A dynamic input allocation paradigm illustrated by applications

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Outline

Weak and strong input redundancy in linear plants

▷ A linear plant with *weak* or *strong* input redundancy

- **Weak**: means that equilibria can be induced by different input patterns
- **Strong**: means that transients can be induced by different input patterns

$$\begin{aligned} \dot{x} &= Ax + Bu + B_d d \\ y &= Cx + Du + D_d d, \end{aligned}$$

Def'n: A plant is input-redundant if one of the following two conditions is satisfied

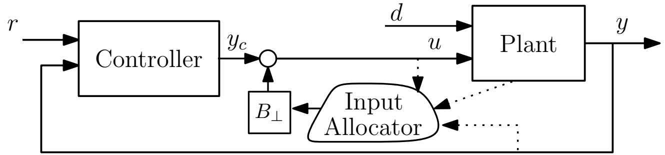
- it is *strongly input-redundant* from u if it satisfies $\mathrm{Ker}\left(\left[\begin{smallmatrix} B \\ D \end{smallmatrix}\right]\right) \neq \emptyset$; denote

$$B_\perp \text{ such that } \mathrm{Im}(B_\perp) = \mathrm{Ker}\left(\left[\begin{smallmatrix} B \\ D \end{smallmatrix}\right]\right);$$

- it is *weakly input-redundant* from u to y if $P^\star := \lim_{s \to 0} (C(sI - A)^{-1}B + D)$ is finite and satisfies $\mathrm{Ker}(P^\star) \neq \emptyset$; denote

$$B_\perp \text{ such that } \mathrm{Im}(B_\perp) = \mathrm{Ker}(P^\star).$$

Allocator dynamics may only act in the $B_\perp$ directions



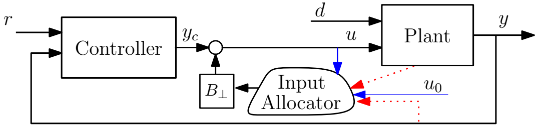
▷ Assume that a controller **has been designed** disregarding input redundancy to obtain a **desirable plant output response** y

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y + B_r r \\ y_c &= C_c x_c + D_c y + D_r r, \end{aligned}$$

▷ Design an input allocator which
- exploits strong redundancy to get *fast reallocation* during transients
- exploits weak redundancy to get *slow reallocation* at the steady-state

▷ The allocator measures controller output y_c and adds compensating signal
- Choose that signal as $B_\perp w$ for some w
- Pick w as the output of a pool of integrators (dynamic solution)

Linear dynamic allocation minimizes $J=(u-u_O)^T \bar{W}(u-u_O)$



▷ Linear solution only relying on the knowledge of the controller output y_c

$$\begin{aligned}\dot{w} &= -2\rho K B_\perp^T \bar{W}(u-u_0) = -\rho K B_\perp^T \nabla J \\ u &= y_c + B_\perp w,\end{aligned}$$

Th'm: With *strong redundancy*, if $K>0$ and $B_\perp^T \bar{W} B_\perp > 0$ then internal stability and output response y unaffected by allocator

Th'm: With *weak redundnacy*, if $K>0$ and $B_\perp^T \bar{W} B_\perp > 0$ then internal stability and steady-state output response y unaffected by allocator for small enough ρ

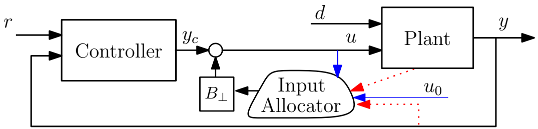
▷ Role of $\bar{W}$: assign the steady-state plant input, solution to:

$$\min_w J(u) := (u-u_0)^T \bar{W}(u-u_0), \quad \text{subject to: } u = y_c^\star + B_\perp w,$$

corresponding to $u^\star = u_0 + \left(I - B_\perp (B_\perp^T \bar{W} B_\perp)^{-1} B_\perp^T \bar{W}\right) y_c^\star$.

▷ u_0 is a useful drift term (e.g., center of saturation range)

Linear dynamic allocation minimizes $J = (u - u_O)^T \bar{W} (u - u_O)$



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Th'm: With *weak redundnacy*, if $K > 0$ and $B_\perp^T \bar{W} B_\perp > 0$ then internal stability and steady-state output response y unaffected by allocator for small enough ρ

▷ Role of K diagonal: promote/penalize different redundant directions while not affecting the steady-state input:

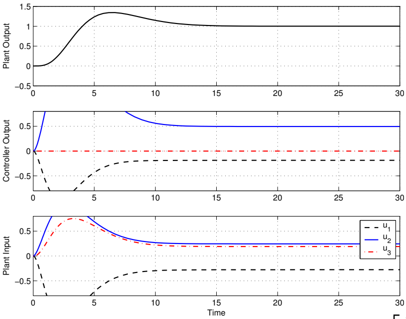
$$u^\star = u_0 + \left(I - B_\perp (B_\perp^T \bar{W} B_\perp)^{-1} B_\perp^T \bar{W}\right) y_c^\star$$

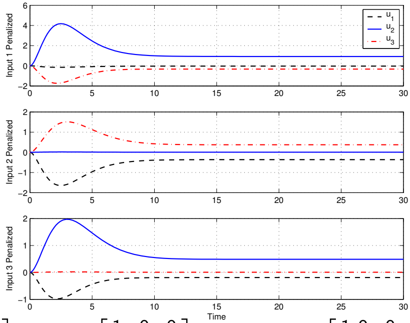
▷ Role of $\rho \in \mathbb{R}_{>0}$ is to assign any (arbitrily fast or slow) allocation speed

Randomly generated academic example (strong)

▷ Plant is strongly input redundant (one direction), controller is LQG

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|ccc} -0.157 & -0.094 & 0.87 & 0.253 & 0.743 \\ -0.416 & -0.45 & 0.39 & 0.354 & 0.65 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right].$$



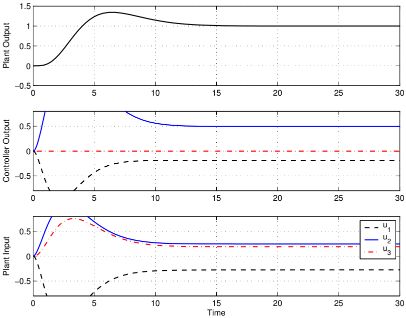


$K = 10I$ and $\bar{W} = I$ $\quad \bar{W} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\bar{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $\bar{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix}$

