PASSIFICATION OF NONLINEAR CONTROLLERS VIA A SUITABLE TIME-REGULAR RESET MAP

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Abstract: For a class of square continuous time nonlinear controllers we design a suitable resetting rule inspired by the resetting rule for Clegg integrators and First Order Reset Elements (FORE). With this rule, we prove that the arising hybrid system with temporal regularization is passive in the conventional continuous time sense with a small shortage of input passivity decreasing with the temporal regularization constant. Based on the passivity property, we then investigate the finite gain stability of the interconnection between this passive controller and a passive nonlinear plant.

Keywords: passivity, reset control system, hybrid system

1. INTRODUCTION

In recent years, much attention has been given to the analysis and design problem of control systems in the hybrid context, namely when the closed-loop dynamics obeys either a continuous law imposing a constraint on the pointwise derivative of the solution when it belongs to the so-called flow set, and/or a discrete law imposing a constraint on the jump that the solution undertakes when it belongs to the so-called jump set. This type of interpretation of hybrid systems, thereby merging classical discrete- and continuous-time concepts in a unifying framework has been pursued in the past years by providing a specific mathematical characterization of the underlying mathematical theory. An extensive survey of the corresponding results can be found in (Goebel *et al.*, 2009).

A specific instance of hybrid systems corresponds to the case analyzed here: continuous-time plants controlled by a hybrid controller, namely a hybrid closed-loop where the jumps only affect the controller states. Within this class of systems a relevant example consists in the reset control systems first introduced in (Clegg, 1958), where a jump linear system (the "Clegg integrator") generalizing a linear integrator was proposed. This generalization was then further developed in (Horowitz and Rosenbaum, 1975) where it was extended to first order linear filters, and therein called First Order Reset Elements (FORE). FORE received much attention in recent years and have been proven to overcome some intrinsic limitations of linear controller (Beker *et al.*, 2001). Moreover, by relying on Lyapunov approaches, suitable analysis and synthesis tools for the stability of a class of reset systems generalizing control systems with FORE have been proposed in (Beker *et al.*, 2004; Nešić *et al.*, 2008) and references therein. Moreover, in the recent paper (Carrasco *et al.*, 2010) the $\mathcal{L}_2$ stability of reset control systems has been addressed in the passivity context, by showing interesting properties of the reset system under

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the assumption that the continuous-time part of the reset controller is passive before resets and that a suitable non-increase condition is satisfied by the storage function at jumps. In (Carrasco *et al.*, 2010) it was also shown by a simulation example that resets do help closed-loop performance in passivity-based closed-loops.

In this paper we further develop the ideas of (Carrasco *et al.*, 2010) by using a specific temporally regularized reset strategy for the reset controller. The reset strategy generalizes the new interpretation of FOREs and Clegg integrators proposed in (Zaccarian *et al.*, 2005; Nešić *et al.*, 2008) and references therein. We show that, with the proposed reset strategy, passification is possible for any continuous-time underlying dynamics under some sector growth assumption on the right hand side of the continuous-time dynamics of the controller. The obtained passivity property is characterized by an excess of output passivity and a lack of input passivity whose size can be made arbitrarily small by suitably adjusting the reset rule. As an example, the proposed reset strategy allows to establish a passivity property for any FORE, including those characterized by an exponentially unstable pole, while the results in (Carrasco *et al.*, 2010) only allow to establish passivity of FOREs with stable poles. This increased potential of the reset rule proposed here is illustrated on a nonlinear simulation example.

The paper is organized as follows. In Section 2 we describe the class of controllers under consideration and the proposed reset rule, together with some notation and preliminaries characterizing the hybrid systems framework of (Goebel *et al.*, 2009). In Section 3 we first state our main passivity result and then establish finite $\mathcal{L}_2$ gain properties of interconnected systems involving the proposed reset controller. Finally, in Section 4 we discuss a simulation example. All the proofs are omitted due to space constraints.

2. A CLASS OF NONLINEAR RESET CONTROLLERS

Consider the following nonlinear controller mapping the input v to the output u,

$$\begin{aligned} \dot{x}_c &= f(x_c) + g(x_c, v) \\ u &= h(x_c), \end{aligned} \tag{1}$$

where $u \in \mathbb{R}^q$, $v \in \mathbb{R}^q$, so that the controller is square and where the following regularity assumption is satisfied by the right hand side.

