Model recovery anti-windup for input-saturated plants illustrated by control applications

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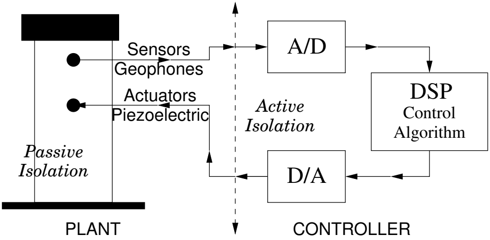
Outline

1. Model recovery anti-windup solution
2. Applications using Linear Model Recovery Anti-Windup
3. Applications using Nonlinear Model Recovery Anti-Windup

Active control provides extreme vibration isolation

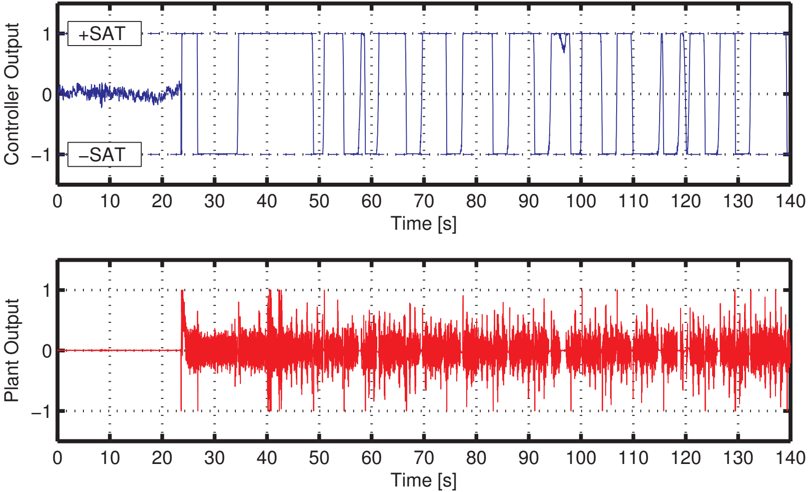
Newport Corporation's Elite 3™ vibration isolation table

- Useful, for example, in
  - high-precision microscopy
  - semiconductor manufacturing
- Actuators: piezoelectric stack
- Sensors: geophones



Input saturation confuses the base control algorithm

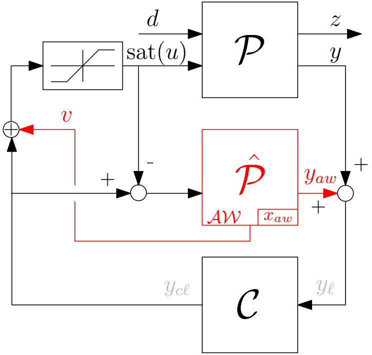
- Extreme vibration suppression (40 dB) up to $t = 23\ s$



- At $t = 23\ s$ someone walks close to the table

Linear Model Recovery Anti-Windup main intuition

Teel and Kapoor [1997], Zaccarian and Teel [2002, 2011]



Model Recovery Anti-Windup *(MRAW)*

- Framework for **nonlinear** $\mathcal{AW}$:
  - $\mathcal{AW}$ is a model $\hat{\mathcal{P}}$ of $\mathcal{P}$
  - $v = k(x_{aw})$ is a (nonlinear) stabilizer whose construction depends on $\mathcal{P}$
- $\mathcal{AW}$ is **controller-independent**:
  - any (nonlinear) $\mathcal{C}$ allowed
- Useful feature of MRAW:
  - $\mathcal{C}$ "receives" linear plant output y_ℓ
  - $\Rightarrow \mathcal{C}$ "delivers" linear plant input $y_{c\ell}$

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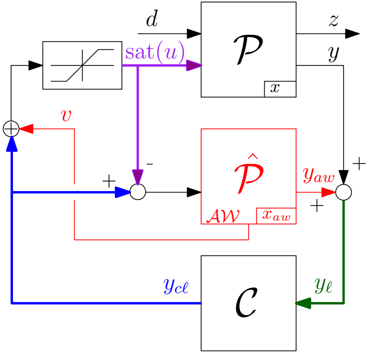

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- Plant $\mathcal{P}$

$$\begin{cases} \dot{x} &= A\,x + B_d\,d + B_u\,\mathrm{sat}(u) \\ z &= C_z\,x + D_{dz}\,d + D_{uz}\,\mathrm{sat}(u) \\ y &= C_y\,x + D_{dy}\,d + D_{uy}\,\mathrm{sat}(u) \end{cases}$$

- Anti-windup filter $\hat{\mathcal{P}}$

$$\begin{cases} \dot{x}_{aw} &= A\,x_{aw} + B_u\,(y_c - \mathrm{sat}(u)) \\ y_{aw} &= C_y\,x_{aw} + D_{uy}\,(y_c - \mathrm{sat}(u)) \end{cases}$$

- Unconstrained dynamics $\mathcal{P} + \hat{\mathcal{P}}$: $\begin{cases} \dot{x}_\ell &= A\,x_\ell + B_d\,d + B_u\,y_c \\ \overset{=y+y_{aw}}{y_\ell} &= C_y\,x_\ell + D_{dy}\,d + D_{uy}\,y_c \end{cases}$

