A cascade interpretation of the Smith predictor and the arising enhanced scheme

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Abstract— **In this paper we propose a novel interpretation of the well known Smith predictor scheme for dead-time plants. By a suitable change of coordinates, it is revealed that the system possesses a useful cascaded structure wherein extra stabilizing signals can be successfully injected to make the approach applicable to non Hurwitz plants. We emphasize here the potential of this type of "enhanced" Smith predictor scheme and suggest some preliminary ways to select the extra stabilizing signals, in addition to clarifying what are the key properties that these signals need to satisfy to induce desirable closed-loop performance.**

I. INTRODUCTION

Dead-time systems represent the simplest class of infinite dimensional systems that often arises in control engineering practice. Indeed, these systems are prevalent in classical process control applications where measurements or actuating signals can be only processed with a delay (i.e. transport delay) or where the chemical reactions in the system naturally occur with a considerable time delay (see [20]). Low order dead-time systems can also be used in model reduction to approximate the behavior of high dimensional delay-free systems (see, e.g., [18]).

The first non-trivial solution to stabilization of dead-time processes was given by O.J.Smith in [21] who presented a controller-predictor structure in which the controller is designed for the delay-free plant. The arising closed-loop performance is then enforced on the actual plant with time-delay via the action of a peculiar filter (the Smith predictor) which has a model-based structure. In general, the classical Smith predictor has some important limitations, the main one being that it can only be applied to open-loop asymptotically stable plants. Already in the 1980s generalizations of the classical Smith predictor scheme were proposed (see, e.g., [23]). Useful extensions of the original Smith predictor scheme were given in [1] to extend its applicability to plants with integrating action, as well as in [6], [13], [12], [9] were these results were also extended to open-loop unstable plants (a nice summary and overview of these advances is given in [20]).

Despite the large literature on stabilization of time-delay systems (see, e.g., [19] for an extensive survey of a research area where Linear Matrix Inequalities are used to design stabilizing compensators), in recent years the Smith predictor has been a preferred technique for the design of high performance control schemes for dead-time processes (see, e.g., [17], [3], [11], [2]). Motivated by the increasing demand for high-performance solutions that overcome the limitations of the original scheme of [21], several constructive solutions have been proposed in recent years: in [10], [4], [22], [24], [5] schemes for automatic tuning of the Smith predictor are proposed. In [14], [16] control design approaches for dead time systems are proposed which rely on analysis tools that address (and characterize) the time delay in different ways. Moreover, in [8], [25], and references therein, important characterizations of intrinsic properties of control systems for dead time plants are given, mainly aimed at bringing classical linear design and analysis tools to bear within the dead time setting.

In this paper we revisit the classical Smith predictor schemes and reveal a special cascade structure in the scheme by way of a suitable coordinate transformation. This cascaded structure allows to focus on the possibility of enhancing the classical scheme with extra stabilizing signals so that the standard approach (known to be only applicable to Hurwitz plants) can easily extend to more complicated settings. We do not pursue fully constructive tools for the design of these extra stabilizing signals here but show that they can be designed based on the solution of a stabilization problem for a dead-time plant. As an example, the design approach proposed in [15] actually falls within this framework wherein the extra stabilizing loop that we characterize here is selected using a static feedback from an observed state. The paper is organized as follows. In Section II we give the problem definition. In Section III we discuss the problem solution by first discussing the classical Smith predictor and then discussing the enhanced Smith predictor. Finally, in Section IV we illustrate the potential of the scheme on a simple simulation example.

II. PROBLEM STATEMENT

Consider the following MIMO linear plant with delays at its control inputs and measured outputs

$$\mathcal{P}\begin{cases} \dot{x}(t) &= Ax(t) + B_u u(t-\tau_I) + B_d d(t) + \psi_x(t) \\ y(t) &= C_y x(t-\tau_O) + D_{uy} u(t-\tau_I-\tau_O) \\ & \quad +D_{dy} d(t-\tau_O) + \psi_y(t-\tau_O) \\ z(t) &= C_z x(t) + D_{uz} u(t-\tau_I) + D_{dz} d(t) + \psi_z(t), \end{cases} \qquad (1)$$

where τ_I is a uniform delay at the plant control input, τ_O is a uniform delay at the plant output. Moreover, y represents

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the measured output, z represents the performance output (without loss of generality we can assume that this output is not delayed) and d represents a disturbance input. The three extra signals ψ_x, ψ_y, ψ_z can be stacked in a single vector Ψ representing the output of the following linear system

$$\Psi(t) = \begin{bmatrix} \psi_x(t) \\ \psi_y(t) \\ \psi_z(t) \end{bmatrix} = \Delta(s) \begin{bmatrix} x(t) \\ u(t) \\ d(t) \end{bmatrix} \qquad (2)$$

which may be infinite dimensional (it may have internal delays) and represents unmodeled dynamics and/or parameter uncertainties in the model (1). We will need the following assumption to hold for the system (2).

Assumption 1: The pair (C_z, A) is detectable. [1] The linear system (2) is an exponentially stable linear retarded delay-differential system with finite $\mathcal{L}_2$ gain equal to γ_Δ.