Assumption 1. *The functions $f(\cdot)$ and $h(\cdot)$ are continuous and sector bounded, namely there exist two constants L_f and L_h such that for all x_c, $|f(x_c)| \le L_f|x|$ and $|h(x_c)| \le L_h|x_c|$.*

Moreover, $g(\cdot,\cdot)$ is continuous in both its arguments and satisfies the following sector condition: there exists a constant L_g such that for all x_c and all v, $|g(x_c, v)| \le L_g(|x_c| + |v|)$.

In this paper we propose a hybrid modification of the controller (1) aimed at making it passive from v to u, regardless of the properties of the original dynamics in (1). In particular, the modified controller follows the continuous-time dynamics of (1) at times when the input/output pair belongs to a certain subset of the input/output space. When the input/output pair exits that subset, the state of the controller is reset to zero, intuitively re-initializing the controller within the set where it is allowed to flow.

To avoid Zeno solutions, namely solutions that exhibit infinitely many jumps in a bounded time interval, we also embed the hybrid modification with a temporal regularization clock, imposing that the controller cannot be reset to zero before ρ times after the previous reset (see also (Nešić *et al.*, 2008; Johansson *et al.*, 1999).

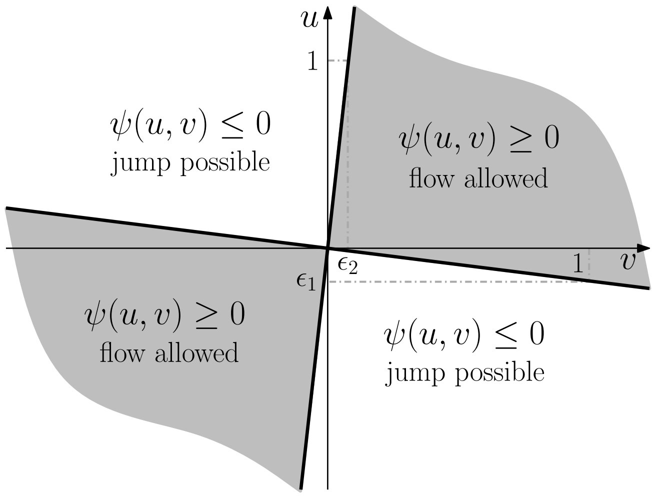

Fig. 1. Input/output space of the controller (2) and subsets where $\psi(u, v) \gtreqless 0$. The grey area defines the set where flow can always happen. The blank area defines the set where a jump is *possible*, provided that $\tau \ge \rho$ (time regularization).

The proposed hybrid controller is given by

$$\begin{aligned} &\begin{cases} \dot{x}_c = f(x_c) + g(x_c, v) \\ \dot{\tau} = 1 \end{cases} && \textit{if } \tau \le \rho \textit{ or } \psi(u, v) \ge 0 \\ &\begin{cases} x_c^+ = 0 \\ \tau^+ = 0 \end{cases} && \textit{if } \tau \ge \rho \textit{ and } \psi(u, v) \le 0 \\ &\ u = h(x_c) \end{aligned} \tag{2a}$$

where $\psi(u, v)$ is defined as

$$\psi(u, v) = (u + \epsilon_1 v)^T (v - \epsilon_2 u) \tag{2b}$$

and ϵ_1 and ϵ_2 are some (typically small) non-negative scalars. As usual in the hybrid system framework, we call C the set $\{(x_c, \tau, v) : \tau \le \rho \textit{ or } \psi(h(x_c), v) \ge 0\}$ and D the set $\{(x_c, \tau, v) : \tau \ge \rho \textit{ and } \psi(h(x_c), v) \le 0\}$.

