Input allocation: hierarchical design paradigm with redundant actuators and its aerospace applications

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The fascinating experience of scientific exchange


D Arzelier


A Franchi


M Furci


D Invernizzi


M Lovera


M Maggiore


C Nainer


D Peaucelle


C Pittet


M Sassano

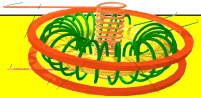
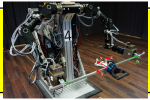

A Seuret


JF Tregouet

Thanks also to: L Boncagni, M Cocetti, G De Tommasi, S Galeani, F Mecocci, A Pironti, A Serrani, G Varano, V Vitale, R Vitelli, A Zambelli

Nonlinear cascades as hierarchical behaviors

- ▷ Practical experience with several applications
- ▷ Abstraction reveals a pervasive pattern of control specs hierarchies

Application	High Priority Task	Perturbation →	Low Priority Task
Hybrid Cars	Driveability: Accelerator Response		State of Charge of Battery
Tokamak Plasmas	Plasma Position		Plasma Elongation
Cooperative Manipulation	Motion Control		Internal Forces

- ▷ Stability/Hierarchy analysis stems from nonlinear cascades

Historical results on nonlinear cascades by E. Sontag

- Cascaded systems intrinsically represent hierarchical tasks

$$\boxed{\dot{x}_U = f_U(x_U)} \xrightarrow{x_U = u} \boxed{\dot{x}_L = f_L(x_L, u)} \xrightarrow{x_L}$$

- Stability analysis results date back to the 1980's

Theorem ISS

If (U) is GAS and (L) is 0-GAS and ISS, then cascade is GAS

Theorem SON

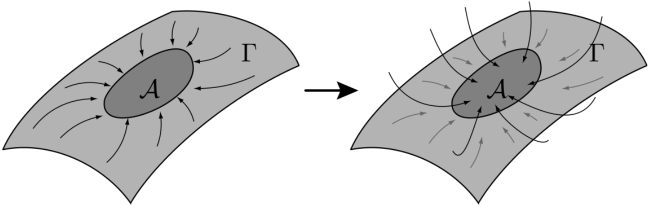
If (U) is GAS and (L) is 0-GAS, then cascade is LAS with basin of attraction $\mathcal{B}_{\mathcal{A}} = \{$largest set from where solutions don't diverge$\}$.

Corollary SON

If (U) is GAS and (L) is 0-GAS, and all solutions are bounded, then cascade is GAS

Cascades generalize to reduction theorems useful next

▷ It is not always possible to write “cascaded-like” coordinates: in reduction theorems, the upper system (U) comprises convergence to a closed set Γ



▷ Redeuction theorems for continuous-time discrete-time and hybrid dynamics

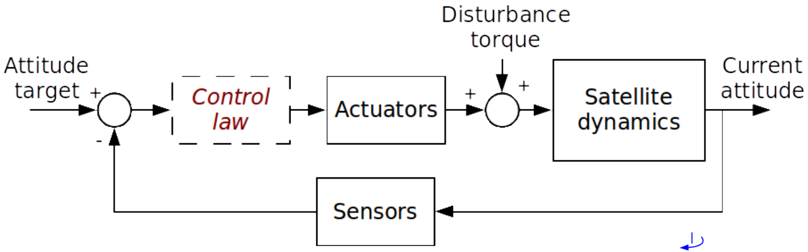
Theorem RED

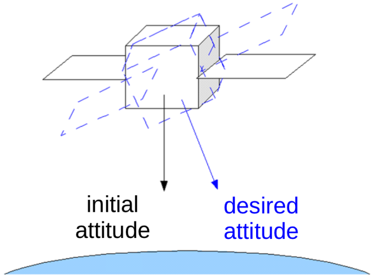
If Γ is GAS and $\mathcal{A}$ is GAS starting from Γ, then $\mathcal{A}$ is LAS with basin of attraction $\mathcal{B}_{\mathcal{A}} = \{$largest set from where solutions don't diverge$\}$.