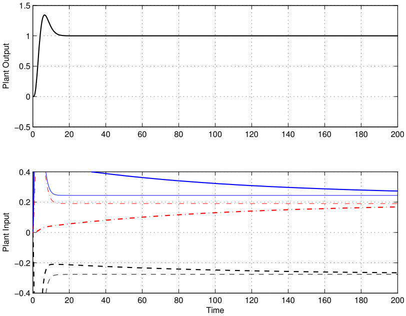
Randomly generated academic example (strong)

▷ Plant is strongly input redundant (one direction), controller is LQG

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|ccc} -0.157 & -0.094 & 0.87 & 0.253 & 0.743 \\ -0.416 & -0.45 & 0.39 & 0.354 & 0.65 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right].$$



$K = 10I$ and $\bar{W} = I$

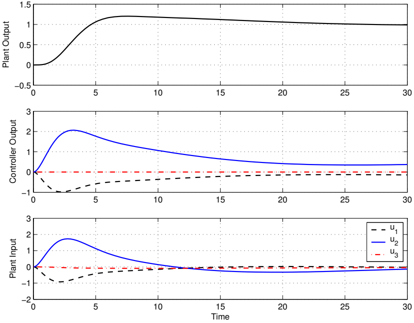


$K = 10$ (solid) and $K = 0.01$ (dash-dotted)

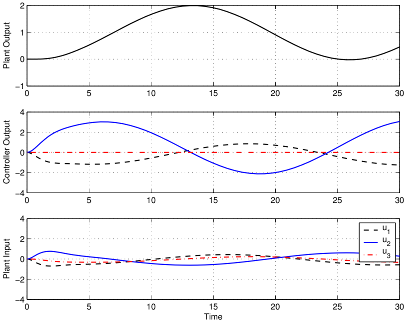
Randomly generated academic example (weak)

▷ Plant is weakly input redundant (two directions), controller is LQG

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|ccc} -0.157 & -0.094 & 0.87 & 0.253 & 0.743 \\ -0.416 & -0.45 & 0.39 & 0.354 & 0.65 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right].$$



$\rho K = 0.1I$ and $\bar{W} = I$: OK!

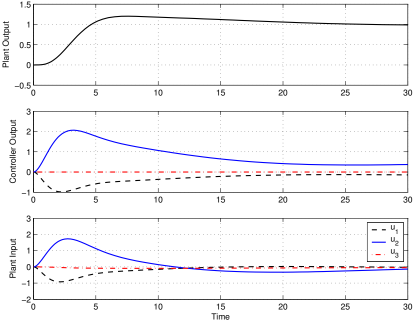


$\rho K = 1I$ and $\bar{W} = I$: unstable!

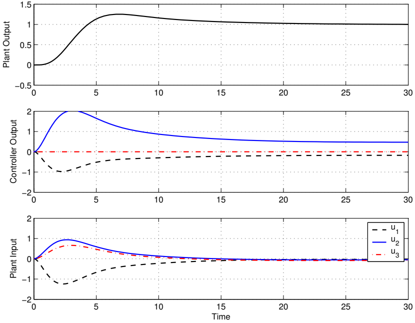
Randomly generated academic example (weak)

▷ Plant is weakly input redundant (two directions), controller is LQG

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|ccc} -0.157 & -0.094 & 0.87 & 0.253 & 0.743 \\ -0.416 & -0.45 & 0.39 & 0.354 & 0.65 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right].$$



$\rho K = 0.1 I$ and $\bar{W} = I$: OK!

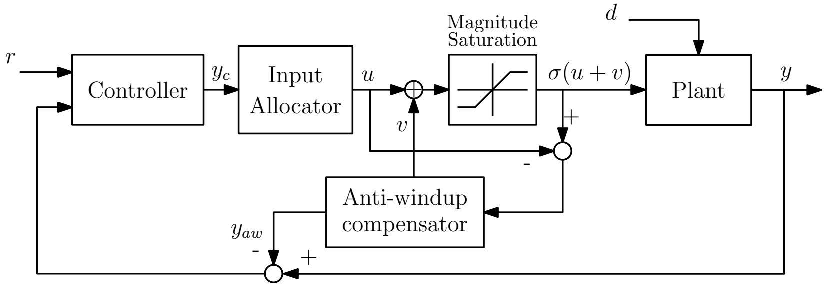


$\rho K = \begin{bmatrix} 100 & 0 \\ 0 & 0.1 \end{bmatrix}$ and $\bar{W} = I$: Better!

Nonlinear allocation with magnitude saturation

- Select nonlinear $W(\cdot)$ to increasingly penalize each actuator as it approaches its magnitude saturation limit M

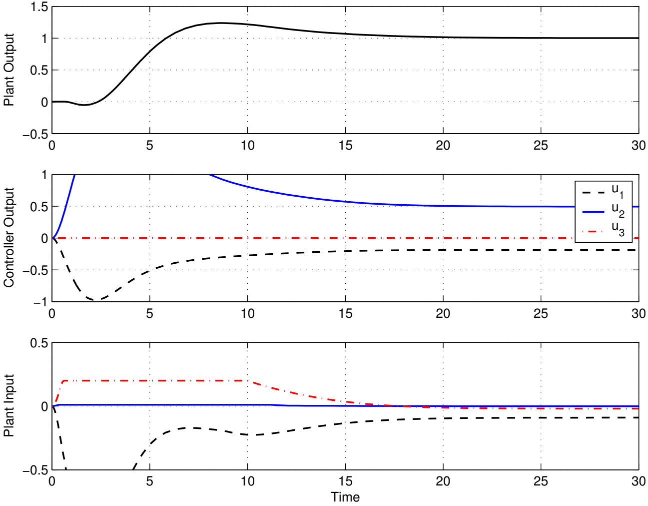
$$W(u) = \left(\mathrm{diag}((1+\epsilon)M - \mathrm{abs}(\mathrm{sat}_M(u)))\right)^{-1}$$



- **Interpretation**: *anti-windup deals* with saturation during transients; *dynamic allocation* avoids saturation at the steady-state

Example 1 (revisited with magnitude saturation)

▷ Input usage after allocation [9.5 3.37 7]% (note $u_2^* \approx 0.5 \gg m_2 = 0.01$)

