Set-point stabilization of SISO linear systems using First Order Reset Elements

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***Abstract*— In this paper we further develop on a novel representation of First Order Reset Elements (FORE) control systems for SISO plants. We study here the problem of guaranteeing asymptotic tracking of constant references for general plants, which may or may not contain an integrator (namely, an internal model of the constant reference signal). We propose a generalization of the FORE which allows to guarantee asymptotic tracking of constant references when the plant parameters are perfectly known. Robustness of the scheme follows from the $\mathcal{L}_\infty$ stability properties of the FORE control schemes. The proposed approach is successfully illustrated on a simulation example.**

I. Introduction

First Order Reset Elements (FOREs) are a special class of reset linear systems used in stabilization of single input single output (SISO) plants. Perhaps the breakthrough idea on reset linear systems was that of Clegg who in 1958 [5] proposed a reset implementation of the integrator (the so-called "Clegg integrator") already discussing its advantages using the mathematical tools available at that time. Reset controllers were then further investigated in the 1970s (see, e.g., [8], [7]) and recently they have been shown to overcome intrinsic limitations of linear control loops [6], [1].

Reset controllers correspond to control systems involving an element (a SISO element in the typical case) whose state is reset to zero whenever the input and the state itself satisfy a suitable algebraic condition. When the Clegg integrator was first proposed (in 1958 [5]), its behavior was not specified in terms of mathematical equations but only in terms of some modifications to the well-known analog circuit implementing a linear integrator (by way of an operational amplifier). Subsequently, the Clegg integrator idea was generalized to the so-called First Order Reset Element (FORE) [7] which corresponds to generalizing the Clegg integrator to a linear reset system with a real pole (that pole is at the origin for the Clegg integrator). Reset control systems involving FOREs and Clegg integrators have been formally shown to overcome intrinsic limitations of linear control systems (see, [6], [1]) and have been recently the subject of renewed interest by the control community (see, e.g., [3], [2] and references therein).

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It was recently pointed out in [9], [10] that when carefully analyzing the modified circuit proposed in 1958 by Clegg, the arising dynamical equations reveal the fact that the closed-loop system is only allowed to flow in a strict subset of the overall state space. Ruling out a non trivial subset of the state space (where the state can never flow) allowed to show guaranteed asymptotic stability and performance properties in cases where the underlying linear dynamics without resets was even exponentially unstable. As an example, the typical case of a Clegg integrator connected to an integrator was shown in [10] to be globally exponentially stable and to guarantee finite $\mathcal{L}_2$ gain from d to y.

In this short note, we further develop on the novel representation of SISO reset control systems proposed in [9], [10] and study the problem of set-point stabilization. We show here that FOREs can be well employed for set-point stabilization also when an integrator is not present in the forward control path and show how the FORE should be implemented to guarantee asymptotic tracking of constant references. In Section II we introduce the model of SISO reset systems with FOREs and given the main result of the paper. In Section III we provide a simulation example and discuss the effectiveness of the proposed control strategy.

II. Set-point regulation for SISO reset systems with FOREs

Consider a strictly proper SISO linear plant whose dynamics is described by

$$\mathcal{P}\left\{\begin{array}{rcl} \dot{x}_p & = & A_p x_p + B_{pu} u + B_{pd} d, \\ y & = & C_p x_p, \end{array}\right. \tag{1}$$

where u is the control input, d is a disturbance input and y is the measured plant output (A_p, B_{pu}, B_{pd} and C_p are matrices of appropriate dimensions).

For the plant (1), assume that a control system is designed for stabilization purposes only, with $r = 0$, where the FORE element dynamics and the interconnection equations correspond to:

$$\text{FORE}\left\{\begin{array}{ll} \dot{x}_r = \lambda_r x_r + e, & \text{if } e x_r \geq 0 \\ x_r^+ = 0, & \text{if } e x_r \leq 0, \end{array}\right. \tag{2}$$

$$\text{Interconnection}\left\{\begin{array}{l} u = k x_r, \\ e = -y \end{array}\right. \tag{3}$$

where $k > 0$ denotes the loop gain and $\lambda_r \in \mathbb{R}$ denotes the pole of the FORE.

The closed-loop (1), (2), (3) has been extensively studied in the literature, already from the time of the introduction of the Clegg integrator [5], which corresponds to selecting $\lambda_r = 0$ in (2). Our recent developments of [9], [10] showed a new understanding of these interconnections by way of a revised model of the Clegg integrator and the corresponding generalization to a generic FORE element.

Remark 1: (Temporal regularization and Zeno solutions) It has been shown in [10] that reset linear systems are prone to the presence of Zeno solutions (namely solutions that jump infinitely many times in a compact time interval). To avoid this phenomenon, we used in [9], [10] the notion of temporal regularization already used, e.g., in [4]. To keep the notation simple, we avoid using temporal regularization in this paper, although it remains evident that any implementation of the reset control systems here discussed would require that modification or alternative ways to rule out Zeno solutions. ○

Proposition 1: (FORE set point stabilizer) Suppose that the transfer function of the plant (1) from u to y does not have zeros at the origin. Also suppose that $k \neq 0$ and the origin of the reset control system (1), (2), (3) with temporal regularization is asymptotically stable. Then, the following quantity is well defined:

$$F = \begin{cases} \dfrac{1}{C_p A_p^{-1} B_{pu} k}, & \text{if } A_p \text{ is invertible,} \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Moreover, for any constant reference $r \in \mathbb{R}$, the following FORE implementation

$$\text{FORE} \begin{cases} \dot{x}_r = \lambda_r x_r + e, & \text{if } (r-y)(x_r + Fr) \geq 0 \\ x_r^+ = -Fr, & \text{if } (r-y)(x_r + Fr) \leq 0, \end{cases} \quad (5)$$

$$\begin{matrix}\text{Inter} \\ \text{conn.}\end{matrix} \begin{cases} u = k x_r, \\ e = (1+\lambda_r F) r - y \end{cases} \quad (6)$$

guarantees asymptotic stability of the equilibrium point $x^* = (x_p^*, x_r^*)$, where $y^* = C_p x_p^* = r$.

