathrm{d}\tau - \alpha\big(k_I(t) - k_I^0\big), \quad k_I(0) = k_I^0. \quad (12)$$

Secondly, consider application of FKYL to design of the variable-structure systems (VSS) [25] and the signal-parametric adaptive controllers [16], [21], [24]. Consider the LTI plant (1) and the control objective $\lim_{t\to\infty} x(t) = 0$. Let the subsidiary objective be taken as maintaining the *sliding mode* on plane $\sigma = Gy = 0$, where G is a $l \times n$-matrix. Let us take the control action in the form

$$u = -\gamma \operatorname{sign}\sigma, \quad \sigma = Gy \quad (13)$$

where $\gamma > 0$ is the gain parameter. As it is shown in [18], [22], the above goal is achieved in the system (1), (13) if there exist matrix $P = P^T > 0$ and vector K_* such that $PA_* + A_*{}^T P < 0$, $PB = LC$, $A_* = A + BK_*{}^T C$. As it is clear from FKYL, the mentioned condition is fulfilled if and only if the function $GW(s)$ is SMP, where $W(s) = C\big(sI_n - A\big)^{-1}B$, and the sign of the high frequency gain GCB is known. In that case for sufficiently large γ holds $\lim_{t\to\infty} x(t) = 0$. To eliminate dependence of system stability from initial conditions and plant parameters it was suggested to use instead of (13) the following "signal-parametric", or "combined", adaptive control law [21]

$$u = -K^T(t)y(t) - \gamma \operatorname{sign}\sigma, \quad \sigma(y) = Gy$$
$$\dot{K}(t) = -\sigma(y)\Gamma y(t), \quad (14)$$

where $\Gamma = \Gamma^T > 0$, $\gamma > 0$.

It should be noted that the convergence $\sigma(y(t))$ to zero at the finite time t_* is essential for VSS systems. It can be shown (see, e.g. [18], [23]) that this property is valid for any bounded region of initial conditions for the system (1), (14).

IV. Adaptive Pitch Angle Control Design for "Helicopter" Benchmark

Based on the above approach two adaptive pitch angle controllers with implicit reference model are developed and implemented in the MATLAB/Simulink and WinCon software environment. The first control law is the *Signal-parametric adaptive control law* (SPAD-IRM1). The second one is the *Adaptive PID control law* (APID-IRM1).

A. Nomenclature

The following notation is used hereafter: $\theta(t)$ stands for the pitch angle, deg;[1] $r(t)$ is the pitch reference signal, deg; $e(t) = r(t) - \theta(t)$ is the pitch reference error, deg; $v_f(t)$ denotes the front motor controlling voltage, V; $v_b(t)$ is the back motor controlling voltage, V; $u(t)$ stands for the pitch (torque) command signal, V; $w(t)$ is the elevation/travel (force) command signal, V.

The front and back motor controlling voltages $v_f(t)$, $v_b(t)$ are calculated through the command signals $u(t)$, $w(t)$ in the following way:

$$v_f = 0.5(w+u), \quad v_b = 0.5(w-u).$$

B. Signal-parametric adaptive control law (SPAD-IRM1)

In accordance with SPAD-IRM1, the pitch command signal is evaluated as follows:

$$u(t) = k_1(t)\operatorname{sat}_E\big(e(t)\big) - k_2(t)\dot{\theta}(t) + \delta \operatorname{sign}\sigma(t), \quad (1)$$

where

$$e(t) = r(t) - \theta(t), \quad \sigma(t) = e(t) - \tau\dot{\theta}(t) \quad (2)$$

where $\sigma(t)$ is the IRM error, $\tau > 0$, $\delta > 0$ are design parameters (δ represents the level of the relay component of the controlling signal). The controller gains $k_1(t)$, $k_2(t)$ are changed by means of the following algorithm:

$$\dot{k}_1(t) = \gamma_1\sigma(t)\operatorname{sat}_E\big(e(t)\big) + \alpha_1\big(k_1^0 - k_1(t)\big) \quad (3)$$
$$\dot{k}_2(t) = -\gamma_2\sigma(t)\dot{\theta}(t) + \alpha_2\big(k_2^0 - k_2(t)\big), \quad (4)$$

where γ_1, γ_2 are the adaptation gains (design parameters); α_1, α_2 are the parametric feedback gains (design parameters); k_1^0, k_2^0 are the base ("guessed") values of the controller gains; $\operatorname{sat}_E(e)$ denotes the *saturation function*

$$\operatorname{sat}_E(e) = \begin{cases} e, & \text{if } |e| \le E, \\ E \operatorname{sign} e, & \text{otherwise.} \end{cases}$$

After a few real-world experiments with the "Helicopter" setup, the following recommended control law parameters are found by trial and error:

$$\tau = 0.5\ s; \quad \delta = 0.5V; \quad E = 25\ deg;$$
$$\gamma_1 = 10^{-3}\ V/(deg^3 \cdot s); \quad \gamma_2 = 10^{-3}\ Vs/deg^3;$$
$$\alpha_1 = 15\ s^{-1}; \quad \alpha_2 = 15\ s^{-1};$$
$$k_1^0 = 0.1\ V/deg; \quad k_2^0 = 0.1\ Vs/deg.$$

C. Adaptive PID controller (APID-IRM1)

The pitch command signal is calculated as

$$u(t) = k_0\int_0^t \operatorname{sat}_E\big(e(\vartheta)\big)\,\mathrm{d}\vartheta + k_1(t)\operatorname{sat}_E\big(e(t)\big) - k_2(t)\dot{\theta}(t), \quad (5)$$

where $e(t)$ is given by (2).

[1] Note that in the present paper the rotation angle of the frame about the long axis is called *the pitch angle* θ, as it is used by the benchmark manufacturer [6], [8]; the pitch angle of the paper [10] in that context is called *the elevation angle*.

TABLE I

SPAD-IRM1 CONTROL PANEL VARIABLES

Variable	r(t)	k_1^0	k_2^0	
Label	pitch_cmd	k_pitch0	k_om0	
Variable	γ_1	γ_2	α_1	α_2
Label	gamma_1	gamma_2	alpha_1	alpha_2
Variable	δ	τ	w	
Label	delta	tau	elevC	

The adaptation algorithm for parameters $k_1(t)$, $k_2(t)$ is same as (4); the value of integral gain k_0 is taken as constant design parameter.

The following control law parameters are found as recommended:

$$\tau = 0.2\ s;\quad \gamma_1 = 10^{-3}\ V/(deg^3 \cdot s);\quad \gamma_2 = 10^{-3}\ Vs/deg^3;$$
$$\alpha_1 = 20\ s^{-1};\quad \alpha_2 = 20\ s^{-1};\quad E = 25\ deg;$$
$$k_0 = 0.05\ V/(deg \cdot s);\quad k_1^0 = 0.1\ V/deg;\quad k_2^0 = 0.1\ Vs/deg.$$

D. Implementation of the Adaptive Controllers in MATLAB*/WinCon Software Environment*

The adaptive controllers are implemented by means of two SIMULINK programme modules:

- *model_ada* — control law SPAD-IRM1, Eqs. (1)–(4);
- *model_adai* — control law APID-IRM1, Eqs. (2)–(5).

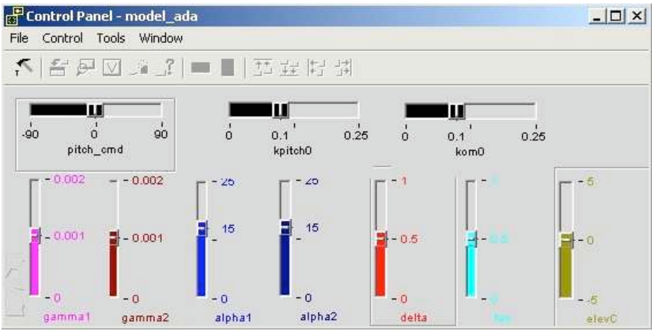

Fig. 1. Control Panel for SPAD-IRM1 controller.