Remark 1: Note that assuming exponential stability of (2) is not too restrictive when addressing the robustness of the control scheme for the plant (1). A sufficient condition for (2) to be exponentially stable is indeed that the number of non asymptotically stable poles of A corresponds to that of the real plant. All other uncertainties are captured by (2), namely unmodeled dynamics, uncertainties in the input and output delays, uncertainties on the entries of all the matrices in (1) and uncertainties in the position of the poles of A (regardless of their stability). $\circ$

Consider the plant (1) and assume that a (nonlinear, in general) controller has been designed to guarantee desirable stability and performance specifications on its *undelayed closed-loop interconnection* with the nominal undelayed plant, namely

$$\mathcal{P}_0 \begin{cases} \dot{x}_u(t) &= Ax_u(t) + B_u u_u(t) + v_1(t) \\ y_u(t) &= C_y x_u(t) + D_{uy} u_u(t) + v_2(t) \\ z_u(t) &= C_z x_u(t) + D_{uz} u_u(t), \end{cases} \qquad (3a)$$

$$\mathcal{C} \begin{cases} \dot{x}_c(t) &= f(x_c(t), u_c(t), r(t)) \\ u(t) &= g(x_c(t), u_c(t), r(t)), \end{cases} \qquad (3b)$$

$$u_u(t) = u(t), \qquad u_c(t) = y_u(t), \qquad (3c)$$

where v_1 and v_2 are suitable signals to be specified later and $f(\cdot,\cdot,\cdot)$ and $g(\cdot,\cdot,\cdot)$ are Lipschitz functions so that uniqueness of solutions is guaranteed. We will assume that the following assumption holds for the undelayed nominal closed-loop system. (Note that this assumption is trivially satisfied when the controller (3b) is a linear stabilizing controller for (3a).)

Assumption 2: The *undelayed nominal closed-loop system* (3a), (3b), (3c) is globally asymptotically stable, finite gain $\mathcal{L}_2$ stable from r to x_u, u_u and finite gain incrementally $\mathcal{L}_2$ stable from (v_1, v_2) to x_u, u_u, namely there exists a positive number γ_U such that for any selection of the reference signal $r(\cdot)$ in $\mathcal{L}_2$ (assuming zero initial conditions, for simplicity),

1) if $x_{u0}(\cdot), u_{u0}(\cdot)$ is the response of the system with $v_1(\cdot) \equiv 0$ and $v_2(\cdot) \equiv 0$, then
$$\left\| \begin{matrix} x_{u0}(\cdot) \\ u_{u0}(\cdot) \end{matrix} \right\|_2 \leq \gamma_U \|r(\cdot)\|_2; \qquad (4)$$
2) if $x_u(\cdot), u_u(\cdot)$ is the response of the system to any selection of $\mathcal{L}_2$ signals $v_1(\cdot)$ and $v_2(\cdot)$, then [2]
$$\left\| \begin{matrix} (x_u - x_{u0})(\cdot) \\ (u_u - u_{u0})(\cdot) \end{matrix} \right\|_2 \leq \gamma_U \left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \end{matrix} \right\|_2 . \qquad (5)$$

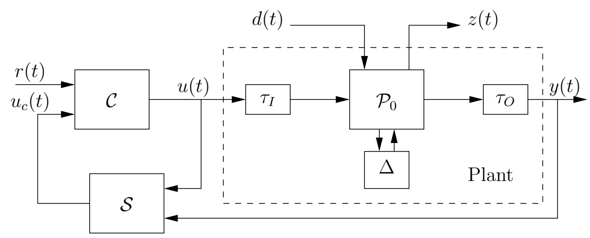

Fig. 1. The compensated closed-loop system.

The goal of this paper is to recover as much as possible the desirable (ideal) output response $z_u(t)$ generated by the undelayed closed-loop system in the case where the nonlinear controller (3b) is interconnected in feedback with the perturbed and delayed plant (1). To this aim, we will consider the augmentation of the delayed control scheme with a suitable model-based dynamic filter $\mathcal{S}$, as represented in Figure 1, which is driven by the plant input and output signals $u(t)$ and $y(t)$ and provides a new input $u_c(t)$ to the controller. The filter $\mathcal{S}$ should be suitably designed with the goal of enforcing the following property on the *compensated closed-loop* (1), (2), (3b).

Definition 1: The delay compensation system $\mathcal{S}$ is said to solve the *nominal prediction problem* if there exists a positive constant τ such that if $d(\cdot) \equiv 0$ and $\Psi \equiv 0$, for any selection of the reference signal $r(\cdot)$, and zero initial conditions, [3] the output responses of the undelayed and of the compensated closed-loops satisfy $z(t) = z_u(t - \tau)$ for all $t \geq \tau$.