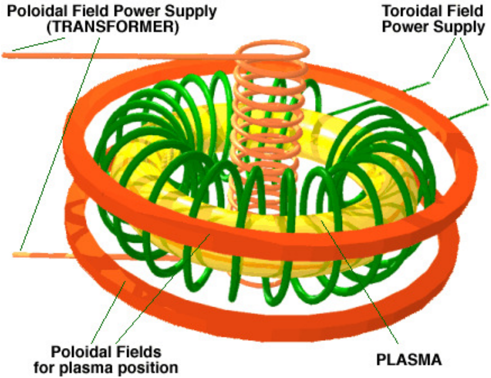


Outline

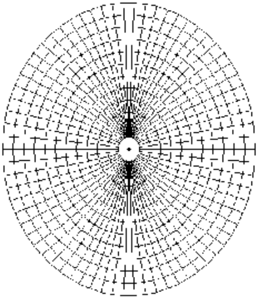
1. Linear results
2. FTU elongation control
3. Adding actuator dynamics
4. JET Current Limit Avoidance

Application: plasma position and elongation control

▷ Frascati Tokamak Upgrade (FTU): a nuclear fusion experiment



Coils and toroidal plasma

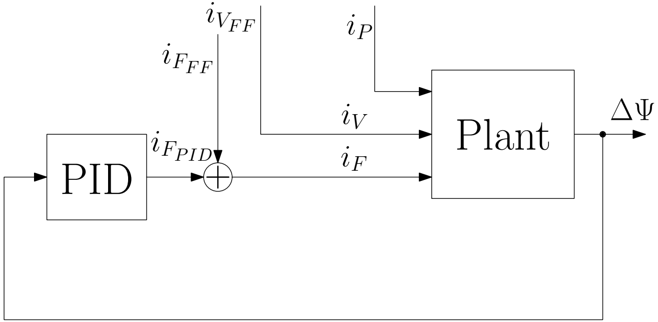


Cross section

▷ Poloidal field coils regulate plasma position and elongation

Current FTU horizontal position regulation

▷ Frascati Tokamak Upgrade: $\Delta\Psi$ = plasma horiz. position, i_P = plasma current



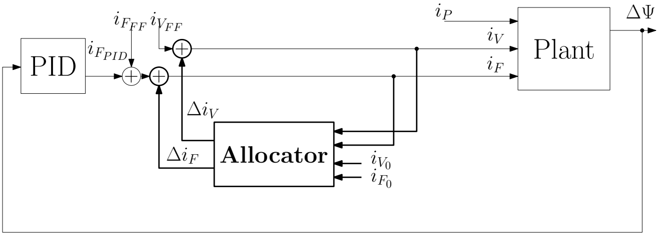
▷ Tools: V coil: very slow and powerful; F coil: fast and squeezes the plasma

▷ Goal: Want to use the F coil to perform two actions:

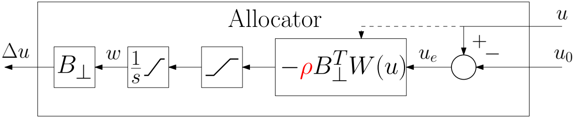
- high bandwith disturbance rejection on $\Delta\Psi$ $(= y)$
- low bandwith elongation, equivalently, $i_F (= u_2)$ regulation

Solution with allocator uses weak redundancy

▷ Transfer (slowly) control authority from F to V using dynamic allocation



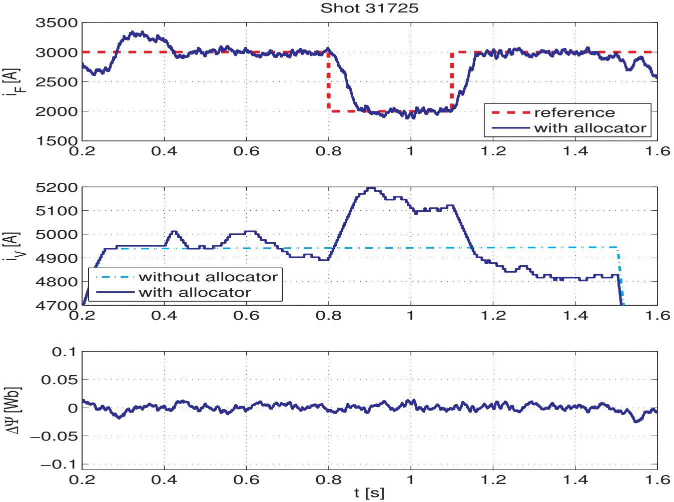
▷ Zoom of the allocator block (note the drift term $u_0 = u_r$ which is now a reference signal for i_F)



Th'm: With *weak redundnacy*, if $K > 0$ then internal stability and steady-state output response $y = \Delta\Psi$ unaffected by allocator for small enough ρ

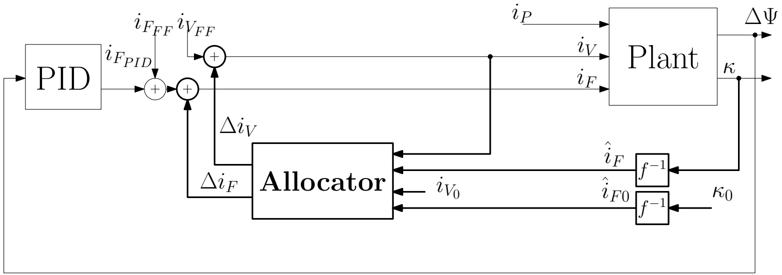
Experiments: F current regulation

- ▷ i_F current is slowly regulated without affecting plant output $y = \Delta\Psi$



From current regulation to elongation regulation

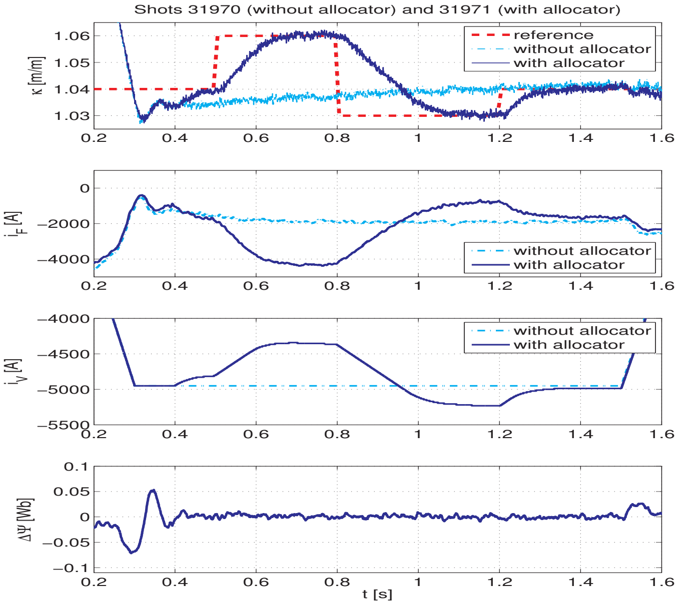
- An approximately known nonlinear static map f relates I_F to the elongation κ