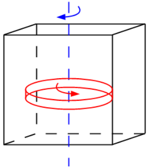
Corollary RED

if Γ is GAS and $\mathcal{A}$ is GAS starting from Γ and all solutions are bounded, then $\mathcal{A}$ is GAS

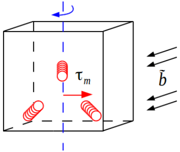
Attitude control performed by two actuators







Reaction wheels

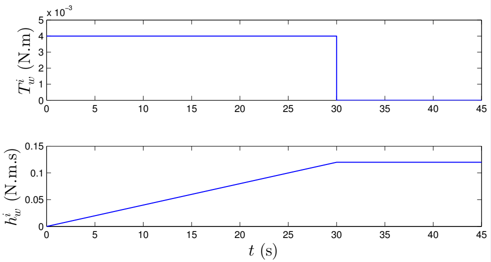


Magnetorquers

Reaction wheels suffer from total momentum problems

Reaction wheels

$$T_w = \dot{h}_w$$



Nomenclature

- $h_w \in \mathbb{R}^3$: angular momentum
- $T_w \in \mathbb{R}^3$: control torque

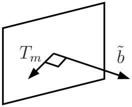
✗ Total momentum can't be modified (wheel turns CW, satellite turns CCW)

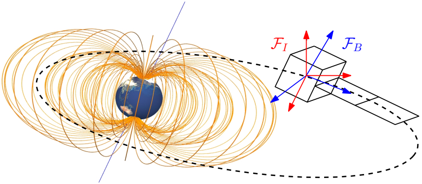
✗ risk of saturation of h_w

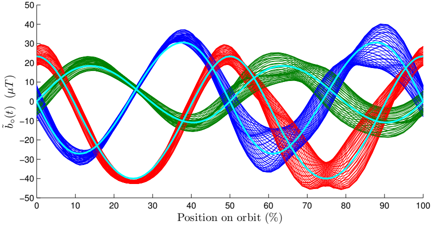
$$\Rightarrow h_w(t) = \int_0^t T_w(\tau)d\tau \text{ needs to be controlled}$$

Magnetorquers confined to exert torque in rotating plane

$$T_m = -\tilde{b}^\times(t,q)\tau_m = -(R(q)\tilde{b}_\circ(t))^\times \tau_m$$







Notation

$$z^\times := \begin{bmatrix} 0 & -z_z & z_y \\ z_z & 0 & -z_x \\ -z_y & z_x & 0 \end{bmatrix}$$

Nomenclature

- $T_m \in \mathbb{R}^3$: control torque
- $\tilde{b} \in \mathbb{R}^3$: magnetic field
- $\tau_m \in \mathbb{R}^3$: magnetic momentum
- $q \in \mathbb{R}^4$: quaternion
- $R \in \mathbb{R}^{3\times 3}$: rotation matrix

✗ $(\ )^\times$: instantaneous controllability restricted to a plane ($z^\times$ is singular)

✗ $\tilde{b}_\circ(t)$: almost periodic and uncertain

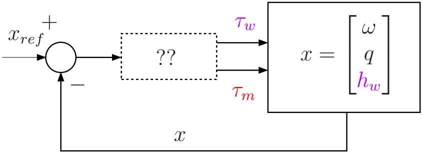
Stabilization problem requires coordination of the actuators

Equations of the attitude motion

$$J\dot{\omega} = -\omega^{\times}(J\omega + h_w) - \tau_w - \overbrace{\widetilde{b}^{\times}(t,q)\tau_m}^{T_m}$$

$$\dot{h}_w = \tau_w$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^{T} & 0 \end{bmatrix}\begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$



Nomenclature

Satellite:

- ω: angular velocity
- $q = (\varepsilon, \eta)$: quaternion
- J: inertia matrix

Reaction wheels:

- h_w: angular momentum
- $\tau_w = T_w$: control torque

Magnetorquers:

- $\widetilde{b}(t,q)$: geomagnetic field
- τ_m: magnetic momentum

$\Rightarrow$ Design goal: find $\tau_w(x)$ and $\tau_m(x)$ such that $x := \begin{bmatrix} \omega \\ q \\ h_w \end{bmatrix} \to \begin{bmatrix} \mathbf{0} \\ q_\circ \\ h_{ref} \end{bmatrix}$

