GRADUATE COURSE ON POLYNOMIAL METHODS FOR ROBUST CONTROL

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Course outline

I Robust stability analysis (Part I)

I.1 General introduction - linear systems, polynomial methods and robust control

I.2 Single parameter uncertainty - eigenvalue criteria

I.3 Interval uncertainty - Kharitonov's theorem

II Robust stability analysis (Part II)

II.1 Polytopic uncertainty - edge theorem

II.2 Multilinear and polynomic uncertainty - mapping theorem

III Robust design and convex optimization

III.1 Robust pole placement - approximation of stability region

III.2 Rank-one robust stabilization - Youla-Kučera and Rantzer-Megretski parametrizations

III.3 Simultaneous stabilization - strong stabilization, Hermite criterion, open problems

IV New results on robust analysis and design

IV.1 Robust stability analysis - Linear matrix inequalities and positive polynomial matrices

IV.2 Robust stability design - Numerical examples

Course Outline

I Robust stability analysis (Part I)

II Robust stability analysis (Part II)

III Robust design and convex optimization

IV New results on robust analysis and design

Scope of the course

Systems • linear • non-linear	Methods • frequential • state-space • polynomial
Control scheme • adaptive • stochastic • robust	Uncertainty • non-parametric • parametric

Linear systems

Theory much better developed than for non-linear systems

Provides background for nonlinearists

Availability of powerful CACSD tools to solve numerical linear algebra problems

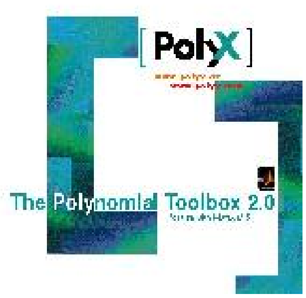
(Old) reference books

- Kailath. Linear systems. Prentice Hall, 1980
- Chen. Linear system theory and design. HRW, 1984

Computer tools used for the course

Matlab 6.0

Polynomial Toolbox 2.5



www.mathworks.com

www.polyx.cz

Polynomial methods

Based on the algebra of polynomials and polynomial matrices, typically involve

- linear Diophantine equations
- quadratic spectral factorization

Pioneered in central Europe during the 70s mainly by Vladimír Kučera from the former Czechoslovak Academy of Sciences

Network funded by the European commission

EUROPOLY

www.utia.cas.cz/europoly

Polynomial matrices also occur in Jan Willems' behavorial approach to systems theory

Alternative to state-space methods developed during the 60s most notably by Rudolf Kalman in the USA, rather based on

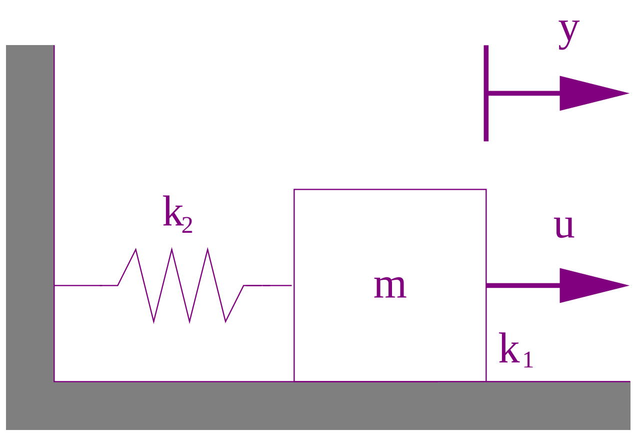
- linear Lyapunov equations
- quadratic Riccati equations

Ratio of polynomials

A scalar transfer function can be viewed as the ratio of two polynomials

Example

Consider the mechanical system



- y displacement
- u external force
- k_1 viscous friction coeff
- k_2 spring constant
- m mass

Neglecting static and Coloumb frictions, we obtain the linear transfer function

$$G(s) = \frac{y(s)}{u(s)} = \frac{1}{ms^2 + k_1 s + k_2}$$

Ratio of polynomial matrices

Similarly, a MIMO transfer function can be viewed as the ratio of polynomial matrices

$$G(s) = N_R(s)D_R^{-1}(s) = D_L^{-1}(s)N_L(s)$$

the so-called matrix fraction description (MFD)