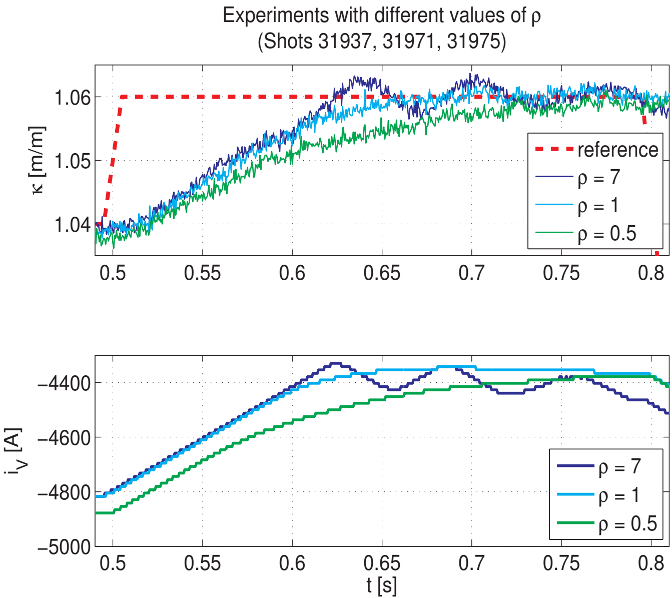
- Invert the map f to perform feedback elongation regulation via allocation
- Experiments confirm that the scheme works only if ρ is sufficiently slow

Th'm: With *weak redundnacy*, if $K > 0$ and map f is invertible, then internal stability and steady-state output response $y = \Delta\Psi$ unaffected by allocator for small enough ρ + elongation regulation $\kappa \to \kappa_0$.

Experiments: Elongation regulation



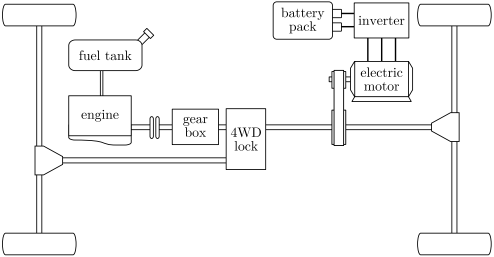
Experiments: loss of stability if parameter ρ too large



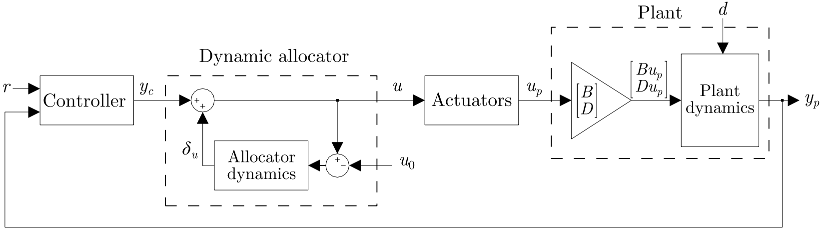
Outline

Hybrid Electric Vehicle has ICE and EM actuators

▷ A prototype built at the "University of Rome, Tor Vergata"

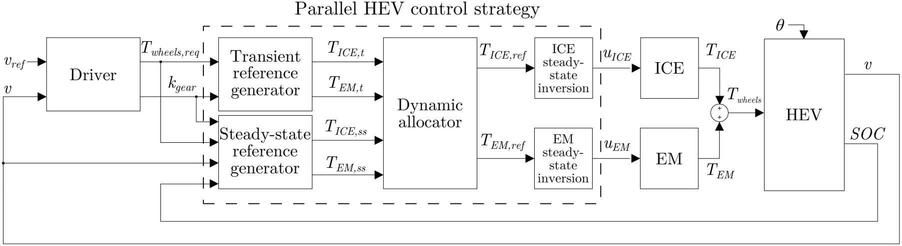


▷ Extension of framework: rendundancy after dynamic actuators

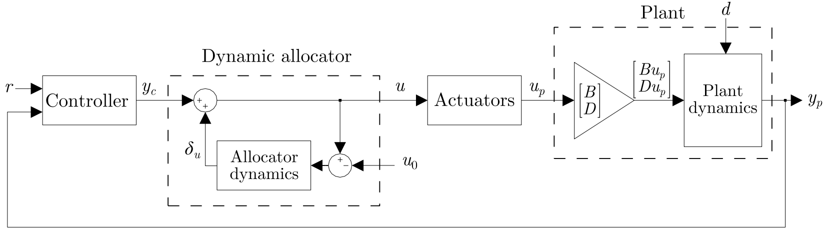


Hybrid Electric Vehicle has ICE and EM actuators

- Redundancy: net torque= ICE torque + EM torque

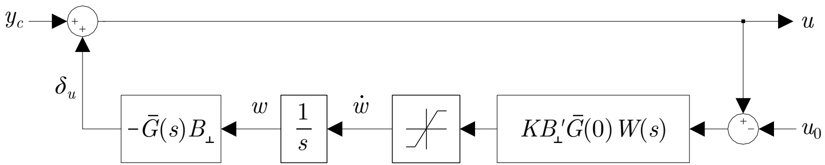


- Dynamic allocator inputs:
  - y_c represents the transient torque request (non-optimized),
  - u_0 represents the steady-state torque allocation (energy efficient)



Dynamic allocation uses LCM of actuators dynamics

▷ Allocator dynamics $\bar{G}(s)$, $W(s)$ designed following a systematic procedure