✗ actuators may badly interact

Global attitude stabilization via hybrid feedback

▷ Ideal attitude feedback u_{att} may be selected as a hybrid control law

$$J\dot{\omega} = -\omega^{\times} J\omega + u_{att}$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^{T} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

- No time-invariant continuous selection $u_{att}(x)$ stabilizes the compact attractor $\mathcal{A} := \{\omega = \varepsilon = 0, \eta = \pm 1\}$ [Bhat et al, 2000]

Hybrid solution available in the literature [Mayhew et al, 2009]

For any scalars $k_p > 0$, $k_d > 0$, $\delta \in (0, 1)$, the attractor $\mathcal{A}$ is globally asymptotically and locally exponentially stabilized by the hybrid PD-like dynamic controller:

$$u_{att}(x_c, \varepsilon, \omega) := -k_p x_c \varepsilon - k_d \omega$$

$$\begin{aligned} \dot{x}_c &= 0, &\text{when } (q, \omega, x_c) \in C := \{(q, \omega, x_c) : x_c \eta \geq -\delta\} \\ x_c^+ &= -x_c, &\text{when } (q, \omega, x_c) \in D := \{(q, \omega, x_c) : x_c \eta \leq -\delta\}, \end{aligned}$$

where the C is the *flow set* and D is the *jump set*.

I. The industrial solution: "cross product control law"

The cross-product control law

$$\tau_w = -\omega^\times h_w - u_{att}, \qquad \tau_m = -\frac{\widetilde{b}^\times(t)}{|\widetilde{b}(t)|^2} k_p (h_w - h_{ref})$$

Ignore the interaction of the two inputs

$$J\dot{\omega} = -\omega^\times J\omega \overbrace{-\tau_w - \omega^\times h_w}^{u_{att}(x_c,\varepsilon,\omega)} + \overbrace{T_m}^{d},$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega^\times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

- loop 1: Attitude control performed by the reaction wheels
- loop 2: Regulation of h_w by the magnetorquers
- the two loops are treated separately

▷ frequency separation between the two loops (= very aggressive attitude stabilizer) gives engineering solution [Camillo,1980; Carrington 1981]

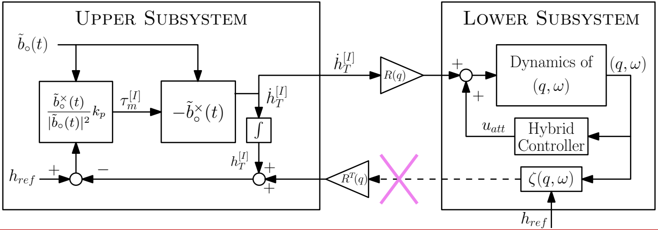
✗ formally proving stability properties of the overall scheme seems hard

II. Revisited "cross product control law" highlights cascade

A revisited version of the cross-product control law

$$\tau_w = -\omega^\times h_w - u_{att}, \qquad \tau_m = -\frac{\widetilde{b}^\times(t)}{|\widetilde{b}(t)|^2} k_p (h_w + J\omega - R(q) h_{ref})$$

- Classical approach reveals quasi cascaded structure where $h_T^{[I]}$ refers to the total angular momentum (satellite + wheels)
  - ✓ the feedback branch (the dashed line) can be avoided by redefining τ_m
  - ✗ attitude dynamics is affected by the momentum dumping action



GAS can be established using Theorem ISS

GAS is proven for any u_{att} under ISS of attitude closed loop if $\widetilde{b}_\circ(t)$ is persistently exciting

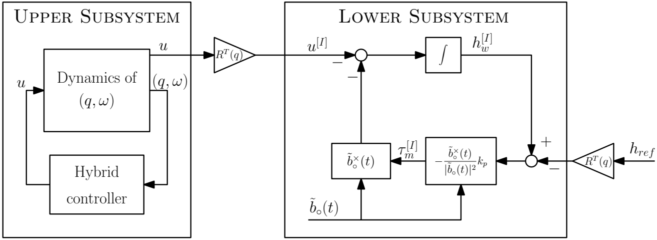
III. Allocation-based controller prioritizes attitude