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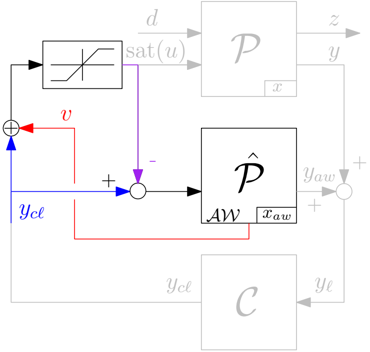

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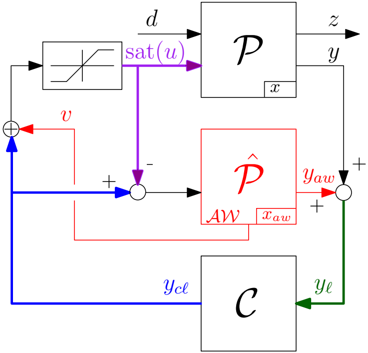
- $x_{aw} = x_\ell - x$ stores useful information about the mismatch response
- Unconstrained recovery: **stabilize** x_{aw} to zero using v
- Anti-windup filter $\hat{\mathcal{P}}$ stabilized by v through time-varying saturation

$$\left\{ \begin{array}{rcl} \dot{x}_{aw} & = & A\,x_{aw} - B_u\,(\mathrm{sat}[y_{c\ell}(t) + k(x_{aw})] - y_{c\ell}(t)) \\ z_{aw} & = & C_z\,x_{aw} - D_{uz}\,(\mathrm{sat}[y_{c\ell}(t) + k(x_{aw})] - y_{c\ell}(t)) \end{array} \right.$$

Linear Model Recovery Anti-Windup main intuition

Pagnotta et al. [2007], Zaccarian and Teel [2005], Forni et al. [2012, 2010], Zaccarian et al. [2005]



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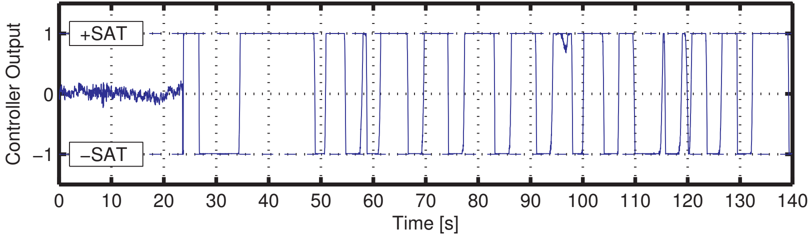
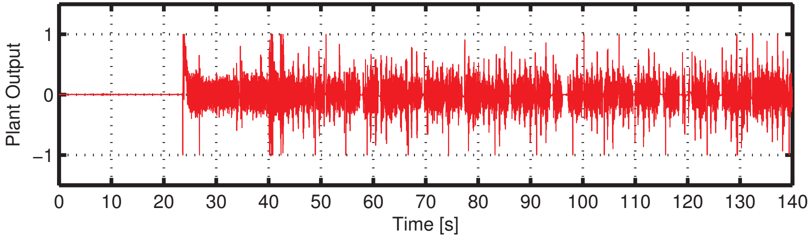
Several **extensions** are possible:

- **Reduced order** $\hat{\mathcal{P}}$ possible (tested on adaptive noise suppression)
- MRAW allows for **bumpless transfer** among controllers
- MRAW generalizes to **rate** and **curvature** saturation
- MRAW generalizes to **dead time** plants (Smith predictor)

Base control algorithm confused (recall)

- Extreme vibration suppression (40 dB) up to $t = 23\ s$



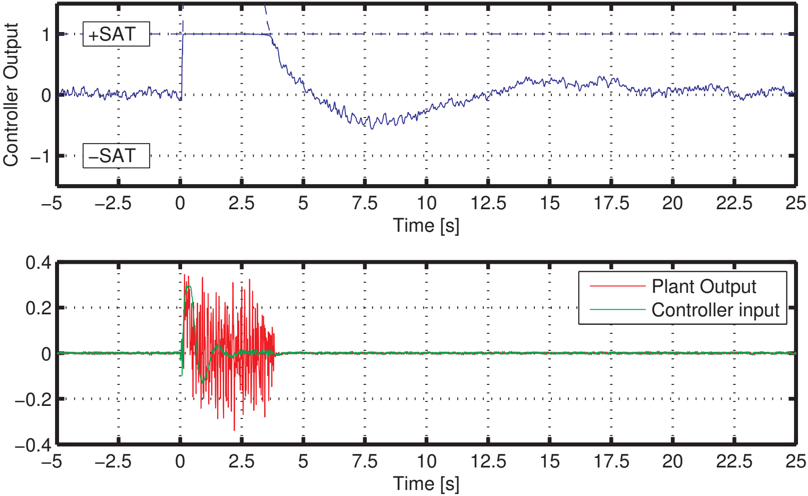


- At $t = 23\ s$ someone walks close to the table

MRAW dramatically reduces isolation recovery time

Teel et al. [2006], Zaccarian et al. [2000]