Lightly damped structures such as oil derricks, regional power models, earthquakes models, mechanical multi-body systems, damped gyroscopic systems are most naturally represented by second order polynomial MFDs

$$(D_0 + D_1 s + D_2 s^2)y(s) = N_0 u(s)$$

Example

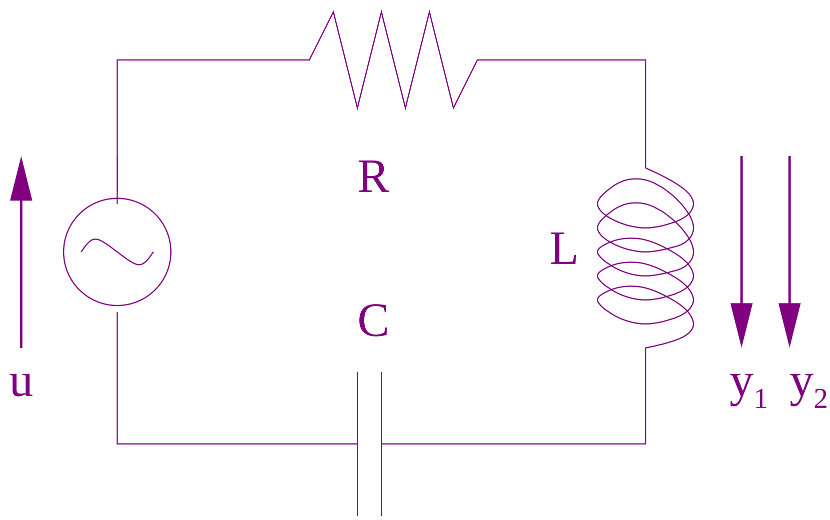
The (simplified) oscillations of a wing in an air stream is captured by properties of the quadratic polynomial matrix [Lancaster 1966]

$$D(s) = \left[\begin{array}{ccc} 121 & 18.9 & 15.9 \\ 0 & 2.7 & 0.145 \\ 11.9 & 3.64 & 15.5 \end{array}\right] + \left[\begin{array}{ccc} 7.66 & 2.45 & 2.1 \\ 0.23 & 1.04 & 0.223 \\ 0.6 & 0.756 & 0.658 \end{array}\right] s + \left[\begin{array}{ccc} 17.6 & 1.28 & 2.89 \\ 1.28 & 0.824 & 0.413 \\ 2.89 & 0.413 & 0.725 \end{array}\right] s^2$$

First-order polynomial MFD

Example

RCL network



- y_1 voltage through inductor
- y_2 current through inductor
- u voltage

Applying Kirchoff's laws and Laplace transform we get

$$\left[\begin{array}{cc} 1 & -Ls \\ Cs & 1+RCs \end{array}\right]\left[\begin{array}{c} y_1(s) \\ y_2(s) \end{array}\right] = \left[\begin{array}{c} 0 \\ Cs \end{array}\right] u(s)$$

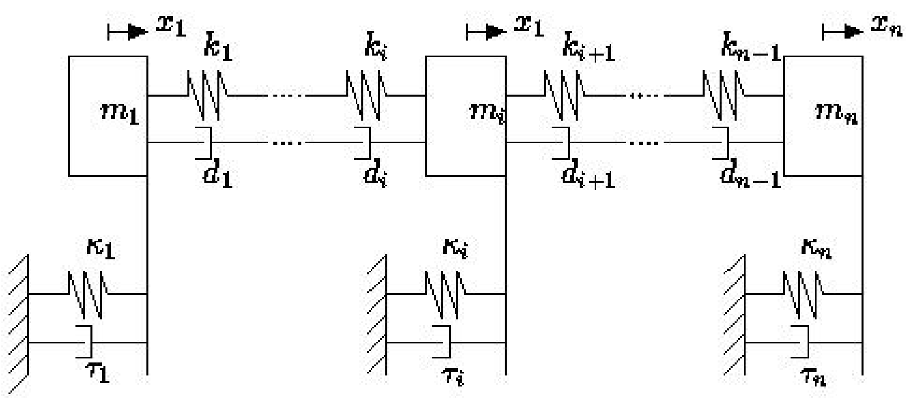
and thus the first-order left system MFD

$$G(s) = \left[\begin{array}{cc} 1 & -Ls \\ Cs & 1+RCs \end{array}\right]^{-1}\left[\begin{array}{c} 0 \\ Cs \end{array}\right].$$

Second-order polynomial MFD

Example

mass-spring system



Vibration of system governed by 2nd-order differential equation $M\ddot{x} + C\dot{x} + Kx = 0$ where e.g. $n = 250$, $m_i = 1, \kappa_i = 5, \tau_i = 10$ except $\kappa_1 = \kappa_n = 10$ and $\tau_1 = \tau_n = 20$

Quadratic matrix polynomial

$$D(s) = Ms^2 + Cs + K$$

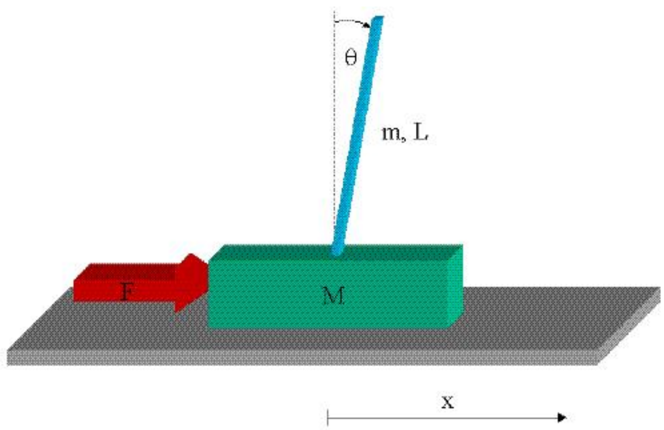
with

$$M = I$$
$$C = \text{tridiag}(-10, 30, -10)$$
$$K = \text{tridiag}(-5, 15, -5).$$