The rationale behind the reset controller (1) is illustrated in Figure 1 where the input/output space of (2) is represented for the case $q = 1$. In the figure, the shaded region corresponds to the set $\psi(u, v) \ge 0$ where the system always flows, regardless of the value of τ. Instead, in the remaining region, where $\psi(u, v) \leq 0$, the system will jump provided that $\tau \geq \rho$. Note also that when $\epsilon_1 = \epsilon_2 = 0$, the shaded region reduces to the first and third quadrant, resembling the resetting rule characterized for the first order reset element (FORE) in (Zaccarian *et al.*, 2005; Nešić *et al.*, 2008). When the reset occurs, since $h(0) = 0$, the u component of the input/output pair will jump at zero thus resulting in a vertical jump to the horizontal axis. Moreover, ϵ_1 and ϵ_2 allow to have extra degrees of freedom in the resetting rule. In particular, the goal of ϵ_1 is to guarantee that the reset rule maps the new input/output pair in the interior of the shaded set whenever $v \neq 0$. Instead, as it will be clear next, the goal of ϵ_2 is to modify the resetting rule to obtain some strict output passivity for the reset controller (2).

Controller (2) will be dealt with in this paper following the framework of (Goebel and Teel, 2006; Goebel *et al.*, 2009; Cai and Teel, 2009). In particular, by Assumption 1, controller (2) satisfies the hybrid basic assumptions (see, e.g., (Cai and Teel, 2009)), which ensure desirable regularity properties of the solutions, such as existence, and robustness to arbitrarily small perturbations (see (Goebel *et al.*, 2009) for details).

As usual in the hybrid system framework, the evolution of the state $\xi = (\xi_x, \xi_\tau)$ either continuously flows through C, by following the dynamics given by $f(\xi_x)+g(\xi_x, v)$ and 1, or jumps from D to $(0, 0)$. Such an alternation of jumps and flow intervals can be conveniently characterized by using a generalized notion of time, called *hybrid time*. By following (Goebel and Teel, 2006), a set $E \subseteq \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a *hybrid time domain* if it is the union of infinitely many intervals of the form $[t_j, t_{j+1}] \times \{j\}$ where $0 = t_0 \leq t_1 \leq t_2 \leq, \ldots,$ or of finitely many such intervals, with the last one possibly of the form $[t_j, t_{j+1}] \times \{j\}$, $[t_j, t_{j+1}) \times \{j\}$, or $[t_j, \infty] \times \{j\}$.

The evolution of the state ξ of the hybrid system (2), depends on the input signal v, so that both ξ and v must be defined on hybrid time domain. By following (Cai and Teel, 2009), we call *hybrid signal* each function defined on a hybrid time domain. A hybrid signal $v : \operatorname{dom} v \to \mathcal{V}$ is a *hybrid input* if $v(\cdot, j)$ is Lebesgue measurable and locally essentially bounded for each j. A hybrid signal $\xi : \operatorname{dom} \xi \to \mathbb{R}^n \times \mathbb{R}_{\geq 0}$ is a *hybrid arc* if $\xi(\cdot, j)$ is locally absolutely continuous, for each j. With the basic assumptions satisfied, a hybrid arc $\xi = (\xi_x, \xi_\tau)$ and a hybrid input v is a *solution pair* (ξ, v) to the hybrid system (2) if $\operatorname{dom} \xi = \operatorname{dom} v$, $(\xi(0, 0), v(0, 0)) \in C \cup D$, and

s.1 for all $j \in \mathbb{N}$ and almost all t such that $(t, j) \in \operatorname{dom} \xi$,

$$\begin{aligned} &(\xi(t, j), v(t, j)) \in C \\ &\dot{\xi}_x(t, j) = f(\xi_x(t, j)) + g(\xi_x(t, j), v(t, j)); \\ &\dot{\xi}_\tau(t, j) = 1; \end{aligned} \tag{3}$$

s.2 for all $(t, j) \in \operatorname{dom} \xi$ such that $(t, j + 1) \in \operatorname{dom} \xi$,

$$\begin{aligned} &(\xi(t, j), v(t, j)) \in D \\ &\xi_x(t, j + 1) = 0; \\ &\xi_\tau(t, j + 1) = 0; \end{aligned} \tag{4}$$

We say that a set of solutions pairs (ξ, v) is *uniformly non-Zeno* if there exists $T \in \mathbb{R}_{>0}$ and $J \in \mathbb{N}$ such that, for any given $(t, j), (t', j') \in \operatorname{dom} \xi$, if $|t - t'| \leq T$ then $|j - j'| \leq J$, that is, in any time period of length T, no more than J jumps can occur. Note that multiple instantaneous jumps are still possible, (Goebel and Teel, 2006).