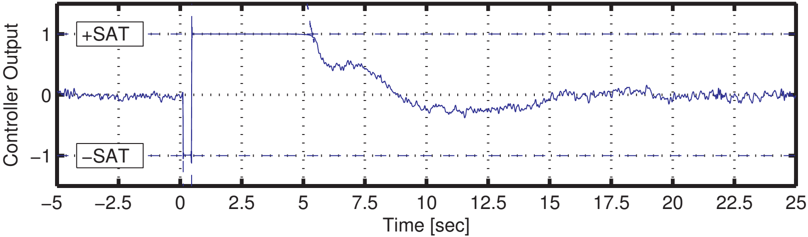
- Effect of a footstep at the side of the table (recovery $\approx$ 4 s)

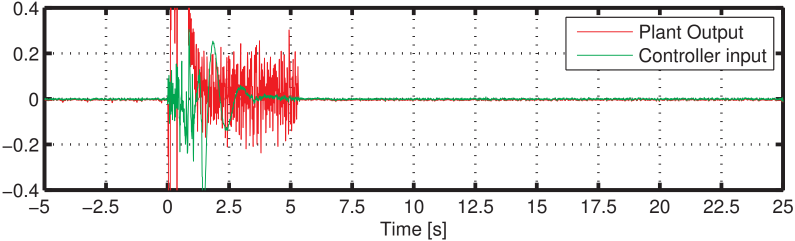


Even a bat strike does not confuse the MRAW controller

Teel et al. [2006], Zaccarian et al. [2000]

- Hitting with a baseball bat the table leg (recovery $\approx$ 5 s)

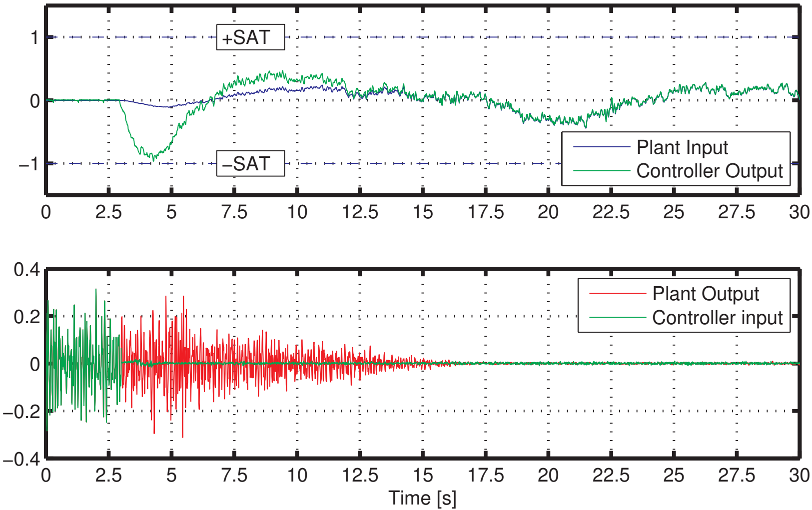




Bumpless transfer enables smooth controller activation

Teel et al. [2006], Zaccarian et al. [2000]

- Controller is gradually activated in bumpless transfer scheme



Anti-windup for open-water irrigation channels

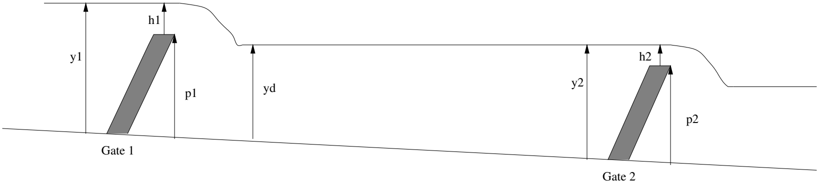
Zaccarian et al. [2007]

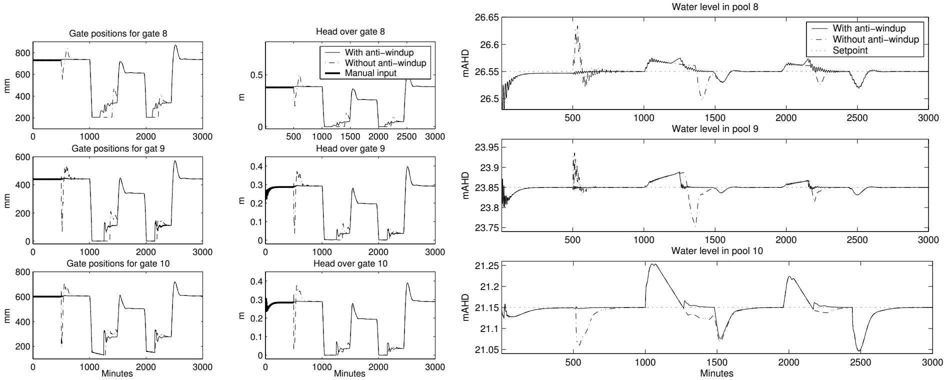
- Open Water Channels: rivers are broken into pools for water saving
- Gate saturation problems:
  - bumpless transfer from manual control to avoid startup transients
  - with small flows in the pools bad lower saturation effects
  - with large disturbances (rain, etc) with overflow to downstream pool
- **Challenge**: plant is not exponentially stable (poles in 0)