The delay compensation system $\mathcal{S}$ is said to solve the *robust prediction problem* if

1) it solves the nominal prediction problem
2) for a sufficiently small gain $\gamma_\Delta > 0$ of the unmodeled dynamics (2) and under Assumption 1, there exists $\gamma > 0$ such that the performance output responses of the undelayed and of the compensated closed-loops satisfy
$$\|z(\cdot) - z_u(\cdot - \tau)\|_2 \leq \gamma(\|d(\cdot)\|_2 + \gamma_\Delta \|r(\cdot)\|_2) . \qquad (6)$$

$\circ$

Remark 2: Note that, in general, to even guarantee the solvability of the only nominal prediction problem, it is necessary that the input delay τ_I is uniform, namely that all the input channels of the plant (1) are delayed of the same quantity. [4] Indeed, consider an undelayed plant $\mathcal{P}_0$

[1] This assumption is only needed to prove the necessity of some of the results in the paper. The sufficiency statements would still hold in the case where (C_z, A) is not detectable.

[2] Different gains could be used in (4) and (5) but we use the same gain here to simplify the notation.

[3] The initial conditions have been omitted to keep the discussion simple.

[4] It is emphasized that small uncertainties in these delays could be incorporated in the perturbation (2).

whose input matrix B_u has full column rank and with non-uniform delays at the input channels. Given an open-loop input signal $u_u(t)$, $t \geq 0$, inducing the performance output response $z_u(t)$, $t \geq 0$, as long as the plant is observable from z and the input matrix is full column rank (namely the inputs are not redundant), there doesn't exist an open-loop signal $u(t)$, $t \geq 0$, for the delayed plant (1) (with $\Psi = 0$) inducing the response $z(t) = z_u(t-\tau)$ for all $\tau < \tau_{Imax}$, where τ_{Imax} is the largest delay at the plant inputs. Moreover, introducing artificial input delays at the input channels to make the overall vector delay uniform (and equal to τ_{Imax}), it is readily seen that the output response $z(t) = z_u(t - \tau_{Imax})$ can be easily obtained by the same input $u(t) = u_u(t)$, therefore the uniform input delay assumption is necessary (and sufficient), in general, for the solvability of the nominal prediction problem and will be used throughout this paper. Note however that the output delay might be taken to be non-uniform. We choose it as uniform here for simplicity, but the special cascade structure pointed out here would also hold for non-uniform output delays. Nevertheless, non-uniform output delays make the task of designing v_1 and v_2 in the next section, quite involved. ◦

III. Problem solution

A. The classical Smith predictor

The so-called "Smith predictor" [21], is a well known solution to the nominal prediction problem specified in the previous section in Definition 1. This scheme was originally formulated for SISO systems but extends in a straightforward way to the case where the delayed plant has uniform input delays in the MIMO case. We revisit in this section the well-known scheme to introduce a new interpretation in terms of the cascade of two relevant subsystems. This interpretation is the baseline for the extensions of the next section. The Smith predictor solution corresponds to selecting the dynamics of the filter $\mathcal{S}$ as a linear time-delay system with the following transfer function:

$$\mathcal{S}\left\{\begin{array}{rcl} \dot{x}_s(t) &=& Ax_s(t) + B_u u(t) \\ y_s(t) &=& C_y x_s(t) + D_{uy} u(t) \\ u_c(t) &=& y_s(t) + y(t) - y_s(t - \tau_I - \tau_O). \end{array}\right. \tag{7}$$

The block diagram corresponding to equation (7) is represented in Figure 2.

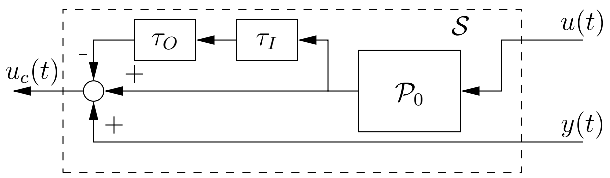

Fig. 2. The classical Smith predictor.

In the following theorem we formalize the well-known result that the classical Smith predictor is a satisfactory solution to the prediction problem as long as the plant is asymptotically stable.

Theorem 1: Given any linear delayed plant (1) and any nonlinear controller (3b), if Assumptions 1 and 2 hold,

1) the Smith predictor (7) (with $x_s(0) = 0$) always solves the nominal prediction problem of Definition 1;
2) the Smith predictor (7) solves the robust prediction problem of Definition 1 if and only if the plant is asymptotically stable.

Proof: Consider the compensated closed-loop system (1), (2), (3b), (7), perform the change of coordinates $(\tilde{x}(t), x_s(t), x_c(t)) = (x(t) - x_s(t - \tau_I), x_s(t), x_c(t))$ and define $\tilde{y}(t) = y(t) - y_s(t - \tau_I - \tau_O)$. Then, the overall dynamics can be written in the following cascaded form:

$$\left\{\begin{array}{rcl} \dot{\tilde{x}}(t) &=& A\tilde{x}(t) + B_d d(t) + \psi_x(t) \\ \tilde{y}(t) &=& C_y\tilde{x}(t - \tau_O) + D_{dy} d(t - \tau_O) + \psi_y(t - \tau_O) \end{array}\right. \tag{8a}$$

$$\left\{\begin{array}{rcl} \dot{x}_s(t) &=& Ax_s(t) + B_u u(t) \\ u_c(t) &=& C_y x_s(t) + D_{uy} u(t) + \tilde{y}(t), \end{array}\right. \tag{8b}$$

$$\left\{\begin{array}{rcl} \dot{x}_c(t) &=& f(x_c(t), u_c(t), r(t)) \\ u(t) &=& g(x_c(t), u_c(t), r(t)), \end{array}\right. \tag{8c}$$

where the first subsystem (8a) drives the second subsystem (8b), (8c) through the signal $\tilde{y}(t)$. Moreover, the performance output equation can be written in the new coordinates as

$$z(t) = \tilde{z}(t) + z_s(t - \tau_I), \tag{9}$$

where

$$\tilde{z}(t) = C_z\tilde{x}(t) + D_{dz} d(t) + \psi_z(t) \tag{10}$$

$$z_s(t) = C_z x_s(t) + D_{uz} u(t), \tag{11}$$

are two additional outputs of the first subsystem (8a) and of the second subsystem (8b), (8c), respectively.

