Discussion on: "Parameter-Dependent Lyapunov Function Approach to Stability Analysis and Design for Uncertain Systems with Time-Varying Delay"

Dimitri Peaucelle and Frédéric Gouaisbaut
LAAS-CNRS
7 avenue du colonel Roche, 31077 Toulouse, FRANCE
Tel. 05 61 33 63 09 fax: 05 61 33 69 69
email: { peaucelle , gouaisbaut }@laas.fr

January 25, 2005

From the paper by Cao and Xue that combines several techniques for contributing to LMI methods for uncertain time-delay systems, we choose to discuss the use of the so-called "slack variables".

This technique has been extensively used these last six years since [9] and has proved to be quite useful. Among papers related to this technique we may point out those that clarify the involved mechanisms. In [13] the additional variables are proved to be related to the elimination lemma technique which is a corollary of the Finsler lemma. In [3] the additional variables are seen as Lagrange multipliers. In [5] they are obtained as a special case of parameter-dependent multipliers involved in a new robust stability condition. In [11] they are proved to define a virtual system with identical properties as the one tested. In [8] they are proved to be multipliers involved in the generalized S-procedure. All these results help the comprehension of the technique and are good guides for possible extensions.

Without anticipating the extensions, the discussion will concentrate on two limitations of the "slack variables" technique as they appear to us currently.

The first limitation is that the technique does reduce conservatism of existing methods only when polytopic parameter-dependent Lyapunov functions (PDLF) are sought. In other cases, such as quadratic stability type conditions, the slack variables imply the augmentation of the numerical burden without any contribution with respect to conservatism.

The second limitation is related to controller design. While there is an implication between analysis conditions with a single Lyapunov matrix and PDLF conditions with slack variables, this implication is not always respected in design. More precisely, such a implication exists for discrete-time systems but cannot be obtained with LMI conditions for continuous-time systems.

When are additional degrees of freedom useful ?

This question is motivated by Remark 1 in the paper by Cao and Xue that states *"Stability condition (18) is expected to be less conservative than (16) because 3 slack variables are provided to increase the freedom to optimize the solution in (18)"*.

Unfortunately this expectation does not hold. This can be proved with the same arguments as in [14]. In this last paper, Theorem 1 gives a delay-dependent LMI condition for asymptotic stability of systems without uncertainties where a bound is known on the delay and its derivative. This first theorem involves four $n \times n$ "slack variables". Then Theorem 2 gives a totally equivalent LMI result that avoids the use of the slack variables. These are hence totally artificial and augment the numerical burden without purpose as long as no polytopic uncertainty is involved.

To conclude this section, we may state that the "slack variables" technique seems to be useful essentially for one purpose : getting PDLF-based conditions for systems with polytopic uncertainties. By itself, the technique is not a contribution for reducing conservatism of techniques used for the analysis of delayed systems.

Parametrization of conservative design conditions

In order to develop state-feedback controller design results based on the PDLF analysis results, the authors decide arbitrarily to constrain the slack variables to be proportional: $P_3 = \epsilon P_2$ and $P_d = \epsilon_d P_2$. Such a choice is not new and can also be found in [5, 7] and others. Although Cao and Xue elude the problem of choosing *a priori* the scalars ϵ and ϵ_d, this problem is far from being trivial.

The first question is on the conservatism to constrain the slack variables to be proportional. In [5] it is proved that this choice is always possible when a unique Lyapunov matrix proves the robust stability, but no conclusion is made for the case of PDLF conditions, which is the only relevant situation for using slack variables.

The second and significant question is on the choice of proportion coefficients (ϵ, ϵ_d) such that the PDLF design conditions are proved to be satisfied if the less demanding classical state-feedback design conditions based on a unique Lyapunov function are fulfilled. In [4, 13] it is proved that such a choice is possible for some problems such as stabilizing discrete-time systems. No such result exists for continuous-time systems. All that can be proved, as in [5], is the hierarchy between the classical conditions and the PDLF conditions when $\epsilon_d = 0$ and $\epsilon \to 0^+$. More precisely, [7] proves that asymptotically the PDLF conditions reduce to the classical ones. Taking ϵ close to zero has therefore of no relevance.