Note that any continuous-time signal $\overline{v} : \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ can be rewritten as hybrid signal with domain E, for any given hybrid domain E. In fact, suppose that $E = \bigcup [t_j, t_{j+1}] \times \{j\}$ is an hybrid time domain. Then, we can define a hybrid signal v *lifted* from $\overline{v}$ on E as follows: $v(t, j) = \overline{v}(t)$ for each $(t, j) \in E$. Conversely, suppose that (ξ, v) is a solution pair to the hybrid system (2). Then, the output signal $u = h(\xi_x)$ is a hybrid signal and $\operatorname{dom} u = \operatorname{dom} \xi$. From u we can construct an continuous-time signal $\overline{u} : \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ *projected* from u on $\mathbb{R}_{\geq 0}$ as follows: $\overline{u}(t) = u(t, j)$ for each $(t, j) \in \operatorname{dom} u$ such that $(t, j + 1) \notin \operatorname{dom} u$, and $\overline{u}(t) = u(t, j + 1)$ otherwise.

We denote by $\|\overline{v}\|_p$ the $\mathcal{L}_p$ gain of a continuous-time signal $\overline{v}$. The $\mathcal{L}_p$ gain of a hybrid signal v, related to the continuous part of its domain, will be denoted by $\|v\|_{c,p} = \left(\sum_{j=0}^{J} \int_{t_j}^{t_{j+1}} |v(t, j)|^p dt \right)^{1/p}$. Note that for any continuous-time signal $\overline{v}$ projected from a hybrid signal $\overline{v}$ on $\mathbb{R}_{\geq 0}$, we have that $\|\overline{v}\|_q = \|v\|_{c,p}$. Conversely, for any hybrid signal v lifted from a continuous-time signal $\overline{v}$ on a given hybrid time domain E, we have that $\|v\|_{c,p} = \|\overline{v}\|_p$.

Finally, the following lemma characterizes regularity of the solutions to (2).

Lemma 1. *Under Assumption 1, all the solutions to (2) are uniformly non-Zeno. Moreover, for each $\mathcal{L}_p$ integrable input signal $\overline{v}$, a solution pair (ξ, v) where v is the hybrid input signal lifted from $\overline{v}$ on $\operatorname{dom} \xi$, is a complete solution pair.*

Proof. For a solution pair (ξ, v), define $t_j = \min\{t \,|\, (t, j) \in \operatorname{dom} \xi\}$. By the definition of C and D given after (2), given any solution pair $(\xi, v) = ((\xi_x, \xi_\tau), v)$ of (2), $t_j - t_{j-1} \geq \rho$ for all $(t, j) \in \operatorname{dom}(x)$, $j \geq 2$. This implies that the uniformly non-Zeno definition in (Goebel and Teel, 2006) (see also (Collins, 2004)) is satisfied with $T = \rho$ and $J = 2$.

By $C \cup D = \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathcal{V}$, $\operatorname{dom} \xi$ is bounded only if ξ blows up in finite time. Looking at the dynamics of the system in (2a), by Assumption 1, $|\dot{x}_c| \leq |f(x_c) + g(x_c, v)| \leq L_f |x_c| + L_g |v|$ and $|\dot{\tau}| = 1$. Therefore, if $|v|$ is $\mathcal{L}_p$ integrable, $|\xi|$ is bounded in any given compact subset of $\mathbb{R}_{\geq 0} \times \mathbb{N}$. □

3. MAIN RESULTS

3.1 Passivity of the reset controller

The following theorem shows that the hybrid controller (2) is almost passive with a shortage of input passivity proportional to the temporal regularization constant ρ plus ϵ_1. Moreover, the slight modification of the function $\psi(\cdot,\cdot)$ enforced by ϵ_2 induces some excess of output passivity.