Proof of item 1. In this case, since Ψ and d are identically zero, then $x(0) = 0$ implies $x(\tau_I) = 0$, therefore $\tilde{x}(\tau_I) = x(\tau_I) - x_s(0) = 0$ and by uniqueness of solutions $\tilde{x}(\cdot) \equiv 0$ and $\tilde{y}(\cdot) \equiv 0$. Moreover, by (10), also $\tilde{z}(\cdot) \equiv 0$.

Since $\tilde{y}(\cdot) \equiv 0$, the closed-loop (8b), (8c) coincides with the undelayed closed-loop and by (11), $z_s(t) = z_u(t)$ for all times. Finally, by (9), since $\tilde{z}(\cdot) \equiv 0$, then $z(t) = z_s(t - \tau_I) = z_u(t - \tau_I)$ for all times, thus proving item 1.

Proof of item 2. Based on the result of the previous item, we don't need to address the nominal property which is always satisfied. We will then only focus on robustness. To show the necessity assume that A is not Hurwitz and select $d(\cdot) \equiv 0$ and $\Psi \equiv 0$. Then from (1) and (7) the Smith predictor and plant state equations correspond to

$$\begin{array}{rcl} \dot{x}(t) &=& Ax(t) + B_u u(t - \tau_I) \\ \dot{x}_s(t) &=& Ax_s(t) + B_u u(t), \end{array}$$

where A is non Hurwitz. It is then apparent that the states x and x_s are not stabilizable through the input u (this property is invariant of the input delay), therefore by the detectability condition in Assumption 1, the output $z(\cdot)$ will grow unbounded for arbitrarily small initial conditions and since $z_u(\cdot)$ is bounded by Assumption 2, then the necessity of the item follows.

To show the sufficiency, first observe that the second subsystem (8b), (8c) coincides with the undelayed closed-loop and that by the incremental stability property at item 2 of Assumption 2 (evaluated with $v_1(\cdot) \equiv 0$ and $v_2(\cdot) \equiv \tilde{y}(\cdot)$), $\|z_u - z_s(\cdot)\| \leq \gamma^* \|\tilde{y}(\cdot)\|_2$, where $\gamma^* = \gamma_U(\|C_z\| + \|D_{uz}\|)$. Therefore, by (9),

$$\begin{array}{rcl} \|z(\cdot) - z_u(\cdot - \tau_I)\|_2 & \leq & \|z_s(\cdot - \tau_I) - z_u(\cdot - \tau_I)\|_2 \\ & & + \|z(\cdot) - z_s(\cdot - \tau_I)\|_2 \\ & \leq & \gamma^* \|\tilde{y}(\cdot)\|_2 + \|\tilde{z}(\cdot)\|_2 \end{array} \tag{12}$$

Since A is Hurwitz by assumption, then the subsystem (8a) is exponentially stable and, by linearity, finite-gain input-output stable from $(d, \psi_x, \psi_y, \psi_x)$ to $\tilde{x}, \tilde{y}, \tilde{z}$. Denote by $\tilde{\gamma}$ the $\mathcal{L}_2$ gain of this system. Then,

$$\left\| \begin{array}{c} \tilde{x}(\cdot) \\ \tilde{y}(\cdot) \\ \tilde{z}(\cdot) \end{array} \right\|_2 \leq \tilde{\gamma} \left\| \begin{array}{c} d(\cdot) \\ \psi_x(\cdot) \\ \psi_y(\cdot) \\ \psi_z(\cdot) \end{array} \right\|_2 \tag{13}$$

Since $x(t) = \tilde{x}(t) - x_s(t - \tau_I)$, then, by Assumption 2, the following holds:

$$\left\| \begin{array}{c} \psi_x(\cdot) \\ \psi_y(\cdot) \\ \psi_z(\cdot) \end{array} \right\|_2 \leq \gamma_\Delta \left\| \begin{array}{c} \tilde{x}(\cdot) - x_s(\cdot - \tau_I) \\ u(\cdot) \\ d(\cdot) \end{array} \right\|_2 \tag{14}$$

Therefore, applying the small-gain theorem to (13) and (14) we get

$$\left\| \begin{array}{c} \tilde{y}(\cdot) \\ \tilde{z}(\cdot) \end{array} \right\|_2 \leq \tilde{\gamma} \frac{1+\gamma_\Delta}{1-\gamma_\Delta} \left\| d(\cdot) \right\|_2 + \gamma_\Delta \frac{\tilde{\gamma}}{1-\gamma_\Delta} \left\| \begin{array}{c} x_s(\cdot - \tau_I) \\ u(\cdot) \end{array} \right\|_2 .$$