Allocation-based controller equations

$$\tau_w = -\omega^\times h_w - (R(q)\widetilde{b}_\circ(t))^\times \tau_m - u_{att}, \quad \tau_m = -\frac{(R(q)\widetilde{b}_\circ(t))^\times}{|\widetilde{b}_\circ(t)|^2} k_p(h_w - h_{ref})$$

- ▷ Reversing the cascaded structure giving priority to the attitude stabilization
- ▷ stemming from a different partition of the effected input:

$$J\dot{\omega} + \omega^\times J\omega = \overbrace{-\tau_w - \omega^\times h_w + T_m}^{u_{att}(x_c,\varepsilon,\omega)}.$$



GAS can be established using Corollary SON

GAS is proven for any u_{att} (No ISS needed) if $\widetilde{b}_\circ(t)$ is persistently exciting. Boundedness from LES of (U) and Gronwall Lemma.

Simulations reveal advantages of the proposed controller

Context of the simulations

- Mission: micro-satellite Demeter by CNES, the French space agency
- $\widetilde{b}_{\circ}(t)$ evaluated by the IGRF model of the geomagnetic field
- rest-to-rest maneuvers with non-nominal h_w

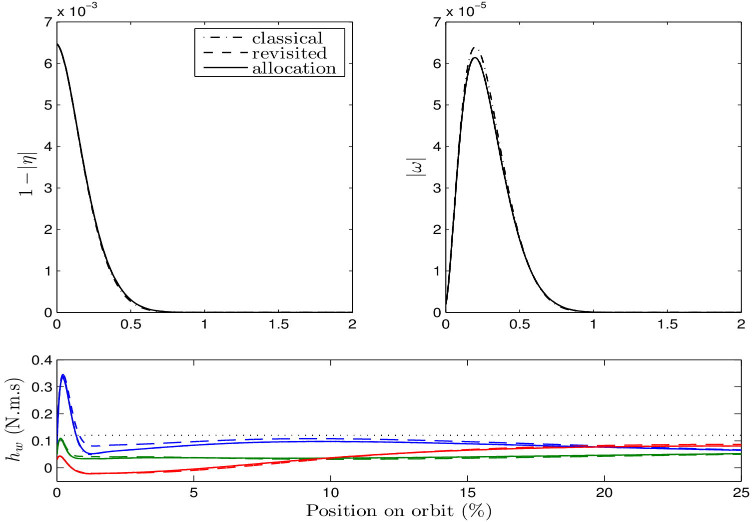
Controllers used

- **Classical** "cross product control law" controller
- **Revisited** version of the classical controller
- **Allocation**-based controller

Simulation tests

- **Nominal**: Shows that the **classical** solution diverges
- **Perturbed** J: **Allocation** outperforms **Revisited**
- **Periodic disturbances**: **Allocation** outperforms **Revisited**

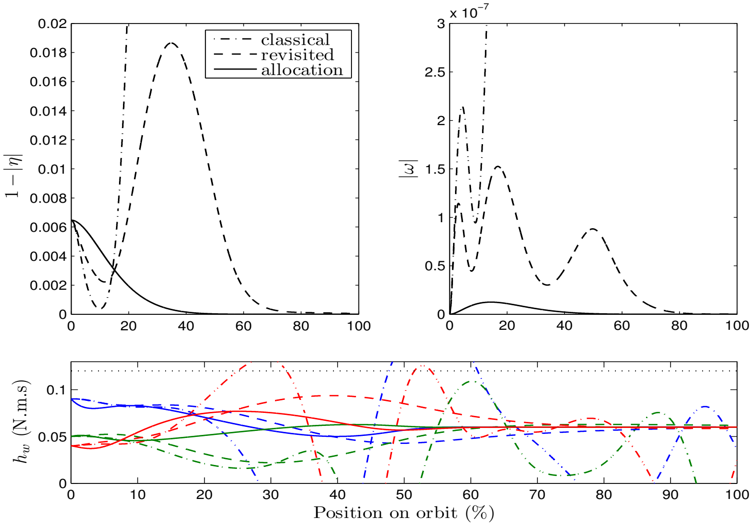
Stabilization transients with aggressive controller