Theorem 1. *Consider the hybrid controller* (2) *satisfying Assumption 1. Define*

$$\varepsilon_1 := \frac{\epsilon_1}{1-\epsilon_1\epsilon_2}\,, \qquad \varepsilon_2 := \frac{\epsilon_2}{1-\epsilon_1\epsilon_2}$$

$$k(\rho) = \rho L_h L_g \max\{1, \rho e^{(L_f+L_g)\rho}\} \tag{5}$$

$$\overline{k}(\rho) = k(\rho)(1+\varepsilon_2 k(\rho))$$

Given a $\mathcal{L}_2$ integrable input signal $\overline{v} \in \mathbb{R}_{\geq 0} \to \mathcal{V}$ and a solution pair (ξ, v) to (2)*, with v lifted from $\overline{v}$ on* $\operatorname{dom}\xi$*, then*

$$\int_0^\infty \overline{u}(t)^T\overline{v}(t) \geq -\left(\varepsilon_1 + \overline{k}(\rho)\right)\|\overline{v}(\cdot)\|_2^2 + \varepsilon_2\|\overline{u}(\cdot)\|_2^2 \tag{6}$$

where the output signal $\overline{u} \in \mathbb{R}_{\geq 0} \to \mathbb{R}^q$ is projected from the hybrid output signal $u : \operatorname{dom} u \to \mathbb{R}^q$ corresponding to the solution pair (ξ, v),

Remark 1. Note that Theorem 1 establishes the passivity of (2) based on the norm $\|\cdot\|_{c,2}$, namely only taking into account the continuous-time nature of the hybrid solutions. This type of passivity concept is relevant because of Lemma 1 and, moreover, allows to rely on standard passivity results (van der Schaft, 1999) to conclude properties of the closed loop between (2) and a plant, as detailed in Section 3.2.

Remark 2. The passivity of the controller (1) induced by the reset policy in (2) is robust to small variations of the dynamics of the controller (1). This is based on the fact that the passivity result in Theorem 1 is inferred from the L_f, L_g and L_h bounds on the functions f, g and h of the controller dynamics (1) (specified in Assumption 1). Thus, small variations of the dynamics of the controller (1) can be taken into account by an appropriate selection of those bounds.

Remark 3. We emphasize the generality of the controller dynamics in (1). Despite this generality, the hybrid controller (2) is passive. Intuitively, such a generality in the controller dynamics is related to the fact that passivity is obtained primarily via a suitable selection of the jump and flow sets D and C, which ensure that the controller state only flows in regions where a passive behavior occurs. Roughly speaking, the passive behavior of the controller can be considered as an effect of the definition of $\psi(u, v)$, that forces a particular shape of the sets C and D. Following this intuition, while $\psi(u, v)$ constrains C and D to induce passivity, time regularization adds some extra constraint on C and D possibly destroying part of this passivity property. This results in a shortage of passivity parameterized with ρ.

3.2 Application to feedback systems

In this section we use the *passivity theorem* (van der Schaft, 1999) to establish useful stability properties of the reset controller (2) interconnected to any passive nonlinear plant: [3]

$$\begin{aligned} \dot{x}_p &= f_p(x_p, u+d) \\ y &= h_p(x, u+d), \end{aligned} \tag{7}$$

via the negative feedback interconnection $v = w - y$, where w is an external signal. In (7), d is an additive disturbance acting at the plant input. The following statement directly follows from the properties of (2) established in Theorem 1.

Proposition 1. *Consider the hybrid controller* (2) *satisfying Assumption 1 in feedback interconnection $v = w - y$ with the plant (7).*

For any $\epsilon_1 \geq 0$, $\epsilon_2 > 0$ and $\rho > 0$, given ε_1 and $\overline{k}(\rho)$ as in (6), if the plant is output strictly passive with excess of output passivity $\delta_P > \varepsilon_1 + \overline{k}(\rho)$, then the closed-loop system (2), (7) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

In Proposition 1 we require a specific excess of output passivity from the plant because we assume that the controller requires implementation with certain prescribed selections of ϵ_1 and ρ. In the case where it is possible to reduce arbitrarily these two parameters, it is possible to relax the requirements of Proposition 1 as follows:

Proposition 2. *Consider the hybrid controller* (2) *satisfying Assumption 1 in feedback interconnection $v = -y$ with the plant (7).*

If the plant (7) is output strictly passive, then for any $\epsilon_2 > 0$, there exist small enough positive numbers ϵ_1^* and ρ^* such that for all $\epsilon_1 \leq \epsilon_1^*$ and all $\rho \leq \rho^*$, the closed-loop system (2), (7) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

Proof. The proposition is a straightforward consequence of Proposition 1 noting that for a fixed ϵ_2, the lack of output passivity established in Theorem 1 decreases monotonically to zero as ϵ_1 and ρ go to zero. Then it is always possible to reduce the two parameters to match the passivity condition in (van der Schaft, 1999). □

Both Propositions 1 and 2 either require an explicit bound on the excess of output passivity of the plant or

[3] See also (Carrasco *et al.*, 2010) for a similar application of the passivity theorem to reset controllers.