Moreover, by both items of Assumption 2 (evaluated with $v_1(\cdot) \equiv 0$ and $v_2(\cdot) \equiv \tilde{y}(\cdot)$), we can write

$$\left\| \begin{array}{c} x_s(\cdot - \tau_I) \\ u(\cdot) \end{array} \right\|_2 \leq 2 \left\| \begin{array}{c} x_s(\cdot) \\ u(\cdot) \end{array} \right\|_2 \leq 2\gamma_U \left(\left\| \tilde{y}(\cdot) \right\|_2 + \left\| r(\cdot) \right\|_2 \right),$$

which, applying once again the small-gain theorem, can be combined with the previous inequality to yield

$$\left\| \begin{array}{c} \tilde{y}(\cdot) \\ \tilde{z}(\cdot) \end{array} \right\|_2 \leq \gamma_d \left\| d(\cdot) \right\|_2 + \gamma_\Delta \gamma_r \left\| r(\cdot) \right\|_2 ,$$

where

$$\begin{array}{rcl} \gamma_d & = & \left(1 - \frac{2\gamma_\Delta \tilde{\gamma} \gamma_U}{1-\gamma_\Delta}\right)^{-1} \frac{1+\gamma_\Delta}{1-\gamma_\Delta} \tilde{\gamma} \\ \gamma_r & = & \left(1 - \frac{2\gamma_\Delta \tilde{\gamma} \gamma_U}{1-\gamma_\Delta}\right)^{-1} \frac{2\gamma_\Delta \tilde{\gamma} \gamma_U}{1-\gamma_\Delta}. \end{array} \tag{15}$$

Finally, by equation (12) we get

$$\begin{array}{rcl} \left\| z(\cdot) - z_u(\cdot - \tau_I) \right\|_2 & \leq & (1+\gamma^*)\gamma_d \left\| d(\cdot) \right\|_2 \\ & & + \gamma_\Delta (1+\gamma^*) \gamma_r \left\| r(\cdot) \right\|_2 \end{array}$$

which implies equation (6) with $\gamma = (1+\gamma^*) \max(\gamma_d, \gamma_r)$. ■

B. The enhanced Smith predictor

In this section we will propose a generalization of the classical Smith predictor recalled in the previous section which is aimed at addressing the lack of robustness established in Theorem 1 (in particular, see item 2). The general class of systems that we will consider is represented in Figure 3 and corresponds to the following equation

$$\mathcal{S} \left\{ \begin{array}{rcl} \dot{x}_s(t) & = & A x_s(t) + B_u u(t) + v_1(t) \\ u_c(t) & = & C_y x_s(t) + D_{uy} u(t) + v_2(t) \\ \tilde{y}(t) & = & y(t) - C_y x_s(t - \tau_I - \tau_O) \\ & & \quad - D_{uy} u(t - \tau_I - \tau_O), \end{array} \right. \tag{16}$$

where the two signals $v_1(t)$ and $v_2(t)$ represent the proposed "enhancement" and are feedback signals from the prediction error $\tilde{y}$ (so that, among other things, $\tilde{y}(\cdot) \equiv 0$ implies $v_1(\cdot) \equiv 0$ and $v_2(\cdot) \equiv 0$). Note that the classical Smith predictor (7) corresponds to a specific selection of these signals, namely $v_1(t) = 0$ and $v_2(t) = \tilde{y}(t)$.

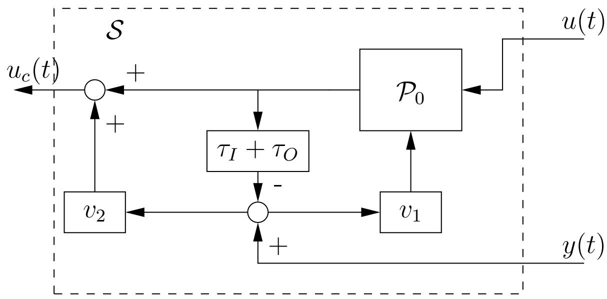

Fig. 3. The enhanced Smith predictor.

The enhanced Smith predictor (16) represents a useful generalization of the classical Smith predictor structure of Figure 7 because as long as v_1 and v_2 are zero when $\tilde{y}$ is zero, the nominal prediction property of Definition 1 will always be satisfied regardless of the selection of these two signals. On the other hand, all the possible selections of v_1 and v_2 that arise from a (static or dynamic, linear or nonlinear) feedback loop from the signal $\tilde{y}$, parametrize a family of enhanced Smith predictors that, as shown in the following sections, is large enough to allow to solve the robust prediction problem of Definition 1 for non exponentially stable plants (thus reaching beyond the potentials of the classical Smith predictor).

