# Scalable algorithms for computational mechanics – new results and numerical experiments

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In this talk we shall review some recent results related to the development of theoretically supported scalable algorithms for the solution of large scale problems of mechanics, including the contact problems of elasticity. The algorithms combine the Total FETI/BETI based domain decomposition methods adapted to the solution of 2D and 3D multibody contact problems with optional preconditioning by conjugate projector or dual scaling with the in a sense optimal algorithms for the solution of resulting quadratic programming or QCQP problems [1]. We first give some new bounds on the rate of convergence of the FETI/BETI algorithms showing that the number of matrix vector multiplications that is necessary to get the relative precision $p$ increases proportionally with $1+|\log(p)|$ [2]. The theoretical results are thus nearly the same as the classical results on the scalability for linear elliptic problems, i.e., the inequality constraints are treated nearly for free. Then we discuss the generalization of these results to problems discretized by non-matching grids. We consider the implementation of nonpenetration conditions by the variationally consistent discretization introduced recently by B. I. Wohlmuth and study performance of related algorithms. We give bounds on the spectrum of the related matrices for some mortar discretizations and compare them with the numerical values obtained for some special cases. Finally we discuss the massively parallel implementation and provide the results of numerical experiments including the solution of large problems discretized by hundreds millions of nodal variables.

[1] Z. Dostál, *Optimal Quadratic Programming Algorithms*, with Applications to Variational Inequalities, Springer US, New York 2009, SOIA 23.

[2] Z. Dostál, D. Horák, T. Kozubek, P. Vodstrčil, *On R-linear convergence of semi-monotonic inexact augmented Lagrangians for bound and equality constrained quadratic programming problems with application*. Mathematics & Computers with Applications, 2013.