✓ **Similar results**

✗ **saturation of h_w**

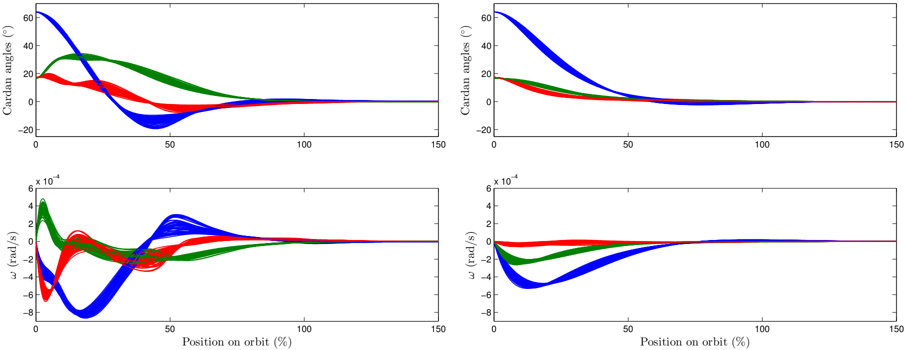
Stabilization transients with non aggressive controller



✓ **revisited and allocation controllers preserve stability**

Monte-Carlo with uncertainties on J: improved transients

▷ Clear advantages emerge from swapping the cascaded structure



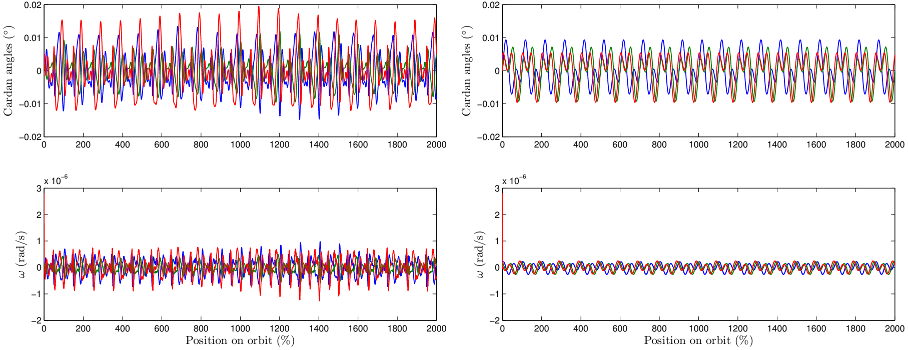
✓ **Improved attitude transients with allocation-based controller** (right)

✓ **Robustness rigorously established by intrinsic results of well-posed (hybrid) feedbacks**

Periodic disturbances are best handled by allocator

▷ No rigiorous analysis has been performed for this case

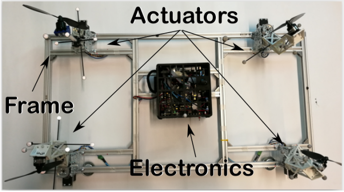
- Interesting direction of future development (regulation theory, contraction theory/convergent dynamics)

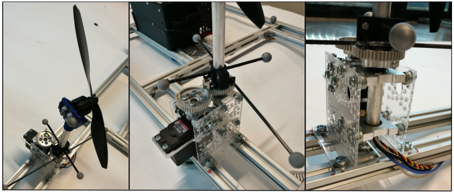


✓ **Improved attitude response with allocation-based controller** (right)

ROSPO represents key challenges in UAV allocation

- ROtor graSPing Omnidirectional (ROSPO) ground platform developed at the LAAS-CNRS in Toulouse (France)