The selection of $v_1(\cdot)$ and $v_2(\cdot)$ within the enhanced Smith predictor (16) can be carried out in several different ways with the ultimate goal of robustifying the (pseudo)-cascaded structure exploited in the proof of Theorem 1. In many cases of practical interest, a very natural selection is

$$v_1 = K_s \tilde{y}(t), \quad v_2 = \tilde{y}(t). \tag{17}$$

where K_s can often be tuned experimentally (or by simulation) to guarantee desirable performance, as it is done in our case study. In general for the minimum requirements on v_1 and v_2 are that they are produced by a static or dynamic filter driven by $\tilde{y}$ with Lipschitz right hand side and such that the following property is satisfied:

Property 1: The delay-differential system

$$\begin{array}{rcl} \dot{\tilde{x}}(t) & = & A\tilde{x}(t) + v_1(t) + \eta_x(t) \\ \tilde{y}(t) & = & C_y\tilde{x}(t - \tau_I - \tau_O) + \eta_y(t), \end{array} \quad (18)$$

is finite gain $\mathcal{L}_2$ stable from $(\eta_x(\cdot), \eta_y(\cdot))$ to $(\tilde{x}(\cdot), v_1(\cdot), v_2(\cdot))$.

Remark 3: (On the selection of v_1) We don't pursue here explicit selections of the signals v_1 and v_2 but in simple cases, the selection can be very straightforward. For example, when the plant is Hurwitz, a natural way is to select them as in (17), with $K_s = P^{-1}X$ and where P and X satisfy the following linear matrix inequality:

$$\begin{bmatrix} A^TP + PA + R & -XC_y & P & X & I \\ \star & -R & 0 & 0 & 0 \\ \star & \star & -\gamma_y^2 I & 0 & 0 \\ \star & \star & \star & -\gamma_x^2 I & 0 \\ \star & \star & \star & \star & -I \end{bmatrix} < 0,$$

(this result can be easily proved by a standard Lyapunov-Krasovskii approach). Alternative solutions can be derived by modifying typical LMI approaches for static state feedback for dead-time systems. Indeed, system (18) that has to be stabilized by v_1 is characterized by an interesting output feedback peculiarity, where v_1 has only output measurement but full authority over the state equation (this resembles an observer design structure). Approaches for v_1 could be for example derived from the LMI conditions in [7] or from the static state feedback methods of [14] combined with a dynamic observer (this last approach was actually used in [15]). We don't pursue here a detailed treatment of all the possible approaches for the selection of v_1 because we regard it as later work. We rather concentrate here on the key features of this extra signal and show on a simple simulation example how even very intuitive selections of v_1 can parametrize a wide variety of (more or less desirable) closed-loop behaviors. ◦

Remark 4: (On the selection of v_2) Note that when designing Smith predictor schemes for systems with step disturbances it is always convenient to select $v_2 = \tilde{y}(t)$, as in the classical Smith predictor. Since rejecting constant load disturbances is typically required in control schemes with Smith predictors, we will always make this selection. It is important to emphasize, though, that alternative selections may be desirable when constant disturbance are not present. The reason why the selection $v_2 = \tilde{y}(t)$ helps with constant disturbance is easily seen by taking the unmodeled dynamics to be zero in the cascaded structure (8) in the proof of Theorem 1. Then $\tilde{y}(\cdot)$ is generated by the following system arising from (8a):

$$\begin{array}{rcl} \dot{\tilde{x}}(t) & = & A\tilde{x}(t) + B_d d(t) \\ \tilde{y}(t) & = & C_y\tilde{x}(t - \tau_O) + D_{dy}d(t - \tau_O), \end{array}$$

which, taking $\hat{x}(t) = \tilde{x}(t - \tau_O)$ can be written as

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + B_d d(t - \tau_O) \\ \tilde{y}(t) & = & C_y\hat{x}(t) + D_{dy}d(t - \tau_O). \end{array} \quad (19)$$

Then, by linearity, equation (19) can be combined with the second subsystem (8b), (8c) to obtain an exact (delayed) copy of the undelayed closed-loop subject to the shifted disturbance $d(t - \tau_O)$. Indeed, by defining $\bar{x}(t) = x_s(t) + \hat{x}(t)$, we can replace (8b) by

$$\begin{array}{rcl} \dot{\bar{x}}(t) & = & A\bar{x}(t) + B_u u(t) + B_d d(t - \tau_O) \\ u_c(t) & = & C_y\bar{x}(t) + D_{uy}u(t) + D_{dy}d(t - \tau_O), \end{array} \quad (20)$$

and the shifted property of (20), (8c) becomes evident. Note also that when d is not constant, one can add the quantity $d(t) - d(t - \tau_O)$ and characterize the incremental version of the disturbance as an undesired perturbation. ◦

Remark 5: (On measuring the disturbance d) It is well known (see, e.g., [20]) that the classical Smith predictor can be modified to achieve improved performance in cases when the disturbance can be measured. Indeed, in that case it is especially convenient to include the disturbance also in the dynamics (16). This fact is convenient because the arising cascaded structure (used in the proof of Theorem 1) allows to fully recover (in a retarded way) the control performance achieved on those measured disturbances. In this paper we are not directly addressing this case to reduce the notational burden, but we are showing its advantage in our simulation example. ◦

Remark 6: (Controller-independence of the scheme) Note that the enhanced Smith predictor in (16) preserves a key feature of the original Smith predictor scheme, which is the fact that the additional dynamics is independent of the controller. Therefore, any controller can be used with the same compensation scheme, even after the design of v_1. As a matter of fact, the requirements on v_1 established in Property 1 only involve the plant dynamics. ◦

Theorem 2: Given any linear delayed plant (1) and any nonlinear controller (3b), if Assumptions 1 and 2 hold and $v_1(\cdot)$ and $v_2(\cdot)$ are selected to satisfy Property 1, then the enhanced Smith predictor (16) solves the (nominal and) robust prediction problem of Definition 1.