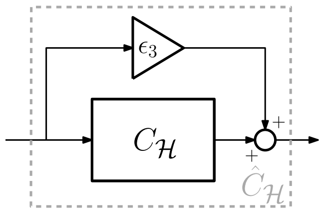

Fig. 2. The very strictly passive version (8) of the reset controller ($C_{\mathcal{H}}$ corresponds to (2)).

constrain the controller parameters ϵ_1 and ρ to be small enough. An alternative solution to this is to add an extra feedforward loop to the reset controller (2), following the derivations in (Khalil, 2002, page 233), to guarantee that the arising reset system is very strictly passive, namely it is both input strictly passive and output strictly passive. To this aim, we modify the output equation of (2) by adding the feedforward term $\epsilon_3 v$, as represented in Figure 2. The corresponding reset controller can then be written as:

$$\begin{cases} \dot{x_c} = f(x_c) + g(x_c, v) \\ \dot{\tau} = 1 \end{cases} \quad \text{if } \tau \le \rho \text{ or } \hat{\psi}(\hat{u}, v) \ge 0$$
$$\begin{cases} x_c^+ = 0 \\ \tau^+ = 0 \end{cases} \quad \text{if } \tau \ge \rho \text{ and } \hat{\psi}(\hat{u}, v) \le 0 \qquad (8a)$$
$$\hat{u} = h(x_c) + \epsilon_3 v$$

where $\hat{\psi}(\cdot,\cdot)$ is defined as

$$\hat{\psi}(\hat{u}, v) = ((\hat{u} + (\epsilon_1 - \epsilon_3)v)^T((1 + \epsilon_2\epsilon_3)v - \epsilon_2\hat{u}) \qquad (8b)$$

and $\epsilon_3 > 0$ is suitably selected as specified below. When using the modified reset controller (8), the following result holds.

Proposition 3. *Consider the hybrid controller* (8) *satisfying Assumption 1 in feedback interconnection $v = w - y$ with a passive plant (7).*

For any $\epsilon_1 \ge 0$, $\epsilon_2 > 0$ and $\rho > 0$, given ε_1 and $\overline{k}(\rho)$ as in (6), if $\epsilon_3 > \varepsilon_1 + \overline{k}(\rho)$, then the closed-loop system (8), (7) with $v = w - y$ is finite-gain $\mathcal{L}_2$ stable from (w, d) to (u, v).

Proof. Define a new output $\hat{u} = u + \epsilon_3 v$ and denote by $\overline{\hat{u}}$ the output signal projected from $\hat{u}$ on $\mathbb{R}_{\ge 0}$. Then, from (6), we have that

$$\int_0^\infty \overline{\hat{u}}(t)^T \overline{v}(t) \ge \epsilon_2 \int_0^\infty \overline{u}^T \overline{u} + \left(\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)\right) \int_0^\infty \overline{v}^T \overline{v}$$
$$\ge \frac{1}{1+2\epsilon_2\epsilon_3} \left(\epsilon_2 \int_0^\infty \overline{\hat{u}}^T \overline{\hat{u}} + \left(\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)\right) \int_0^\infty \overline{v}^T \overline{v} \right).$$

It follows that

$$\int_0^\infty \overline{\hat{u}}(t)^T \overline{v}(t) \ge \eta_1 \|\overline{\hat{u}}\|_2^2 + \eta_2 \|\overline{v}\|_2^2 \qquad (9)$$

with $\eta_1 = \frac{\epsilon_2}{1+2\epsilon_2\epsilon_3} > 0$ and $\eta_2 = \frac{\epsilon_3 - \varepsilon_1 - \overline{k}(\rho)}{1+2\epsilon_2\epsilon_3} > 0$.