Simulations save days of transient response

Zaccarian et al. [2007]

Simulations with model of Haughton Main Channel (Queensland, Aus)





Rate Saturated McDonnell Douglas TAFA dynamics

Barbu et al. [2005]

- Linearized longitudinal dynamics (α=angle of attack; q=pitch rate)

$$\dot{z} := \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} z + \begin{bmatrix} 0 \\ M_\delta \end{bmatrix} \delta =: A z + B_u \delta$$

- Saturation: $M = 20\ deg$, $R = 40\ deg/s$.

$$\dot{\delta} = R\,\mathrm{sgn}\left[M\,\mathrm{sat}\left(\frac{u}{M}\right) - \delta\right],$$

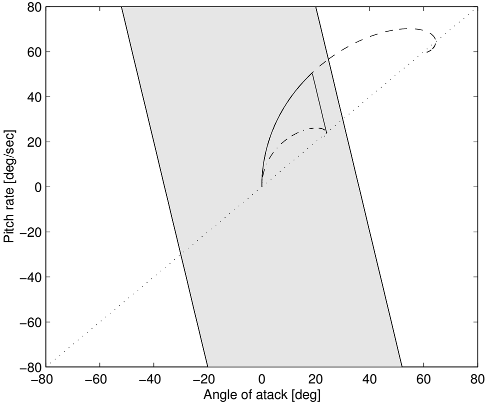
- Study a flight trim condition with one exp unstable mode

$$\dot{x} := \begin{bmatrix} \dot{x}_s \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} b_s \\ b_u \end{bmatrix} \delta$$

Magnitude saturation and exponential instability

Galeani et al. [2007], Teel [1999]

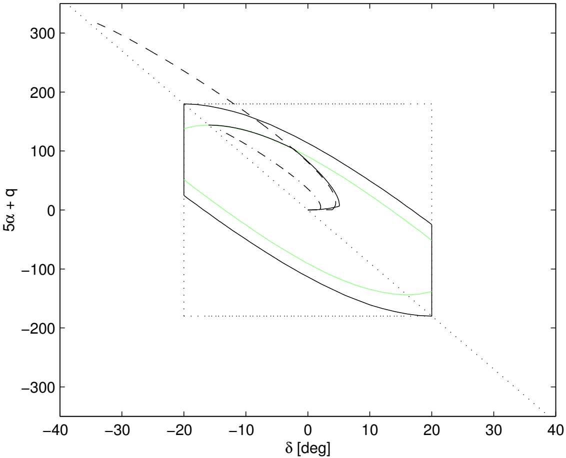
- Unconstrained trajectory may exit the null-controllability region
- To prevent this, AW scheme uses $v = \bar{k}(x_{aw}, x_u)$



- Unconstrained (−−), possible desired trajectories (− and − · −)

Problems due to magnitude+rate saturation

- Unconstrained trajectory may exit the null-controllability region

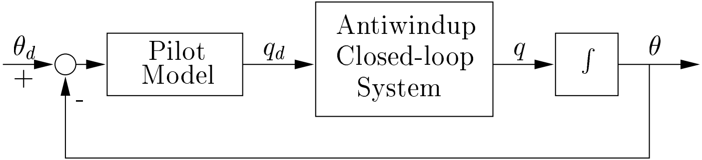


- Unconstrained (−−), possible desired trajectories (− and − · −)

Close the position loop using a pilot model

Barbu et al. [2005]

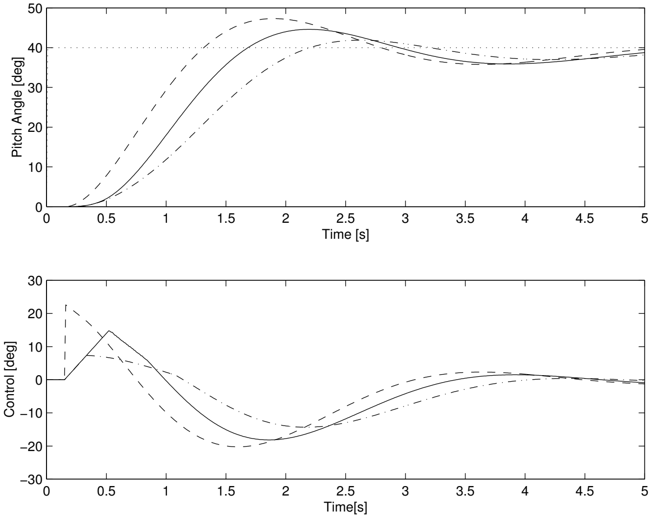
- Use a simple crossover model



- Study the maneuverability of the aircraft with anti-windup
- Study the possible occurrence of PIOs (Pilot Induced Oscillations)
- Compare with static command limiting (saturating q_d)
- Use a step reference $\theta_d = 40\ deg$