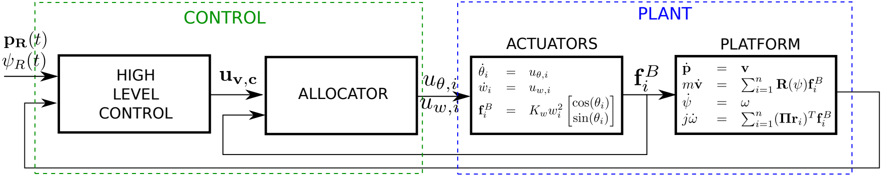
- **3 DoF** Task in SE(2): 2 DoFs position + 1 DoF orientation
  - each turret 2 actuators:
    - propeller: thrust magnitude
    - servo: thrust orientation
  - n turrets: $n = 3,\ 4, \dots$
  - **overactuated** for $n > 1$

$$\begin{cases} \dot{\mathbf{p}} &= \mathbf{v} \\ m\dot{\mathbf{v}} &= \sum_{i=1}^{n} \mathbf{R}(\psi)\mathbf{f}_i^B \\ \dot{\psi} &= \omega \\ j\dot{\omega} &= \sum_{i=1}^{n} (\mathbf{\Pi r}_i)^T \mathbf{f}_i^B \end{cases} \qquad \begin{cases} \dot{\theta}_i &= u_{\theta,i} \\ \dot{w}_i &= u_{w,i} \\ \mathbf{f}_i^B &= k_w w_i^2 \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} \end{cases} \qquad \begin{matrix} \underline{\theta_i} \le \theta_i \le \overline{\theta_i} \\ \underline{w_i} \le w_i \le \overline{w_i} \end{matrix}$$

Platform equations — Actuator dynamics — Constraints

Control Objective and Allocator Hierarchy

▷ Design Goal: Trajectory tracking in position and attitude SE(2)



▷ High level control

- ensures *trajectory tracking* by generating a suitable "commanded virtual input" $\mathbf{u_{v,c}}$ for the allocator

▷ Allocator tasks with their **priorities**

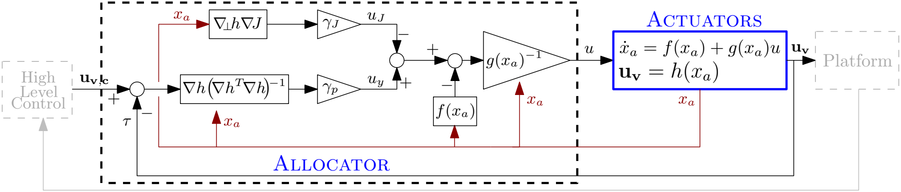
- **HIGH** ensures that the commanded virtual input is *dynamically* exerted on the plant with time constant γ_p

$$\dot{\mathbf{u}}_\mathbf{v} = \gamma_P(-\mathbf{u_v} + \mathbf{u_{v,c}}), \text{ where } \mathbf{u_v} := \left(\sum_{i=1}^n \mathbf{f}_i^B \,,\, \sum_{i=1}^n (\mathbf{\Pi r_i})^T \mathbf{f}_i^B\right)$$

- **LOW** ensures optimal allocation w.r.t a cost function $J(\mathbf{w}, \theta)$ penalizing constraints violation

Input Allocator Exploits Projection for Hierarchy

▷ Allocator Dynamics is based on combined effect of u_J and u_y



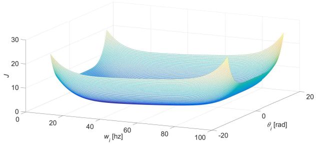
▷ Feedback linearization transforms actuators in $\dot{x}_a = u_y + u_J$, $\mathbf{u_v} = h(x_a)$

▷ u_y takes care of assigning first order dynamics

$$\dot{\mathbf{u}}_\mathbf{v} = \gamma_P(-\mathbf{u_v} + \mathbf{u_{v,c}}), \text{ where } \mathbf{u_v} := \left(\sum_{i=1}^{n} \mathbf{f}_i^B \, , \, \sum_{i=1}^{n} (\mathbf{\Pi r_i})^T \mathbf{f}_i^B \right)$$