Another second-order polynomial MFD

Example

Inverted pendulum on a cart



Linearization around the upper vertical position yields the left polynomial MFD

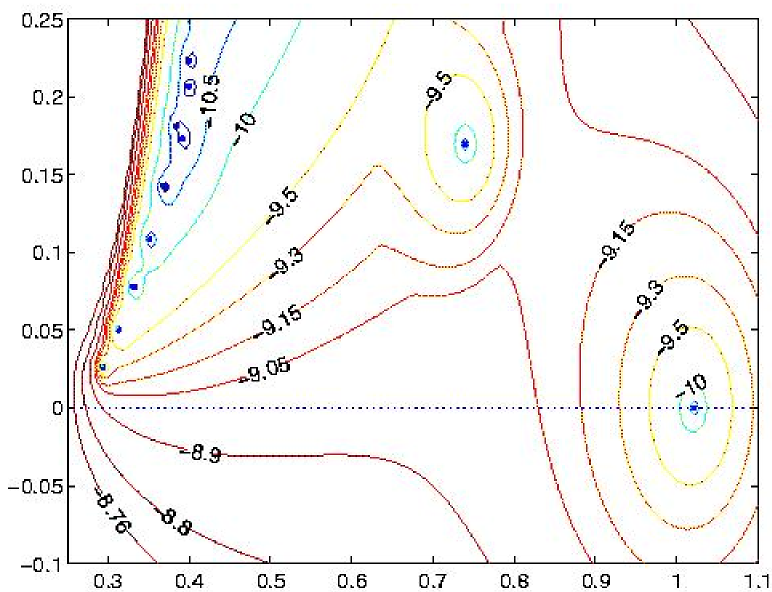
$$\begin{bmatrix} (M+m)s^2+bs & lms^2 \\ lms^2 & (J+l^2m)s^2+ks-lmg \end{bmatrix} \begin{bmatrix} x(s) \\ \phi(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} f(s)$$

With $J = mL^2/12$, $l = L/2$ and $g = 9.8$, $M = 2$, $m = 0.35$, $l = 0.7$, $b = 4$, $k = 1$, we obtain the denominator polynomial matrix

$$D(s) = \begin{bmatrix} 5s+3s^2 & 0.35s^2 \\ 0.35s^2 & -3.4+s+0.16s^2 \end{bmatrix}$$

More examples of polynomial MFDs

Higher degree polynomial matrices can also be found in aero-acoustics (3rd degree) or in the study of the spatial stability of the Orr-Sommerfeld equation for plane Poiseuille flow in fluid mechanics (4rd degree)



Pseudospectra of Orr-Sommerfeld equation

For more info see Nick Higham's homepage at
www.ma.man.ac.uk/∼higham

Uncertainty

When modeling systems we face several sources of uncertainty, including

- non-parametric (unstructured) uncertainty
  - unmodeled dynamics
  - truncated high frequency modes
  - non-linearities
  - effects of linearization, time-variation..
- parametric (structured) uncertainty
  - physical parameters vary within given bounds
  - interval uncertainty (l_∞)
  - ellipsoidal uncertainty (l_2)
  - l_1 uncertainty

How can we overcome uncertainty ?

- model predictive control
- adaptive control
- robust control

Robustness

We seek a control law valid over the whole range of admissible uncertainty

- off-line
- simple
- cheap
- secure

Basically we will study two classes of problems

Robust stability analysis
Parts I, II & IV of this course

Robust controller design
Parts III & IV of this course

Course material

Information on the course can be found at

www.laas.fr/~henrion/courses/polyrobust.html

Most of the material of the first two parts of the course (robust stability analysis) is taken from the textbooks

- J. Ackermann. Robust control: systems with uncertain physical parameters. Springer, 1993
- B. R. Barmish. New tools for robustness of linear systems. MacMillan, 1994
- S. P. Bhattacharyya, H. Chapellat, L. H. Keel. Robust control - The parametric approach. Prentice Hall, 1995

The third part (robust design) describes recent results published from 1992 to 1999 in

- IEEE Transactions on Automatic Control
- IFAC Automatica
- System and Control Letters
- International Journal of Control
- SIAM Journal on Control and Optimization

The last part of the course contains mostly new, previously unpublished material