Replace now the output u of the controller (2) with $\hat{u} = u + \epsilon_3 v = h(x_c) + \epsilon_3 v$. Then, $\hat{\psi}(\hat{u}, v)$ is obtained by substituting $u = \hat{u} - \epsilon_3 v$ in the expression of $\psi(u, v)$ of Equation (2b). By the *passivity theorem* in (van der Schaft, 1999), Proposition 3 follows. □

Remark 4. The results in this section can be seen as a generalization of the results on full reset compensators in (Carrasco *et al.*, 2010), where passivity techniques are used to establish finite gain $\mathcal{L}_2$ stability of the closed-loop between passive nonlinear plants and reset controllers. When focusing on linear reset controllers such as Clegg integrators (Clegg, 1958) and First Order Reset Elements (FORE) (Horowitz and Rosenbaum, 1975; Beker *et al.*, 2004), the novelty of Theorem 1 as compared to the results in (Carrasco *et al.*, 2010) is that those results establish passivity of FORE whose underlying linear dynamics is already passive (namely FORE with stable poles). Conversely, our results of Theorem 1 apply regardless of what the underlying dynamics of the controller is. Therefore, for example, any FORE with arbitrarily large unstable poles would still become passive using the flow and jump sets characterized in (2). Note however that, as compared to the approach in (Carrasco *et al.*, 2010), we are using a different selection of the flow and jump sets. In the example section we illustrate the use of unstable FOREs within (2).

4. SIMULATION EXAMPLE

We consider a planar two-link rigid robot manipulator as modeled and with the parameters selection in (Morabito *et al.*, 2004). Denoting by $q \in \mathbb{R}^2$ the two joint positions and by $\dot{q} \in \mathbb{R}^2$ the corresponding velocities, the manipulator is modeled as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + h(q) = u_p \qquad (10)$$

where $D(q)$ is the inertia matrix, $C(q, \dot{q})\dot{q}$ comprises the centrifugal and Coriolis terms, $h(q)$ is the gravitational vector, and u_p represents the external torques applied to the two rotational joints of the robot.

Given a reference signal $r \in \mathbb{R}^2$ representing the desired joint position, following a standard passivity based approach, it is possible to close a first control loop around the robot (10) to induce the equilibrium point $(q, \dot{q}) = (r, 0)$ while guaranteeing passivity from a suitable input u to the joint velocity output $\dot{q}$. In particular, define $V(q, r) = \frac{k_p}{2}(q - r)^T(q - r)$, where the scalar $k_p > 0$ is a weight parameter on the position error, and choose

$$u_p = -\frac{\partial V(q, r)}{\partial q} + h(q) + u. \qquad (11)$$

Then, the interconnection (10), (11) corresponds to

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + \frac{\partial V(q, r)}{\partial q} = u \qquad (12)$$

and, following similar steps to those in (Fantoni *et al.*, 2000), it can be shown to be passive from u to $\dot{q}$.

For the outer loop, we rely on the very strictly passive controller (8) where the dynamics in (8a) is selected as a pair of decentralized First Order Reset Elements, namely denoting $x_c = [x_{c1} \;\; x_{c2}]^T$, we select $f(x_c) = [\lambda_1 x_{c1} \;\; \lambda_2 x_{c2}]^T$ and $g(x_c, \dot{q}) = \dot{q}$. Moreover, we choose $u = k_{\mathcal{H}} \hat{u}$, where $k_{\mathcal{H}}$ is a positive constant.