Piloted flight simulation

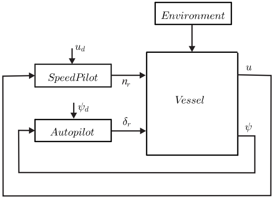
Barbu et al. [2005]



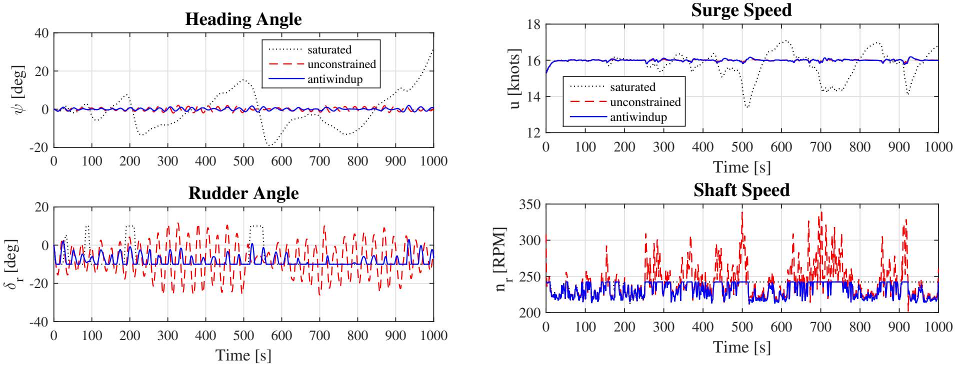
(unconstrained $--$, anti-windup $-$, static limiting $- \cdot -$)

Speed and Heading Control of Ships: approximate models

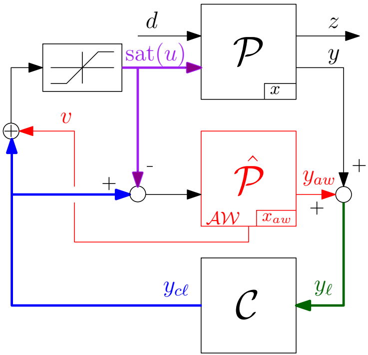
Donnarumma et al. [2016]



- u = surge speed, n_r = (commanded) shaft speed
- ψ = heading angle, δ_r = rudder angle
- Two indepedent loops on nonlinearly coupled plant
- Anti-windup model is linear and decentralized
- Robustness of MRAW provides strong improvement



Recall the Linear MRAW scheme



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- Unconstrained dynamics $\mathcal{P} + \hat{\mathcal{P}}$: $\begin{cases} \dot{x}_\ell &= A\,x_\ell + B_d\,d + B_u\,y_c \\ \overset{=y+y_{aw}}{y_\ell} &= C_y\,x_\ell + D_{dy}\,d + D_{uy}\,y_c \end{cases}$

Unconstrained response information (linear case)

- Plant $\mathcal{P}$

$$\begin{cases} \dot{x} &= A\,x + B_d\,d + B_u\,\mathrm{sat}(u) \\ z &= C_z\,x + D_{dz}\,d + D_{uz}\,\mathrm{sat}(u) \\ y &= C_y\,x + D_{dy}\,d + D_{uy}\,\mathrm{sat}(u) \end{cases}$$

- Anti-windup filter $\hat{\mathcal{P}}$

$$\begin{cases} \dot{x}_{aw} &= A\,x_{aw} + B_u\,(y_c - \mathrm{sat}(y_c + v)) \\ y_{aw} &= C_y\,x_{aw} + D_{uy}\,(y_c - \mathrm{sat}(y_c + v)) \end{cases}$$

- Unconstrained controller $\mathcal{C}$

$$\begin{cases} \dot{x}_c &= A_c\,x_c + B_{cu}\,u_c + B_{cr}\,r \\ y_c &= C_c\,x_c + D_{cu}\,u_c + D_{cr}\,r \end{cases}$$

- Interconnections

$$\begin{cases} u &= y_c + v, \\ u_c &= y + y_{aw} \end{cases}$$

$v = \bar{k}(x_{aw}, x_u)$: to be selected!

- Coordinate transformation: $(x_\ell, x_c, x_{aw}) = (x + x_{aw}, x_c, x_{aw})$

- Unconstrained dynamics $\mathcal{P} + \hat{\mathcal{P}}$: $\begin{cases} \dot{x}_\ell &= A\,x_\ell + B_d\,d + B_u\,y_c \\ y + y_{aw} &= C_y\,x_\ell + D_{dy}\,d + D_{uy}\,y_c \end{cases}$

- $\Rightarrow$ Unconstrained response information embedded within the scheme!

Unconstrained response information (nonlinear case)

- Plant $\mathcal{P}$

$$\begin{cases} \dot{x} & = & f(x, \mathrm{sat}(u)) \\ z & = & h(x, \mathrm{sat}(u)) \end{cases}$$

- Unconstrained controller $\mathcal{C}$

$$\begin{cases} \dot{x}_c & = & g(x_c, u_c, r) \\ y_c & = & k(x_c, u_c, r) \end{cases}$$

- Anti-windup filter $\hat{\mathcal{P}}$

$$\begin{cases} \dot{x}_{aw} & = & f(x + x_{aw}, y_c) - f(x, \mathrm{sat}(y_c + v)) \\ y_{aw} & = & x_{aw} \end{cases}$$

- Interconnections

$$\begin{cases} u & = & y_c + v, \\ u_c & = & x + x_{aw} \end{cases}$$

$v = \bar{k}(x_{aw}, ??)$: to be selected!