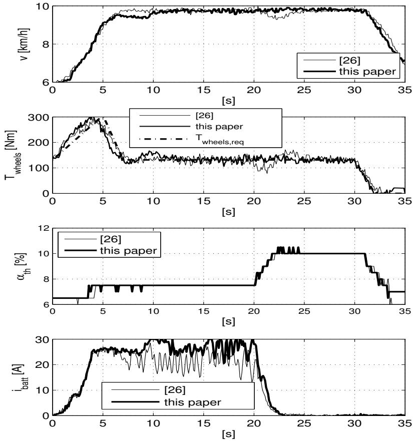


▷ Slow variation of the injected signals ensured by the presence of saturation

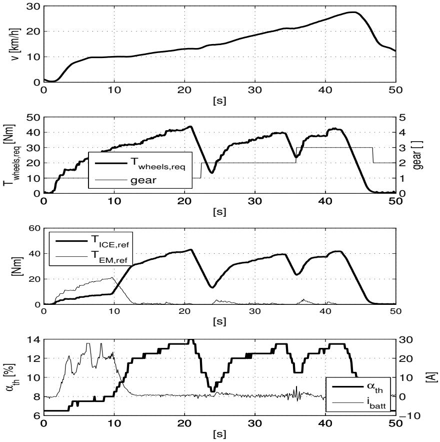
▷ Main result proven using saturated systems techniques

Th'm: If the actuator parameters are designed following the procedure, the **transient response** given by the controller is not modified by the allocator, and the **steady-state torque allocation** u_0 is asymptotically obtained.

Experimental response on the prototype car

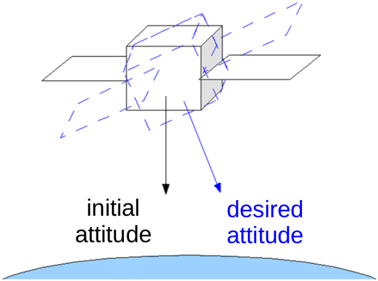


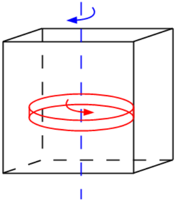
Torque regulation. Steady-state reference u_0 changes at $t = 20$ s



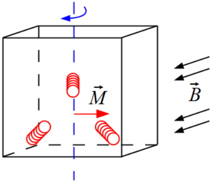
Human driver in the loop. Reference u_0 changes at $t = 10$ s

Attitude control with reaction wheels and magnetorquers





Reaction wheels



Magnetorquers

- Plant dynamics:

$$J\dot{\omega} + \omega^{\times} J\omega = \omega^{\times} h_w - \tau_w - \underbrace{\tilde{b}^{\times}(t, q)\tau_m}_{T_m}$$

$$\dot{q} = S(\omega)q$$

- Actuator dynamics (Reaction Wheels):

$$\dot{h}_w = \tau_w$$

▷ **Reaction wheels**: if $\tau_w = k$ then $h_w = kt \rightarrow$ risk of saturation of h_w

▷ **Magnetorquers**: Controllability issues: $T_m = -\tilde{b}^{\times}(t, q)\tau_m = -(R(q)\tilde{b}_{\circ}(t))^{\times}\tau_m$

Attitude control with reaction wheels and magnetorquers

- ▷ Classical solution: "Cross-product law" uses separate loops and high-gain
- ▷ Proposed-solution: use static allocation in feedback from actuator state

Dynamics:

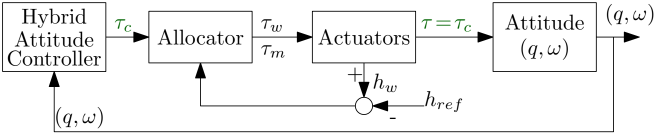
$$
\begin{aligned}
J\dot{\omega} + \omega^\times J\omega &= \underbrace{-\tau_w - \omega^\times h_w + T_m}_{\tau} \\
\dot{q} &= S(\omega)q \\
\dot{h}_w &= \underbrace{-\omega^\times h_w - (R(q)\widetilde{b}_\circ(t))^\times \tau_m - \tau}_{\tau_w}
\end{aligned}
$$

Control law:

$$
\tau_w = -\omega^\times h_w - (R(q)\widetilde{b}_\circ(t))^\times \tau_m - \tau,
$$

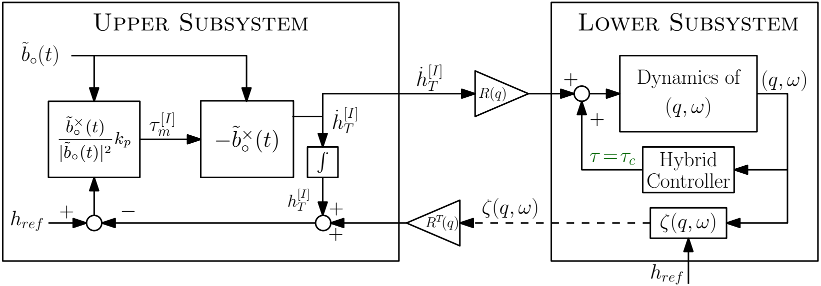
$$
\tau_m = -\frac{(R(q)\widetilde{b}_\circ(t))^\times}{|\widetilde{b}_\circ(t)|^2} k_p(h_w - h_{ref})
$$

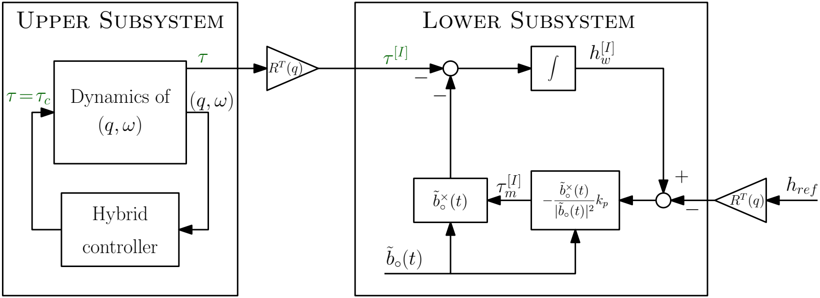
$\tau =$ Hybrid attitude controller command



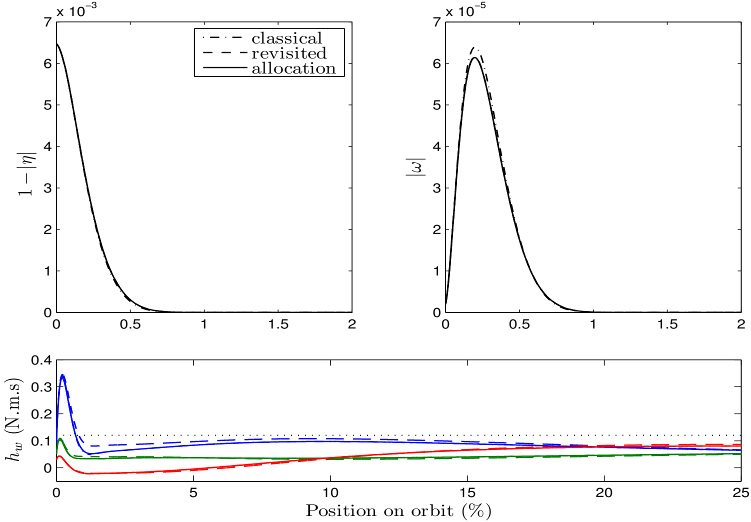
Th'm: If τ ensures GAS of the origin for (q, ω) dynamics, then allocation scheme preserve the same exact (q, ω) response and ensures GAS of $h_w = h_{ref}$.