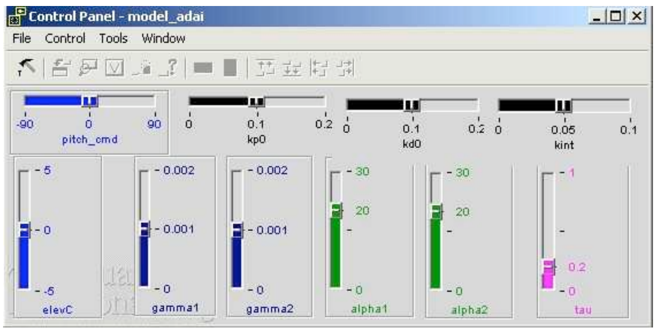

Fig. 2. Control Panel for APID-IRM1 controller.

The lookup tables of the controller variables and the labels on the Control Panel are given in Tabs. I, II.

TABLE II

APID-IRM1 CONTROL PANEL VARIABLES

Variable	k_0	k_1^0	k_2^0	γ_1	γ_2
Label	k_int	k_p0	k_d0	gamma_1	gamma_2
Variable	α_2	α_2	$r(t)$	τ	w
Label	alpha_1	alpha_2	pitch_cmd	tau	elevC

V. EXPERIMENTAL RESULTS

Software modules (*model_ada.mdl* and *model_adai.mdl*), implementing adaptive control algorithms SPAD-IRM1 and APID-IRM1 of pitch angle control of the LAAS 3-DOF Helicopter Benchmark have been successfully tested and added to the software of the LAAS "3-DOF Helicopter" Benchmark.

A number of testing real-world experiments has been provided to evaluate the system performance in the case of parametric uncertainties and unmodeled plant dynamics. Both modules achieve good accuracy. The transient time for command (reference) signal changing within 40 degrees does not exceed 5 s for algorithm SPAD-IPM and 8 s for algorithm APID-IPM. The transient performance of the closed loop system is higher than that of the controller supplied by Quanser Co. The experimental results demonstrate high closed loop system performance and confirm robustness of the suggested control laws with respect to parametric uncertainties and unmodeled plant dynamics as well.

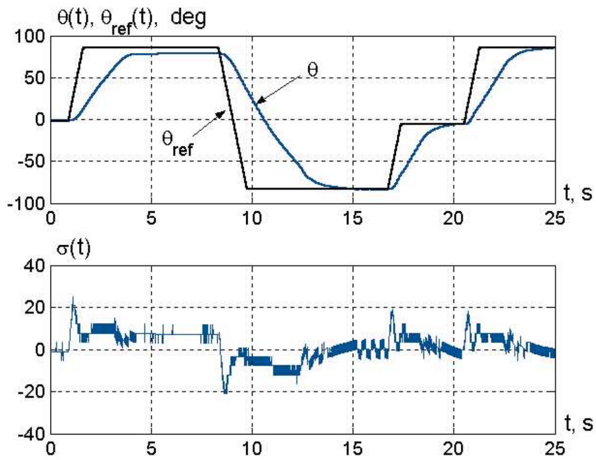

Fig. 1. Pitch angle and the adaptation error time histories. SPAD-IRM1 control law.