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- Unconstrained dynamics $\mathcal{P} + \hat{\mathcal{P}}$: $\begin{cases} \dot{x}_\ell & = & f(x_\ell, y_c) \\ u_c & = & x + x_{aw} = x_\ell \end{cases}$

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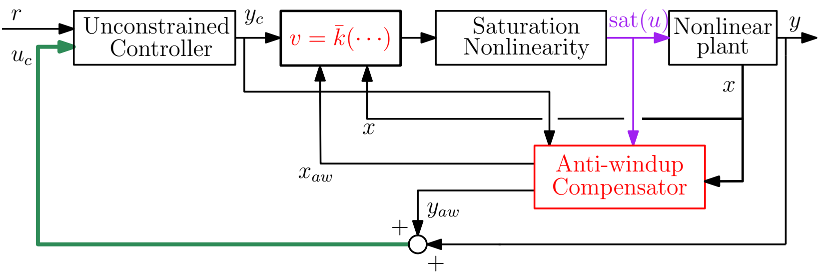
Anti-windup for nonlinear plants: resulting scheme



- Need extra plant state measurements (x generally needed)
- Recall that $x_{aw} = x_\ell - x$: useful for unconstrained response recovery
  - worry about stability looking at x (e.g., x_u for exponential instability)
  - worry about performance looking at x_{aw}
- A few application examples:
  - Anti-windup for robot manipulators Morabito et al. [2004]
  - Anti-windup for Brake-by-Wire systems Todeschini et al. [2016]

A SCARA robot manipulator example

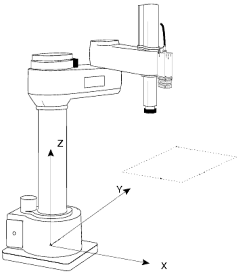
Morabito et al. [2004]

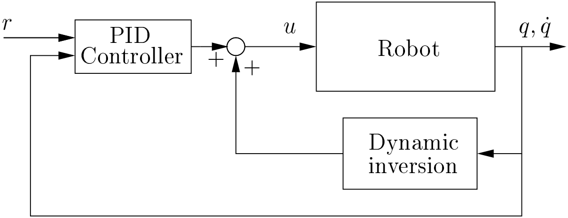
- SCARA robot with limited torque/force inputs

Link	1	2	3	4
m_i	$55\ Nm$	$45\ Nm$	$70\ N$	$25\ Nm$

- General class of systems is:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + h(q) = \mathrm{sat}(u)$$



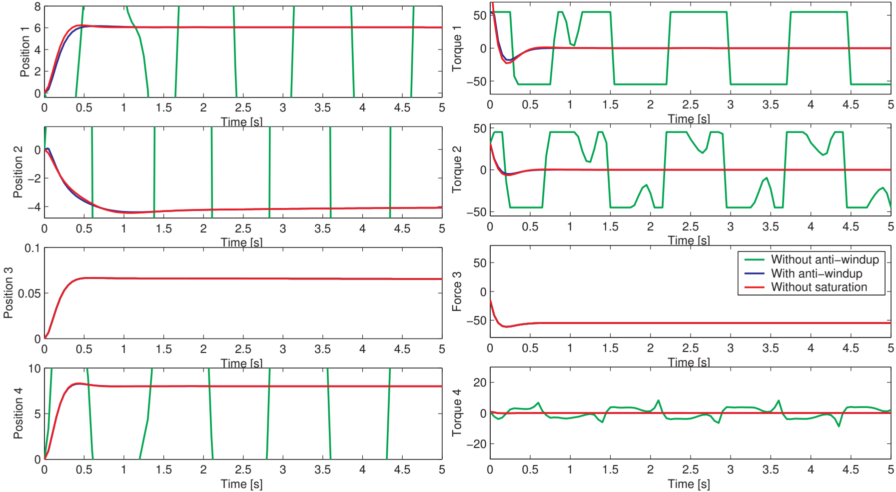


P	I	D
121	7.5	17.8
30	10	8.2
150	1	24.7
150	0.5	20.1

- Feedback linearizing controller+PID action (*computed torque*) induces decoupled linear performance (for small signals)