The discussion indicates that one needs a methodology for choosing the parameters ϵ and ϵ_d. Based on results in [12] a helpful information is that the system $\dot{x} = -\frac{1}{\epsilon}x(t) - \frac{\epsilon_d}{\epsilon}x(t-d(t))$ should be stable for Theorem 3 to have eventually a solution. If this condition on (ϵ, ϵ_d) holds there is no guarantee to have a solution to the design problem, but if not, there cannot be any solution. The choice $(\epsilon, \epsilon_d) = (1, 0)$ made in Examples 3 and 4 is therefore justified.

To conclude this last section we may state that PDLF state-feedback conditions based on "slack variables" technique are parameterized by a virtual stable system, that can eventually be chosen scalar. A choice *a priori* of such a system leads to conditions such as those in [1], that can be seen as based on the stability of $\dot{x} = -x$, or in [6], that can be seen as based on the stability of $\dot{x} = -\frac{1}{2}x$. Then if the choice does not give a positive answer, one may perform a rudimentary line search on ϵ (`fminsearch` in [14]) or take advantage of more sophisticated techniques as in [10, 2].

References

[1] P. Apkarian, P.C. Pellanda, and H.D. Tuan. Mixed H_2/H_∞ multichannel linear parameter-varying control in discrete time. In *American*

Control Conference, pages 1322–1326, Chicago, Il, June 2000.

[2] D. Arzelier, D. Henrion, and D. Peaucelle. Robust D-stabilization of a polytope of matrices. *Int. J. Control*, 75(10):744–752, 2002.

[3] M.C. de Oliveira and R.E. Skelton. *Perspectives in Robust Control*, chapter Stability tests for constrained linear systems, pages 241–257. Lecture Notes in Control and Information Sciences. Springer, 2001. edited by S.O. Reza Moheimani.

[4] M.C. de Oliveira, J. Bernussou, and J.C. Geromel. A new discrete-time stability condition. *Systems & Control Letters*, 37(4):261–265, July 1999.

[5] M. Dettori and C.W. Scherer. New robust stability and performance conditions based on parameter dependent multipliers. In *IEEE Conference on Decision and Control*, pages 4187–4192, Sydney, Australia, December 2000.

[6] Y. Ebihara and T. Hagiwara. New dilated LMI characterizations for continuous-time control design and robust multiobjective control. In *American Control Conference*, pages 47–52, Anchorage, AK, May 2002.

[7] Y. Ebihara and T. Hagiwara. Robust controller synthesis with parameter-dependent lyapunov variables: A dilated LMI approach. In *IEEE Conference on Decision and Control*, pages 4179–4184, Las Vegas, Nevada, December 2002.

[8] Y. Ebihara, K. Maeda, and T. Hagiwara. Robust D-stability analysis of uncertain polynomial matrices via polynomial-type multipliers. In *16th IFAC World Congress*, Praha, Czech Republic, July 2005.

[9] J.C. Geromel, M.C. de Oliveira, and L. Hsu. LMI characterization of structural and robust stability. *Linear Algebra and its Applications*, 285:68–80, 1998.

[10] D. Henrion, D. Arzelier, and D. Peaucelle. Robust stabilization of matrix polytopes with the cone complementarity linearization algorithm: Numerical issues. In *13th Conference Process Control '01*, Strbske Pleso, June 2001.

[11] D. Henrion, D. Arzelier, and D. Peaucelle. Positive polynomial matrices and improved LMI robustness conditions. In *15th IFAC World Congress*, Barcelona, July 2002.

[12] D. Peaucelle. *Formulation Générique de Problèmes en Analyse et Commande Robuste par les Fonctions de Lyapunov Dépendant des Paramètres*. PhD thesis, Université Toulouse III - Paul Sabatier, France, July 2000.

[13] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou. A new robust D-stability condition for real convex polytopic uncertainty. *Systems & Control Letters*, 40(1):21–30, May 2000.

[14] V. Suplin, E. Fridman, and U. Shaked. A projection approach to H_∞ control of time-delay systems. In *IEEE Conference on Decision and Control*, pages 4548–4553, Paradise Island, Bahamas, December 2004.