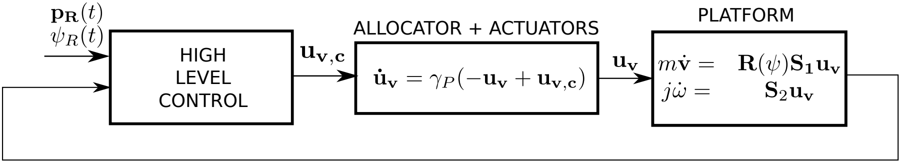
▷ u_J takes care of the cost function J via projection operator $\nabla_\perp \mathbf{h}(\mathbf{x_a})$

▷ Cost Function $J(\mathbf{w}, \theta)$ penalizes:
- approaching actuator saturation
- energy consumptions of propellers



High-level controller ensures trajectory tracking

▷ The block **Allocator + Actuators** externally appears as a first-order filter



▷ Design goal is to track a reference motion $t \mapsto \mathbf{p_R}(t), \psi_R(t)$

▷ Design task is then simplified by allocator

- Simple Feedforward+Feedback scheme ensures PD-like behavior
- The selected gains ensure desirable damping and bandwidth

GAS can be established using Theorem RED

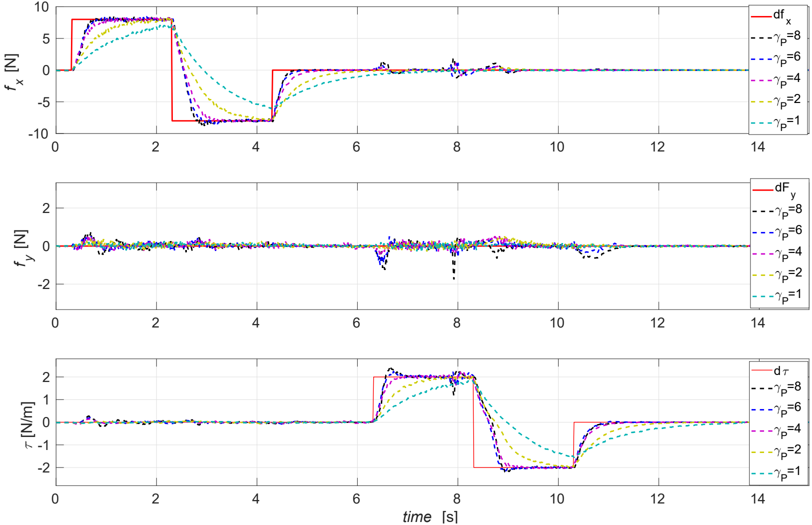
EXP $\mathbf{p}(t), \psi(t)$ converge globally and exponentially to $\mathbf{p_R}(t), \psi_R(t)$

ACT $\mathbf{u_v}(t)$, $\mathbf{u_{v,c}}(t)$ asymptotically satisfy $\dot{\mathbf{u}}_\mathbf{v} = \gamma_P(-\mathbf{u_v} + \mathbf{u_{v,c}})$

OPT if $\mathbf{p_R}(t), \psi_R(t)$ is constant, then $\mathbf{x_a}(t)$ converges to a stationary point of $J(\mathbf{x_a})$ subject to $\mathbf{h}(\mathbf{x_a}) = \mathbf{u_{v,c}}$.

Experiments show allocator-induced "external linearity"

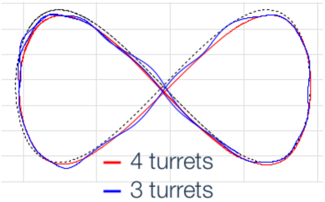
- ▷ Step refs in directions $\mathbf{u_v} = (f_x, f_y, \tau)$ confirm linear $\dot{\mathbf{u}}_\mathbf{v} = \gamma_P(-\mathbf{u_v} + \mathbf{u_{v,c}})$

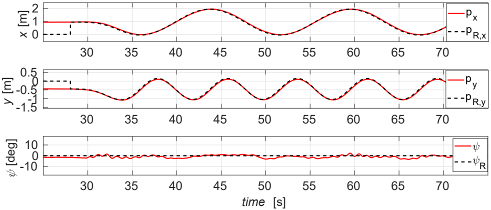


- ▷ If γ_p is too large, input saturation becomes relevant

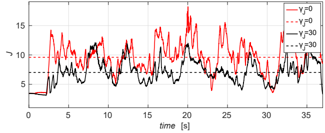
Experiments following an ∞-shaped motion