A slight saturation can be disastrous

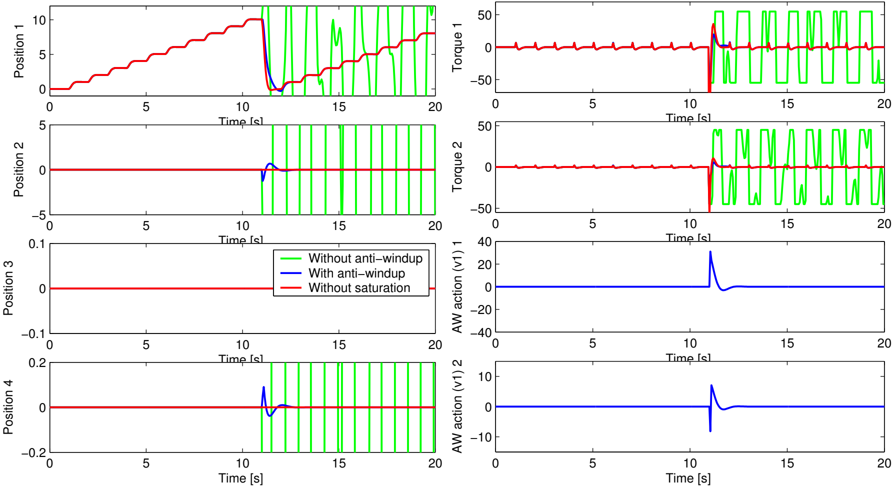
- The reference is $r = [6\ deg,\ -4\ deg,\ 4\ cm,\ 8\ deg]$



- Stability is recovered, performance is almost fully preserved

Anti-windup injects signals and then fades out

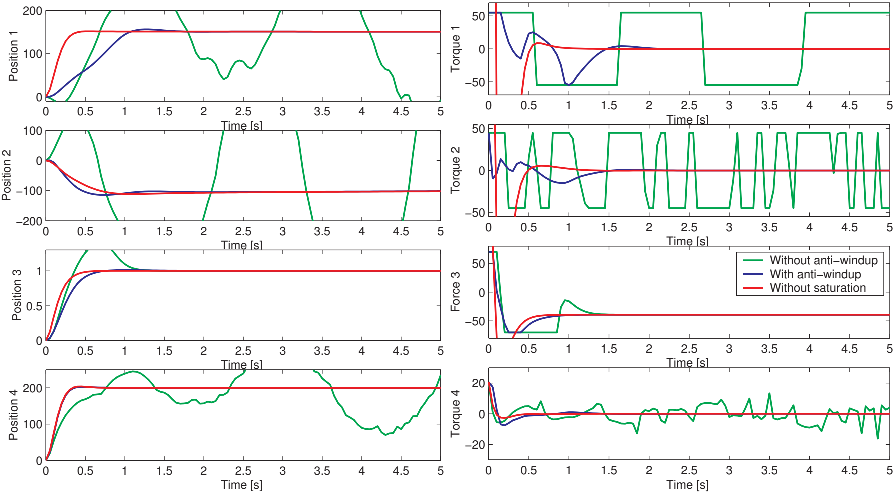
- The reference is a sequence of little steps followed by a large step



- Anti-windup action dies away to recover the unconstrained closed-loop

SCARA: large signals (nonlinear stabilizer $v = \bar{k}(x_{aw}, q, \dot{q})$)

- The reference is $r = [150\ deg,\ -100\ deg,\ 1\ m,\ 200\ deg]$

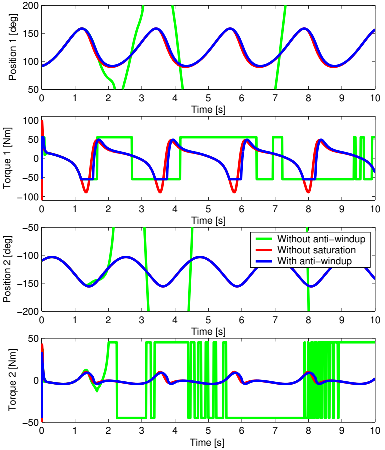


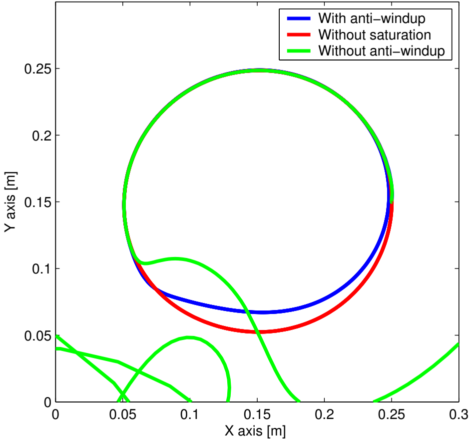
- Performance dramatically improved (input authority well exploited)

MRAW intrinsically addresses tracking recovery

Example: a **SCARA robot** (planar robot) following a circular motion

- Saturated "computed torque" controller goes postal (unstable)
- Nonlinear MRAW provides slight performance degradation

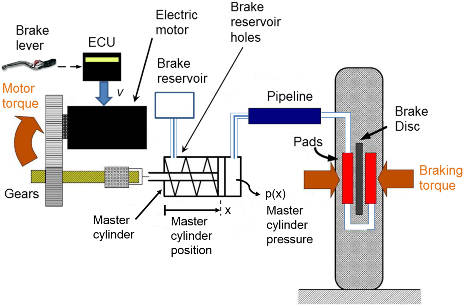


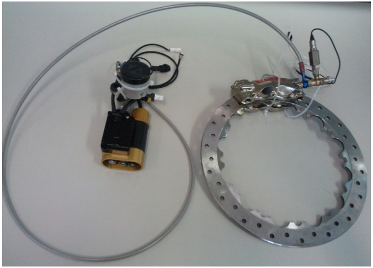


Nonlinear anti-windup for a Brake By Wire System

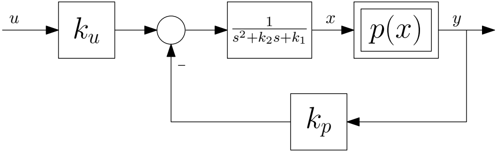
Todeschini et al. [2016]