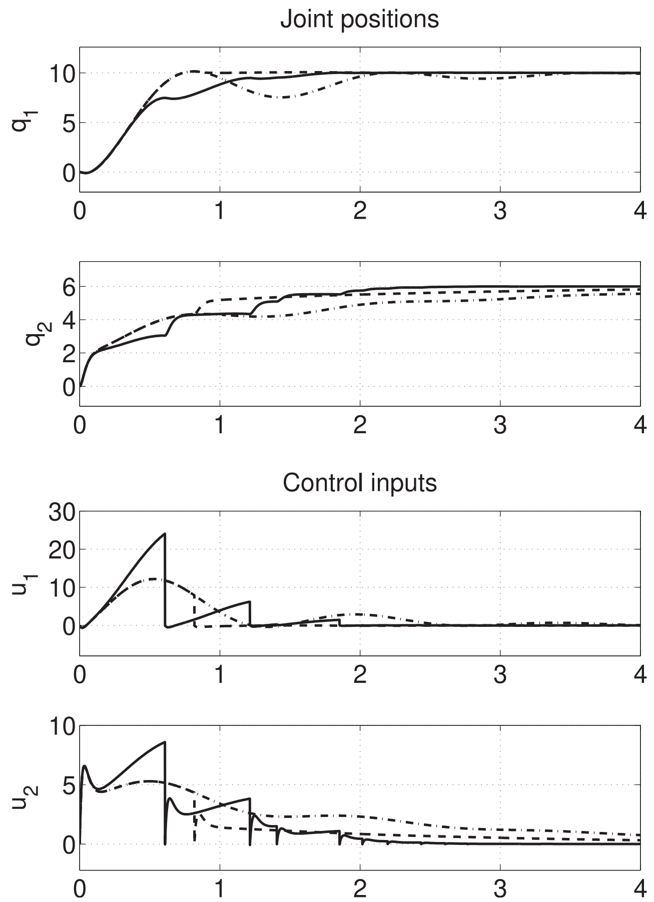

Fig. 3. Simulations results. Stable FORE and no resets (dash-dotted), stable FORE with resets (dashed) and unstable FORE with resets (solid).

By Proposition 3, the closed loop system (10), (11), (8a) with $u = k_{\mathcal{H}} \hat{u}$ is finite-gain $\mathcal{L}_2$ stable. Figure 3 compares several simulation results for this closed-loop using the constant reference signal $r = [10\,6]^T$ and the following values of the parameters: $k_p = 100$, $k_{\mathcal{H}} = 100$ and $\rho = 0.1$. First, we select stable FORE poles $(\lambda_1, \lambda_2) = (-2, -1)$ so that the closed-loop stability can be concluded also using the results in (Carrasco *et al.*, 2010). For this case, when no resets occur, the position output (namely q) and plant input (namely u) responses correspond to the dash-dotted curves in Figure 3. That response is converging because the system without resets is passive due to the stability of the FORE poles. When introducing resets, the response becomes the dashed curves in the figure, where it can be appreciated that a single reset occurring around $t = 0.8\ s$ significantly improves the closed-loop response. A last simulation is carried out by selecting an unstable FORE with $(\lambda_1, \lambda_2) = (2, 1)$. In this case the speed of convergence of the second joint is faster at the price of a reduction of the speed of convergence of the first joint. Note also that the dwell time imposed by the temporal regularization is never active for this specific simulation, as each jump occurs after more than $\rho = 0.1$ seconds from the previous jump. We don't include a simulation with the unstable FORE without resets because this leads to diverging trajectories. Figure 4 compares several simulation results

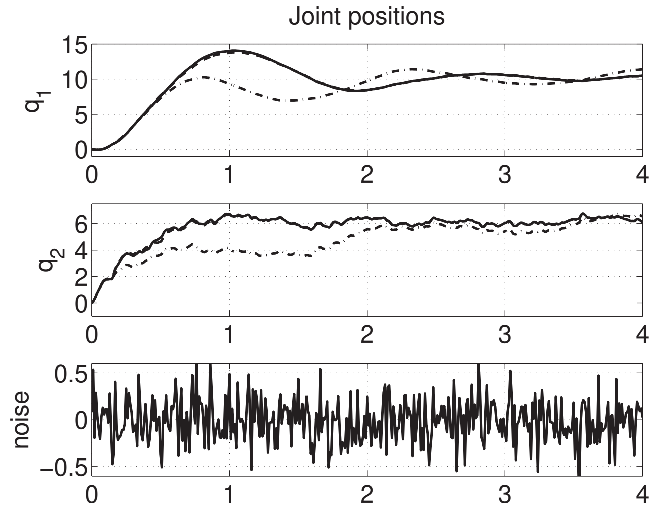

Fig. 4. Simulations results with noise on measured output. Stable FORE and no resets (dash-dotted), stable FORE with resets (dashed) and unstable FORE with resets (solid).

for the closed-loop when a disturbance signal is added to the measured value of $\dot{q}$. The simulations are based on reference signal and parameters of the nominal case. Note that q_1 and q_2 trajectories of the stable and unstable FOREs with resets overlap.

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