VI. CONCLUSIONS

In the paper the design and experiments on passification based adaptive control of the pitch angle for LAAS Helicopter benchmark are presented. The results demonstrate ability of both signal-parametric and adaptive PID controller to control the pitch angle with good performance. The signal-parametric controller provides faster transients, at the price of more forced control regime. The adaptive PID controller in the contrary, ensures more smooth and slow processes

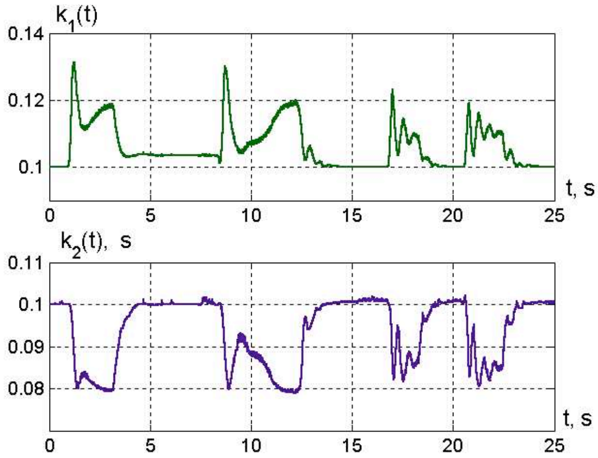

Fig. 2. Controller gains time histories. SPAD-IRM1 control law.

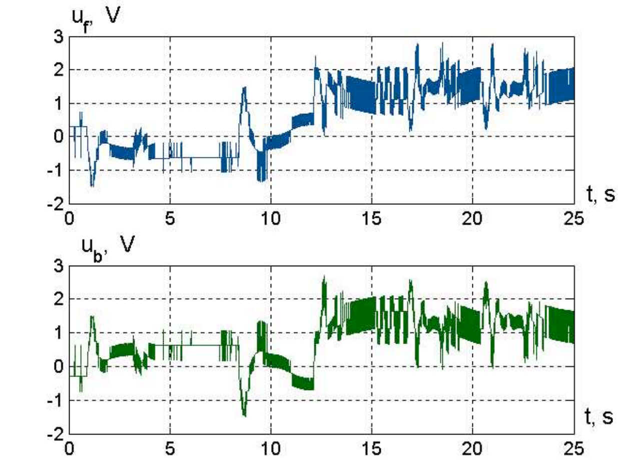

Fig. 3. Controlling voltages time histories. SPAD-IRM1 control law.

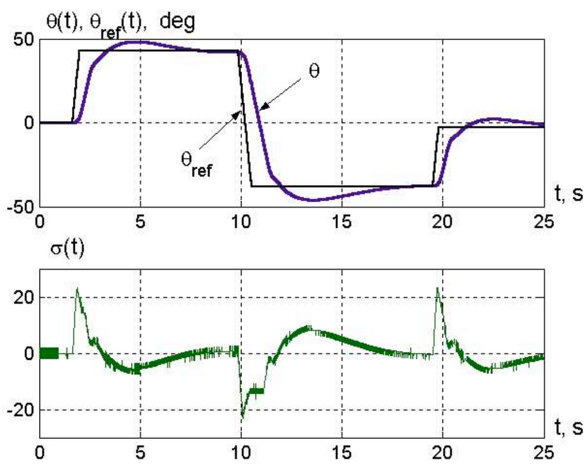

Fig. 4. Pitch angle and the adaptation error time histories. APID-IRM1 control law.

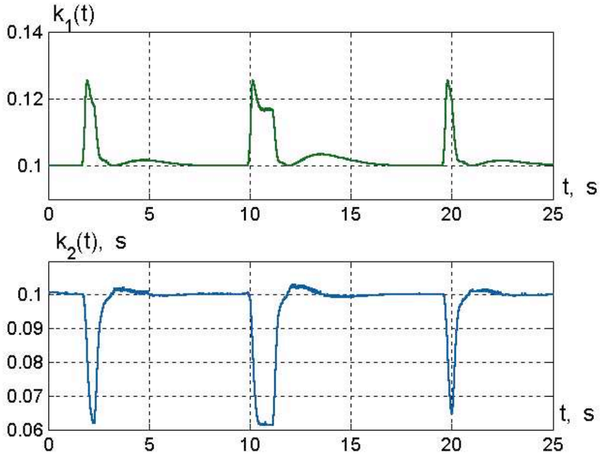

Fig. 5. Controller gains time histories. APID-IRM1 control law.