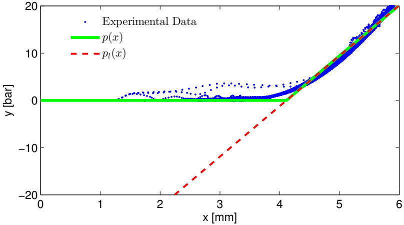
- Brake-by-wire system in motorcycles corresponds to a nonlinear plant





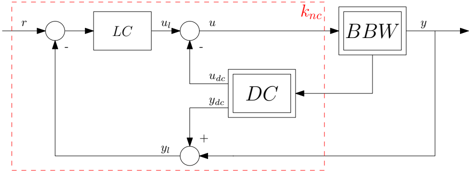
- The main nonlinear effect can be easily isolated in the model:

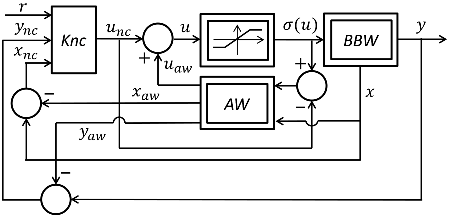


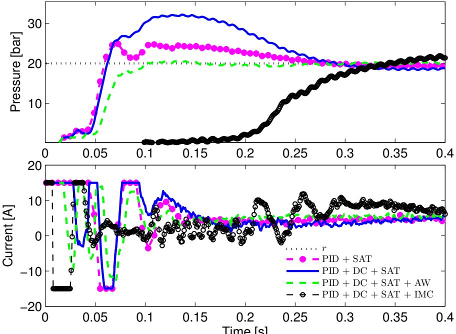


BBW solution uses nonlinear MRAW

- "Deadzone compensation" scheme provides **nonlinear** baseline controller
- **Fully Nonlinear anti-windup** addresses saturation with **nonlinear plant** and **nonlinear controller**







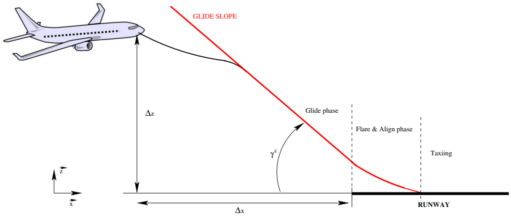
- Step response reveals successful anti-windup action
- Driver would get confused by large overshoots
- Alternative existing solutions (nonlinear IMC-based anti-windup) are unacceptably slow (black)

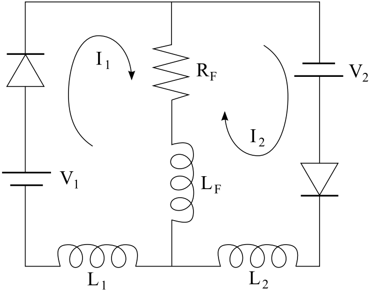
Anti-windup designs apply to additional applications

Vitelli et al. [2010], Burlion et al. [2019]

Image-based visual servoing

- Relevant for plane landing
  - follow reference glide slope
  - position measurement scaled by unknown factor
- **Challenge**: plant is uncertain (need robust approach)





Small signal nonlinearity compensation in high-power circulating current amps

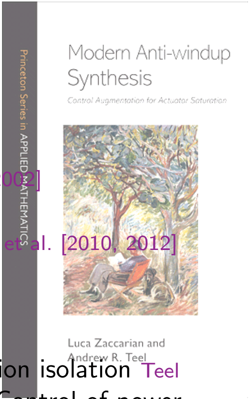
- Thyristors have a min current threshold:
  - below the treshold: circulating current
  - this generates a undesired nonlinearity
  - possibly destabilizing outer feedback
- **Challenge**: reverse anti-windup problem

References

▷ **Summary** of the proposed Model Recovery Anti-Windup in Galeani et al. [2009], Zaccarian and Teel [2011]

▷ **Model-Recovery anti-windup** schemes

- Baseline ideas Teel and Kapoor [1997], Zaccarian and Teel [2002]
- Bumpless transfer extensions Zaccarian and Teel [2005]
- Generalizations to rate and curvature saturations Forni et al. [2010, 2012]
- Dead-time plants (input delays) Zaccarian et al. [2005]



▷ **MRAW Applications** discussed in this talk:

- Linear MRAW: Flight Control Barbu et al. [2005], Vibration isolation Teel et al. [2006], Open Water Channels Zaccarian et al. [2007], Control of power converters Vitelli et al. [2010], Ship control Donnarumma et al. [2016].
- Nonlinear MRAW: Control of Euler-Lagrange systems Morabito et al. [2004], control of Break-by-wire systems Todeschini et al. [2016], Image-based servoing Burlion et al. [2019].

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