Proof: The proof is a generalization of the proof of Theorem 1, therefore it will follow the same steps (note that the nominal prediction property is not directly addressed here because it is automatically implied by the robust prediction property proved next). Consider the compensated closed-loop system (1), (2), (3b), (16), perform the change of coordinates $(\tilde{x}(t), x_s(t), x_c(t)) = (x(t) - x_s(t - \tau_I), x_s(t), x_c(t))$ and define $\tilde{y}(t) = y(t) - C_y x_s(t - \tau_I - \tau_O) - D_{uy}u(t - \tau_I - \tau_O)$. Then, the overall dynamics can be written in the following cascaded form:

$$\left\{ \begin{array}{rcl} \dot{\tilde{x}}(t) & = & A\tilde{x}(t) + B_d d(t) + \psi_x(t) + v_1(t - \tau_I) \\ \tilde{y}(t) & = & C_y\tilde{x}(t - \tau_O) + D_{dy}d(t - \tau_O) + \psi_y(t - \tau_O) \end{array} \right. \quad (21a)$$

$$\left\{ \begin{array}{rcl} \dot{x}_s(t) & = & Ax_s(t) + B_u u(t) + v_1(t) \\ u_c(t) & = & C_y x_s(t) + D_{uy}u(t) + v_2(t), \end{array} \right. \quad (21b)$$

$$\left\{ \begin{array}{rcl} \dot{x}_c(t) & = & f(x_c(t), u_c(t), r(t)) \\ u(t) & = & g(x_c(t), u_c(t), r(t)), \end{array} \right. \quad (21c)$$

where the first subsystem (21a) drives the second subsystem (21b), (21c) through the signals $v_1(t)$, $v_2(t)$. Moreover, the performance output equation can be written in the new coordinates as

$$z(t) = \tilde{z}(t) + z_s(t - \tau_I), \tag{22}$$

where

$$\begin{aligned} \tilde{z}(t) &= C_z\tilde{x}(t) + D_{dz}d(t) + \psi_z(t) \quad (23)\\ z_s(t) &= C_z x_s(t) + D_{uz}u(t), \quad (24)\end{aligned}$$

are two additional outputs of the first subsystem (21a) and of the second subsystem (21b), (21c), respectively.

First observe that the second subsystem (21b), (21c) coincides with the undelayed closed-loop and that by the incremental stability property at item 2 of Assumption 2, $\|z_u - z_s(\cdot)\| \le \gamma_U \left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \end{matrix} \right\|_2$. Therefore, by (22),

$$\begin{aligned} \|z(\cdot) - z_u(\cdot - \tau_I)\|_2 &\le \|z_s(\cdot - \tau_I) - z_u(\cdot - \tau_I)\|_2 \\ &\quad + \|z(\cdot) - z_s(\cdot - \tau_I)\|_2 \\ &\le \gamma_U \left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \end{matrix} \right\|_2 + \|\tilde{z}(\cdot)\|_2 \end{aligned} \tag{25}$$

Note that for suitable selections of η_1 and η_2 the subsystem (21a) coincides with (a time-shifted version of) system (18). Hence, the finite gain $\mathcal{L}_2$ stability stated in Property 1 and equation (23) imply:

$$\left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \\ \tilde{x}(\cdot) \\ \tilde{z}(\cdot) \end{matrix} \right\|_2 \le \tilde{\gamma} \left\| \begin{matrix} d(\cdot) \\ \psi_x(\cdot) \\ \psi_y(\cdot) \end{matrix} \right\|_2 \tag{26}$$

where $\tilde{\gamma}$ is a sufficiently large positive constant dependent on the $\mathcal{L}_2$ gain of the system and on the matrices in (23).

Since $x(t) = \tilde{x}(t) - x_s(t - \tau_I)$, then, by Assumption 2, equation (14) holds. Therefore, applying the small-gain theorem to (26) and (14) we get

$$\left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \\ \tilde{z}(\cdot) \end{matrix} \right\|_2 \le \tilde{\gamma}\frac{1+\gamma_\Delta}{1-\gamma_\Delta} \| d(\cdot) \|_2 + \gamma_\Delta \frac{\tilde{\gamma}}{1-\gamma_\Delta} \left\| \begin{matrix} x_s(\cdot - \tau_I) \\ u(\cdot) \end{matrix} \right\|_2 .$$