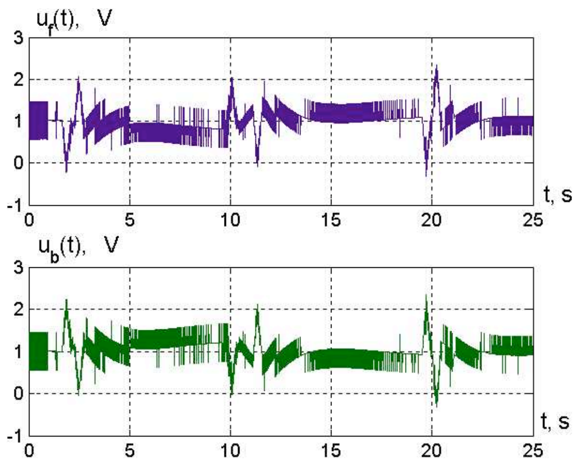

Fig. 6. Controlling voltages time histories. APID-IRM1 control law.

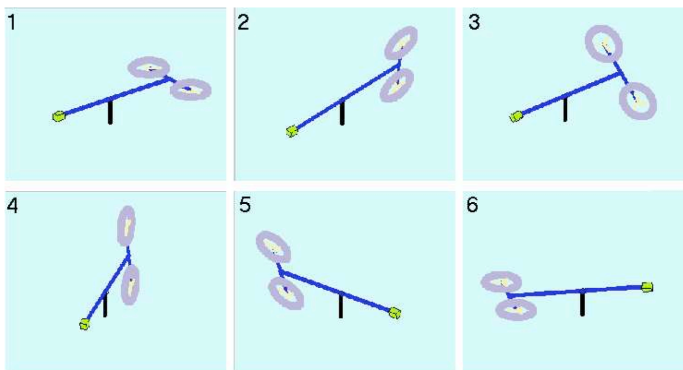

Fig. 7. Experimental results (MATLAB visualization).

with moderate control level. Its transients are typically 1.5 times longer than those provided by the signal-parametric controller.

An important advantage of adaptive control is simplicity of the design procedure compared to conventional model based approach [8]. Further research should be aimed at design of adaptive controller for full 3D motion control.

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References

[1] *Fantoni, I., Lozano, R.* Global stabilization of the cart-pendulum system using saturation functions. Proc. 42nd IEEE Conf. on Decision and Control, 9-12 Dec., 2003. V. 5, pp. 4393–4398.

[2] *Gromov, D. Raisch, J.* Hybrid control of a cart-pendulum system with restrictions on the travel. Proc. Int. Conf. Physics and Control, St.Petersburg, 20-22 Aug. 2003, v. 4, pp. 1231-1235.

[3] *Furuta, K., Yamakita, M., Kobayasji, S. and M. Nishimura* A new inverted pendulum apparatus for education. IFAC Symp. on Advances Contr. Education. Tokyo, 1994, pp.191–194.

[4] *Spong, M. W., Block, D.* The Pendubot: A mechatronic systems for control research and education. *Proc. IEEE CDC*, New Orleans. 1996, pp. 555–556.

[5] *Unger, B.J., Nicolaidis, A., Thompson, A., Klatzky, R.L., Hollis, R.L., Berkelman, P.J., Lederman, S.* Virtual peg-in-hole performance using a 6-DOF magnetic levitation haptic device: Comparison with real forces and with visual guidance alone. 10th Symp. Haptic Interfaces for Virtual Environment and Teleoperator Syst. March 24–25, 2002, p. 263.

[6] *Quanser Co*, web-site: `http://www.quanser.com/choice.asp`.