Allocation scheme enables inverting the cascade



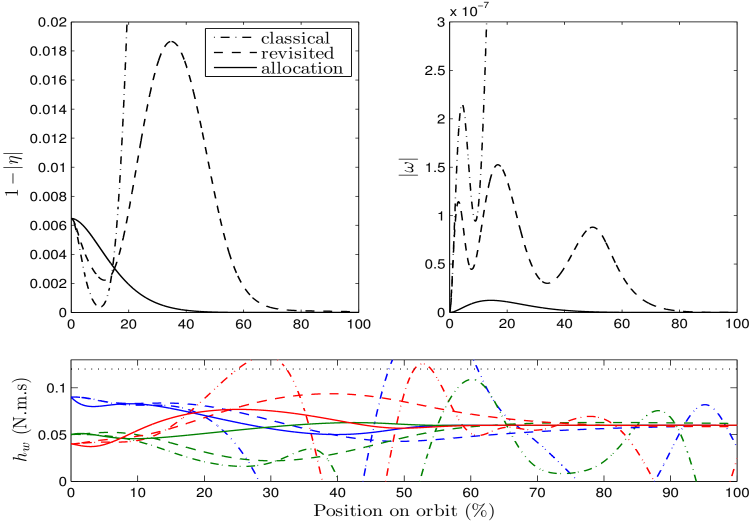


Stabilization transients with aggressive controller



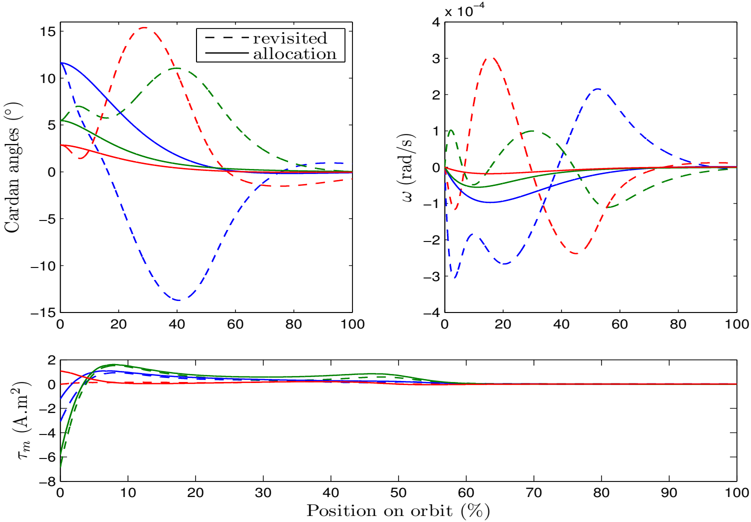
✓ **Similar results** ✗ **saturation of h_w**

Stabilization transients with non aggressive controller



✓ **revisited and allocation controllers preserve stability**

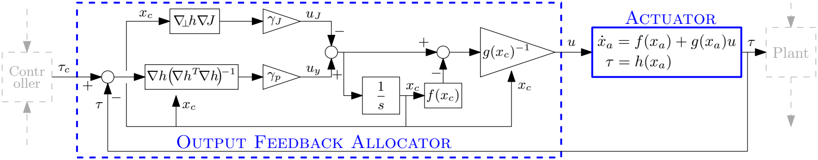
Attitude transient decoupled from the h_w transient



✓ **allocation-based strategy gives more regular attitude transient**

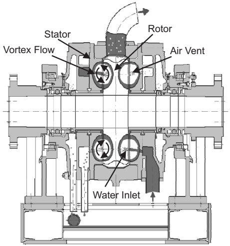
Nonlinear allocation with partial actuator measurements

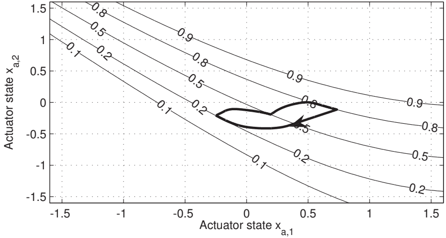
- In some applications may be able to only access *virtual input* τ
- If $g(x_a)$ is invertible and $f(x_a)$ is incrementally stable, may use scheme



Th'm: Under stated assumptions, we have $\dot{\tau} = -\gamma_p(\tau - \tau_c)$ and (slow) convergence of x_a to the minimum of $J(x_a)$.

- Hydrodynamic dynamometer uses two valves with nonlinear output map h





Outline

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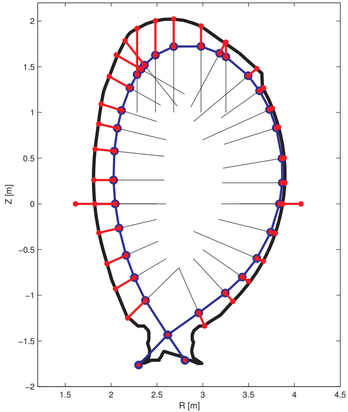
2. FTU elongation control
3. Adding actuator dynamics
4. JET Current Limit Avoidance

Joint European Torus (JET) plasma shape control

- We want to control the plasma shape on a poloidal cross section.
- Shape is described by a finite number of geometrical parameters called **gaps**.