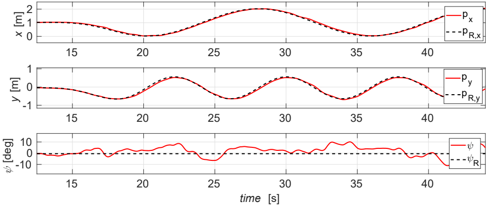
- ▷ ∞-shaped motion with $n = 3$ turrets and $n = 4$ turrets configurations
- ▷ Allocator parameters do not change between $n = 3$ and $n = 4$





$n = 4$



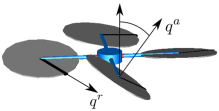


$n = 3$

- ▷ Lower precision in the case $n = 3$ as compared to $n = 4$
- ▷ Cost Function J highly improved by the allocator action

Beyond ISS cascades in hierarchical UAV control

▷ In **underactuated** UAVs, the cascaded sequence enforced by the dynamics

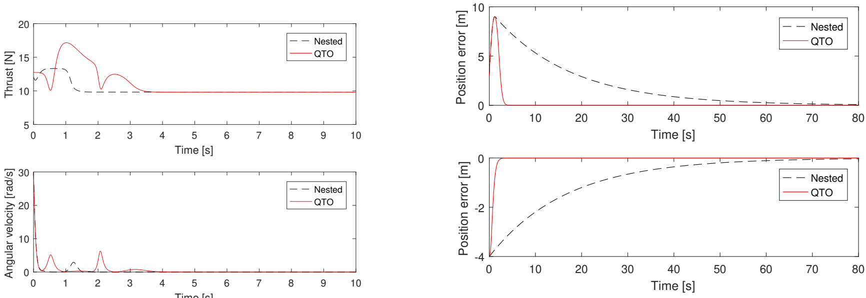


$$\begin{cases} \dot{x} &= v \\ m\dot{v} &= -mge_3 + R{\color{blue}f_c} \\ \dot{R} &= R\hat{\omega} \\ J\dot{\omega} &= (J\omega)^\times \omega + {\color{blue}\tau_c} \end{cases} \Rightarrow \begin{cases} \dot{x} &= v \\ m\dot{v} &= {\color{green}\gamma(x,v)} + {\color{red}\Delta_\gamma(R,x,v,}{\color{blue}f_c}{\color{red})} \\ \dot{R} &= R\hat{\omega} \\ J\dot{\omega} &= (J\omega)^\times \omega + {\color{blue}\tau_c} \end{cases}$$

▷ Cascade interconnection shows undesirable position feedback perturbation

$$\boxed{\begin{array}{c}\text{Attitude dynamics controlled} \\ \text{to guarantee } {\color{red}\Delta_\gamma \to 0}\end{array}} \xrightarrow{\color{red}\Delta_\gamma} \boxed{\begin{array}{c}\text{Perturbed Position} \\ \text{Stabilization}\end{array}}$$

▷ New iISS quasi-time-optimal stabilizer outperforms historical ISS approach



Acknowledgements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Main references related to the presented works

Key works for this presentation:

▷ Reduction Theorems for Hybrid Dynamical Systems Maggiore et al. [2019]

▷ Applications of static allocation:
- Satellite attitude stabilization Trégouët et al. [2015]
- ROSPO experimental platform Nainer et al. [2017]

▷ Hierarchical paradigms for UAV control Invernizzi et al. [2018, 2019]

Additional related references

▷ Dynamic allocation paradigms for linear systems Zaccarian [2009], Cocetti et al. [2018], Galeani et al. [2015]

▷ Applications of dynamic allocation:
- Internal wrenches control in interacting robots Zambelli Bais et al. [2015]
- Tokamak plasma shape control Boncagni et al. [2012], De Tommasi et al. [2011, 2012]
- Hybrid Electric Vehicle control Cordiner et al. [2014]
- Hydrodynamic dynamometer application Passenbrunner et al. [2016]

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