Moreover, by both items of Assumption 2, we can write

$$\left\| \begin{matrix} x_s(\cdot - \tau_I) \\ u(\cdot) \end{matrix} \right\|_2 \le 2 \left\| \begin{matrix} x_s(\cdot) \\ u(\cdot) \end{matrix} \right\|_2 \le 2\gamma_U \left(\left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \end{matrix} \right\|_2 + \| r(\cdot) \|_2 \right),$$

which, applying once again the small-gain theorem, can be combined with the previous inequality to yield

$$\left\| \begin{matrix} v_1(\cdot) \\ v_2(\cdot) \\ \tilde{z}(\cdot) \end{matrix} \right\|_2 \le \gamma_d \| d(\cdot) \|_2 + \gamma_\Delta \gamma_r \| r(\cdot) \|_2 ,$$

with γ_d and γ_r defined in (15). Finally, by (25) we get

$$\begin{aligned} \| z(\cdot) - z_u(\cdot - \tau_I) \|_2 &\le (1+\gamma_U)\gamma_d \| d(\cdot) \|_2 \\ &\quad + \gamma_\Delta(1+\gamma_U)\gamma_r \| r(\cdot) \|_2 \end{aligned}$$

which implies equation (6) with $\gamma = (1+\gamma_U)\max(\gamma_d, \gamma_r)$. ■

IV. SIMULATION EXAMPLE

We use a very simple example taken from [1] to illustrate the potentials of the new feedback loop given by v_1. The example consists of a SISO linear plant with transfer function $P(s) = \frac{e^{-5s}}{s}$ driven by a PID controller with constants $K_p = 0.3$, $K_i = 0.3/18$ and $K_d = 0.3/3$. The closed-loop system is driven by a unit reference at time $t = 0$ and affected by a load disturbance of size -0.1 at the plant input at time $t = 70$.

Since the plant under consideration is a scalar SISO plant, it is quite simple to select K_s in (17) to satisfy Property 1. In particular, necessarily, K_s is a positive scalar and it can be shown that selecting it in the range $(0, 0.2)$ ensures Property 1.

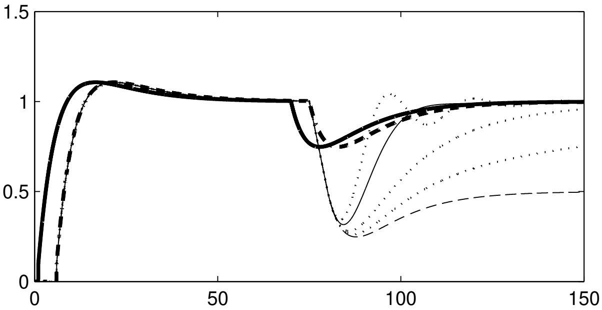

Fig. 4. Nominal responses: Undelayed (bold); classical Smith predictor (dashed); proposed scheme with $K_s = 0.1$ (thin solid); proposed scheme with alternative selections of K_s (dotted); proposed scheme with disturbance measurement (bold dashed).

Figure 4 reports on the closed-loop responses when there's not uncertainty on the plant model. The bold curve represents the undelayed performance output, namely the response of the closed-loop without delay and without Smith predictor. The dashed line represents the response of the dead-time closed-loop when using the classical Smith predictor. This last response will exhibit a constant steady-state error due to the fact that the stable plant mode is unobservable by the controller (due to the presence of the Smith predictor dynamics). The thin solid response shows the behavior of the proposed scheme when using $K_s = 0.1$. Responses arising from alternative selections of K_s are represented by the dotted curves, where the most aggressive (and oscillatory) one corresponds to $K_s = 0.2$ and the sluggish ones correspond to the selections $K_s = 0.03$ and $K_s = 0.01$.

The bold dashed curve shows the closed-loop response in the case where the disturbance can be measured by the control system, so that it is directly accounted for in the enhanced Smith predictor dynamics. Note that in this case the prediction problem is perfectly solved by the scheme (namely, we have a perfect equality $z(\cdot) \equiv z_u(\cdot - \tau)$).

Figure 5 shows the same responses reported in Figure 4 in the perturbed case where the estimate of the dead time has a 50% error (namely, the estimated delay is 7.5). The closed-loop exhibits oscillatory transients but the overall performance is quite well preserved.

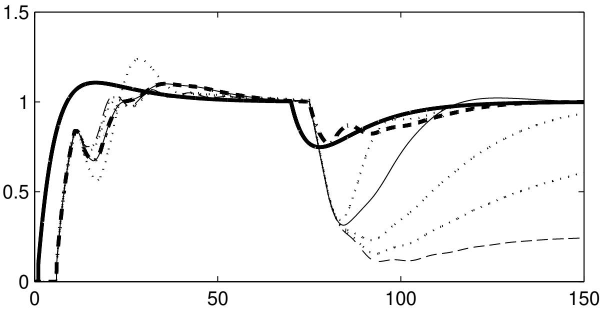

Fig. 5. Perturbed responses: Undelayed (bold); classical Smith predictor (dashed); proposed scheme with $K_s = 0.1$ (thin solid); proposed scheme with alternative selections of K_s (dotted); proposed scheme with disturbance measurement (bold dashed).

V. Conclusions

In this section we proposed a novel interpretation of the classical Smith predictor scheme for dead time processes. This interpretation reveals an intrinsic cascaded structure which allows for extra degrees of freedom in the Smith predictor design. The potential for high performance compensation schemes has been shown via a simple scalar example, but future work will involve more constructive tools for the design of the enhanced schemes herein introduced.

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