- Gaps are defined as the distances between the plasma boundary and the first wall along certain segments.
- Gaps values are evaluated from magnetic sensor measurements by estimation algorithms.
- We want to control:

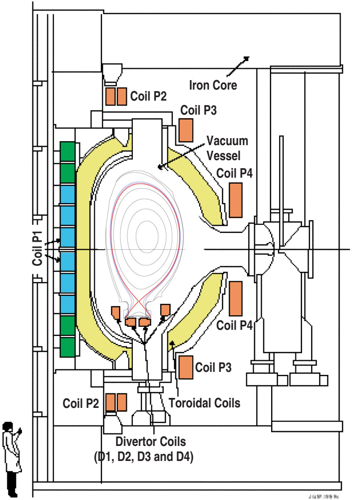
  32 outputs y.



JET shape control has not redundant inputs

- JET has 8 poloidal field (PF) coils available as actuators for plasma shape control.
- JET PF coils are connected to form 9 circuits.
- Control inputs represented by currents flowing in the circuits.
- Inputs available:

  9 control inputs u.

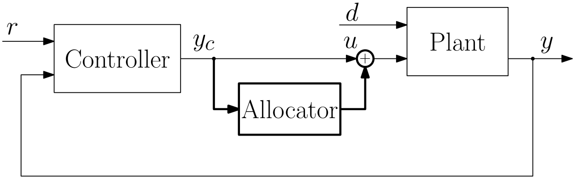


▷ No redundant inputs: still need achieve saturation avoidance!

Recall allocator features for the redundant case

- Essential features of the dynamic allocator seen before

$$\dot{w} = -\rho K B_\perp^T \nabla J$$
$$u = y_c + B_\perp w$$



- The columns of $B_\perp$ correspond to the redundant directions
- K diagonal allows to promote/penalize different redundant directions
- $\bar{W}$ imposes the optimality criterion: u converges to

$$u^\star = \text{argmin}_w (u - u_0)^T \bar{W} (u - u_0), \text{ subject to: } u = y_c^\star + B_\perp w,$$

namely minizes cost $J = (u - u_0)^T \bar{W} (u - u_0)$.

- ρ, positive scalar allows to adjust convergence speed

Extended cost function and new “trade-off” allocator

▷ We introduce a more general **cost function** [before]

$$J_e(u, \delta y) \qquad [J = (u - u_0)^T \bar{W} (u - u_0)]$$

▷ Minimum of J_e is a **trade-off between** ($\star$ denotes steady state values).

- the modified steady state value of the **plant input** $u^\star$ and
- the associated **output modification** $\delta y^\star$ with respect to the original $y^\star$

▷ The new allocator is described by the equations [before] :

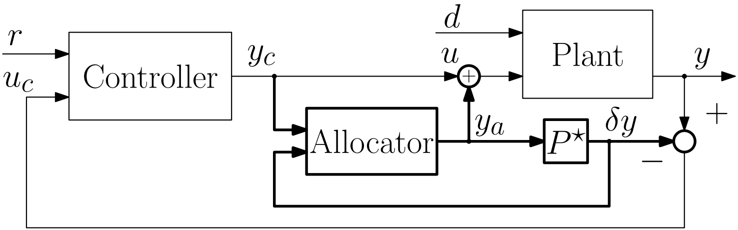
$$\begin{aligned} \dot{w} &= -\rho K B_0^T \begin{bmatrix} I \\ P^\star \end{bmatrix}^T \nabla J_e \\ u &= y_c + B_0 w \end{aligned} \qquad \left[\begin{aligned} \dot{w} &= -\rho K B_\perp^T \nabla J \\ u &= y_c + B_\perp w \end{aligned} \right]$$

▷ B_0 is a suitable full column rank matrix, generalizing the matrix $B_\perp$ (all input directions are potentially “redundant” now).

Allocator now also injects signals at plant output

▷ New allocator injects extra signal $\delta y = P^\star y_a$ so as to not "fight" against the controller at the steady-state:

$$\begin{aligned} u_c &= y - P^\star B_0 w = y - P^\star y_a \\ u &= y_c + B_0 w = y_c + y_a \end{aligned}$$



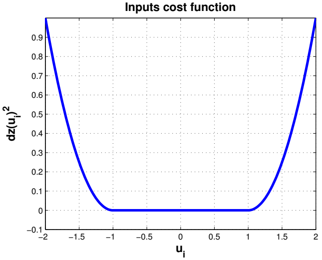
Th'm Under some convexity assumptions on nonlinear cost J_e, for sufficiently small ρ the allocator is such that, under constant inputs, $(u(t), \delta y(t))$ converge to the minimizer of J_e.

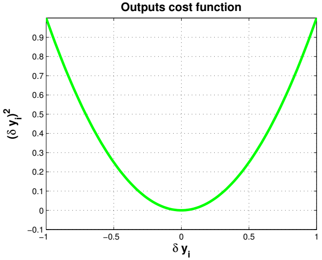
Example of a cost function: penalize u and δy

A possible selection of the cost function is

$$J_e(u, \delta y) = \sum_{i=1}^{n_u} a_i \mathrm{dz}(u_i)^2 + \sum_{i=1}^{n_y} b_i (\delta y_i)^2$$

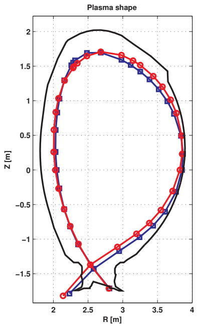
where $\mathrm{dz}(u_i) = \mathrm{sign}(u_i) \max\{0, |u_i| - 1\}$, $a_i \geq 0$, $i = 1, \dots, n_u$ and $b_i > 0$ $i = 1, \dots, n_y$.

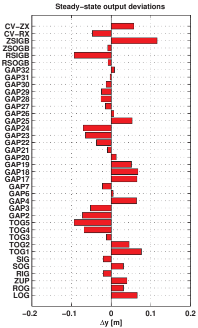




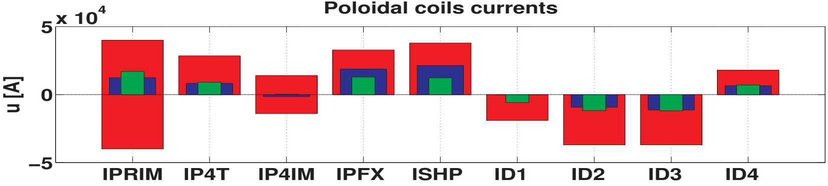
Alternative non symmetric choices are possible