Proof: Since $C_p A_p^{-1} B_{pu} k$ is the static loop gain of the control system, then by the assumption on the zeros of the plant it follows that $C_p A_p^{-1} B_{pu} k > 0$ (note that the absence of zeros at the origin is a necessary assumption to be able to achieve set point regulation from the input u). Therefore, F in (4) is well defined.

Consider now the dynamics (1), (5), (6) and for any $r \in \mathbb{R}$ perform the change of coordinates $(x_p, x_r) \to (\tilde{x}_p, \tilde{x}_r) := (x_p - x_p^*, x_r - x_r^*)$, where x_p^* is a vector satisfying the following (always solvable) set of equations: $A_p x_p^* = B_{pu} k x_r^*$, $C_p x_p^* = r$ (note that if A_p is invertible, then $x_p^* = -A_p^{-1} B_{pu} k x_r^*$, otherwise $x_p \in \ker(A_p)$ such that $C_p x_p^* = r$). The the arising dynamics coincide with the dynamics (1), (2), (3), with (x_p, x_r) replaced by $(\tilde{x}_p, \tilde{x}_r)$. Therefore, by assumption, the origin of the reset closed-loop in the $(\tilde{x}_p, \tilde{x}_r)$ coordinates is asymptotically stable and the equilibrium point $x^* = (x_p^*, x_r^*)$ for (1), (5), (6) is asymptotically stable too. ■

III. Simulation Example

The construction proposed in Proposition 1 generalizes the FORE control system construction to the set point regulation problem. This generalization is trivial when the plant (1) has an integrator in it (such as in the classical case of a FORE controlling an integrator widely studied in the literature). As a matter of fact, in that case $y^* = r$ and $x_r^* = 0$. Therefore, equations (5), (6) reduce to a very intuitive implementation which corresponds to keeping the FORE equations in (2) unchanged and only exchanging the last interconnection equation in (3) to $e = r - y$. However, when the plant does not satisfy $\det(A_p) \neq 0$, this intuitive generalization is no longer effective and can lead to very undesirable closed-loop behavior, including instability. On the other hand, the implementation in (5), (6) always guarantees asymptotic stability of the set point x^*.

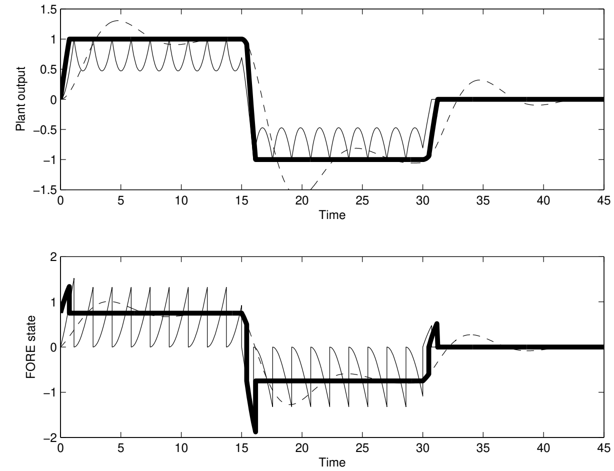

Fig. 1. Example discussed in Section III. Linear response (dashed), incorrect FORE response (solid) and correct FORE response (bold).

The set-point stabilization properties of the FORE implementation (5), (6) is illustrated by the simulations of Figure 1, which report the closed-loop responses to a doublet reference input when the following parameters are used:

$$A_p = -1.5, \quad B_{pu} = 1, \quad C_p = 1, \quad k = 2, \quad \lambda_r = 1,$$

so that $F = -0.75$.

In Figure 1, the dashed line represents the response of the system without resets, which is exponentially stable for these parameters (with a suitable reference scaling to guarantee the desired steady-state value), the solid line reports the response of the FORE control system incorrectly implemented using (2), (3) (replacing the last interconnection equation by $e = r - y$) and the bold line reports the response of the set-point FORE control system correctly implemented using (5), (6).

Remark 2: (On robustness of the set-point controller implementation) When A_p is nonsingular, for the implementation of the FORE set point stabilizer in Proposition 1, it is necessary to exactly know the static gain of the plant from the input u to the output y (this corresponds to the quantity $C_p A_p^{-1} B_{pu}$). It is important to emphasize the effect of uncertainties on this quantity on the closed-loop. This can be done by writing the closed-loop system by taking the change of coordinates $(x_p, x_r) \to (\tilde{x}_p, \tilde{x}_r) := (x_p - x_p^*, x_r + Fr)$, where x_p^* is the desired equilibrium that satisfies $C x_p^* = r$). Then the closed loop in the transformed coordinates coincides with the dynamics (1), (2), (3) with an extra constant disturbance term acting on the plant state equation and corresponding to $d^* := A_p x_p^* - B_{pu} k x_r^*$. Hence, as long as the original reset closed-loop satisfies a suitable $\mathcal{L}_\infty$ property from a disturbance acting at the plant input to the output y (see, e.g., [9] for some directions on Lyapunov based conditions that guarantee this property), the tracking error will be small when the static gain estimation error is small. $\circ$

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