[7] *LAAS-CNRS*, web-site: `http://www.laas.fr/`.

[8] *Apkarian, J.* Internet control. Circuit Cellar, Sept. 1999, Iss.110. www.circuitcellar.com.

[9] *Tanaka, K., Othake, H., Wang, O.* A practical design approach to stabilization of a 3-DOF RC Helicopter. IEEE Trans. Control Syst. Technology, 2004, v. 12, No 2, pp.315–325.

[10] *Kutay, A.T., Calise, A.J., Idan, M., Hovakimyan, N.* Experimental results on adaptive output feedback control using a laboratory model helicopter. IEEE Trans. Contr. Syst. Technol., v. 13, No. 2, pp. 196–202, Mar. 2005.

[11] *Chen, D.R., Chen, H.S., Wang, J.D.* Comparison between the system identification and the neural network methods in identifying a model helicopter's yaw movement. Int. J. Nonlinear Sciences and Numerical Simulation, v. 3–4, 2002, pp. 391–394.

[12] *Chen, H.S., Chen, D.R.* System identification of a model helicopter's yaw movement based on an operator's control. Int. J. Nonlinear Sciences and Numerical Simulation, v. 3–4, 2002, pp. 395–398.

[13] *Dzul, A., Lozano, R., Castillo, P.* Adaptive control for a radio-controlled helicopter in a vertical flying stand. Int. J. Adapt. Control Signal Process., v. 18, 2004, pp. 473–485.

[14] *Fradkov, A.L.*. Synthesis of adaptive system of stabilization of linear dynamic plants. Automation and Remote Control, vol.35, No. 12, 1974, pp. 1960–1966. Translated from: *Avtomatika i Telemekhanika*, No. 12, pp. 96–103, 1974.

[15] *Andrievsky, B.R. and A.L. Fradkov*. Adaptive controllers with Implicit Reference Models based on Feedback Kalman-Yakubovich Lemma. 3rd IEEE Conf. on Control Applications, Glasgow, 1994, pp. 1171–1174.

[16] *Andrievsky, B.R., A.N. Churilov and A.L. Fradkov*. Feedback Kalman-Yakubovich Lemma and its applications to adaptive control. 35th IEEE Conf. Dec. Contr., Kobe, 11-13 Dec., 1996.

[17] *Fradkov, A.L.* Quadratic Lyapunov Functions in the adaptive stability problem of a linear dynamic target. Translated from: *Siberian Mathematical Journal*, 1976, v. 17, No. 2, pp. 436–445.

[18] Fradkov, A.L., Miroshnik, I.V. and V.O. Nikiforov. *Nonlinear and Adaptive Control of Complex Systems*. Dordrecht, Kluwer, 1999.

[19] *Fradkov A.L.* Passification of nonsquare linear systems and Feedback Kalman-Yakubovich Lemma. *Europ. J. Contr.* 2003, v. 9, No 11.

[20] *Landau, J.D.* Adaptive control systems. The model reference approach. N.-Y. Dekker. 1979.

[21] *Andrievsky, B.R., Stotsky, A.A., Fradkov, A.L.* Velocity gradient algorithms in control and adaptation problems. *Autom. Remote Control*. 1989, pp. 1533–1564 (Translated from Automatica i telemechanica, 1988; 12).

[22] *Fradkov, A.L.* Speed gradient scheme and its application in adaptive control problems. *Autom. Rem. Contr.* 1979, v. 40, No 9, pp. 1333–1342.

[23] Fradkov, A.L. *Adaptive control in large-scale complex systems*. Nauka, Moscow, 1990 (in Russian).

[24] *Stotsky, A.A.*. Combined adaptive and variable structure control. *Variable Structure and Lyapunov Control,* (Ed: A.S.I. Zinober), Springer-Verlag, London, 1994, pp. 313–333.

[25] *Utkin, V.I.* Optimization and Control using Sliding Modes. Springer-Verlag, 1992.