Steady-state allocation: penalize input u



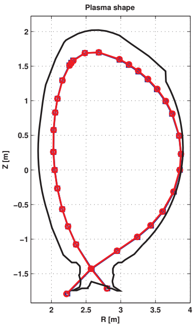


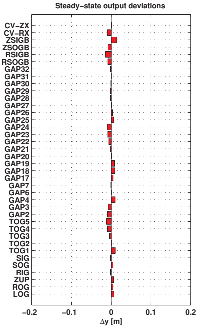
- Allocated shape (red baloon) greatly modified wrt the nominal shape (blue baloon)
- ID1 is moved away from saturation by allocator



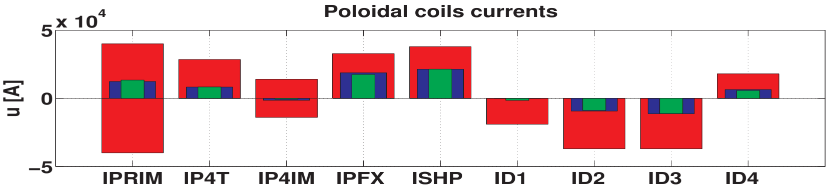
Input ranges (red), controller output y_c (blue), allocated input u (green)

Steady-state allocation: penalize output y



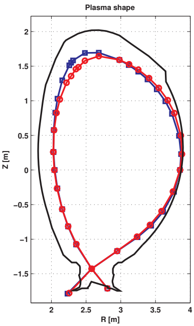


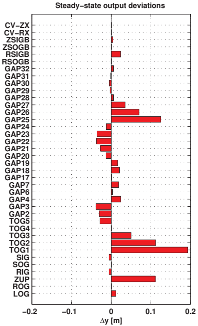
- Allocated shape (red baloon) slightly modified wrt the nominal shape (blue baloon)
- Increasing output penalty, shape modification $\delta y^{\star}$ is reduced
- ID1 comes back very close to saturation level



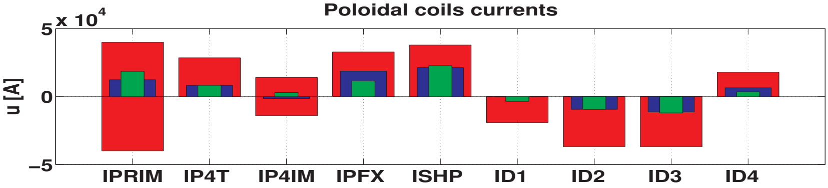
Input ranges (red), controller output y_c (blue), allocated input u (green)

Steady-state allocation: restrict B_0 to nail down outputs





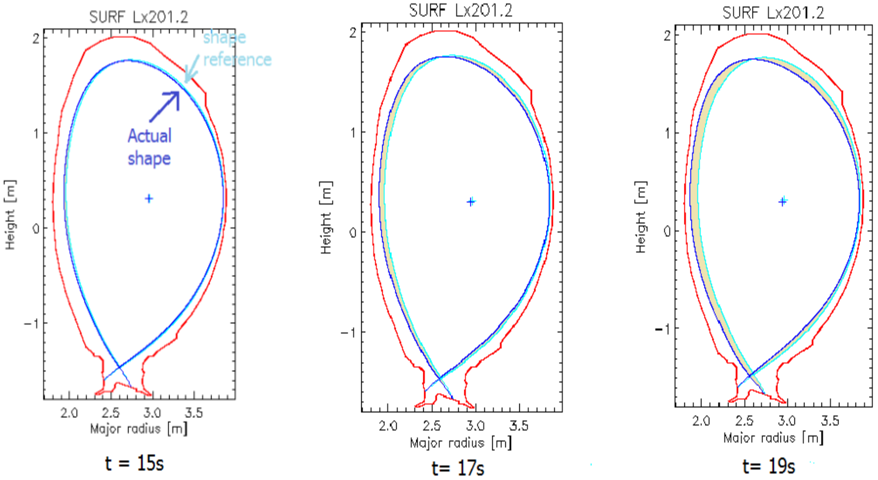
- Allocated shape (red baloon) nominal shape (blue baloon)
- Penalize input u as in first test
- Remove columns from B_0 to fix 5 outputs (CV-RX, CV-ZX, ZSOGB, RSIGB and RSOGB, i.e. X-point and strike points) and one input (IP4T current)
- ID1 again far from saturation level



Input ranges (red), controller output (blue), allocated input (green)

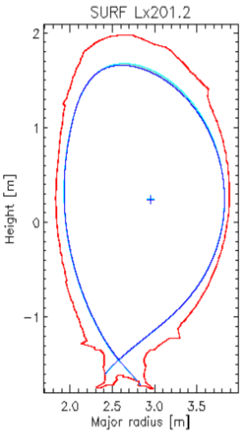
Experiment during current ramp-down *without* allocator

- ▷ X-point and strike points severely compromised at $t = 19\ s$
- ▷ Radial Inner Gap (RIG) also becomes very small

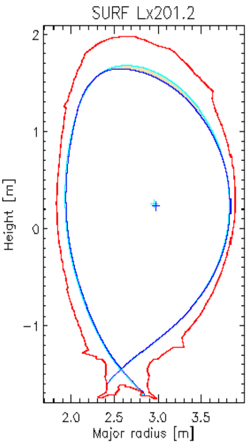


Experiment during current ramp-down *with* allocator

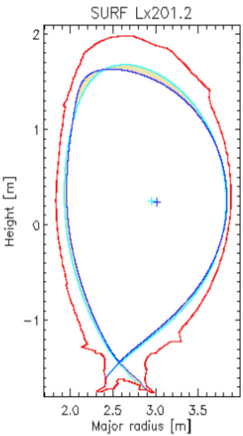
- ▷ X-point, strike points and RIG better behaved in the same conditions
- ▷ Shape is sacrificed in the upper part of the vessel where space is available



t = 15s



t = 16s



t = 18s

Summary of presented works with references

▷ A recent survey about input allocation in Johansen and Fossen [2013]

▷ First ideas behind the presented theory with some nonlinear applications Zaccarian [2007, 2009]

▷ The presented applications are reported in:

- FTU elongation control Boncagni et al. [2012]
- Hybrid Electric Vehicle control Cordiner et al. [2014]
- Satellite attitude stabilization Trégouët et al. [2014]
- Hydrodynamic dynamometer application Passenbrunner et al. [2012]

▷ JET current limiit avoidance system

- Theory of trade-off allocator and first simulations Tommasi et al. [2011]
- Software implementation commissioning Tommasi et al. [2012]
- Closed-loop experimental results Tommasi et al. [2013a,b]

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