An LMI Condition for Robust Stability of Polynomial Matrix Polytopes *

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Abstract

A sufficient LMI condition is proposed for checking robust stability of a polytope of polynomial matrices. It hinges upon two recent results: a new approach to polynomial matrix stability analysis and a new robust stability condition for convex polytopic uncertainty. Numerical experiments illustrate that the condition narrows significantly the unavoidable gap between conservative tractable quadratic stability results and exact NP-hard robust stability results.

Keywords

Polynomial matrix, Parametric uncertainty, Robust stability, Quadratic stability, LMI.

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Introduction

Polynomial matrices appear as a key tool for studying systems control. Dynamics of many systems (e.g. lightly damped structures such as oil derricks or regional power system models, see [11] and references therein) are most naturally represented by polynomial matrices and polynomial matrix fraction descriptions. It is well-known that the denominator polynomial matrix in a matrix fraction description captures essential information on system stability [12, 10]. Quite naturally, the problem of checking robust stability of uncertain linear systems amounts to checking robust stability of uncertain polynomial matrices.

Polytopic uncertainty is probably the most general representation of the parametric uncertainty affecting a linear system [2, 3]. However, this generality has recently led to the somehow disappointing conclusion that most of the stability analysis problems involving polytopic uncertainty are NP-hard [4]. Following the hierarchy described in [2], we now briefly review these results. In increasing order of complexity, we can distinguish between stability of

- Interval polynomials, where each coefficient of the polynomial varies independently in a given interval. Kharitonov showed that stability of an interval family of continuous-time polynomials can be checked efficiently in polynomial time [2, 3]. Unfortunately, there is no broad generalization of Kharitonov's result to other stability regions [4].

- Polytopes of polynomials, sometimes also referred to as affine polynomial families, which are linear combinations of a set of given polynomials. These families have value sets which are convex polygons in the complex plane. The Edge Theorem and polynomial-time algorithms for checking stability of polytopes of polynomials stemmed from this nice property [2, 3].

- Multilinear polynomial families, where each coefficient of the polynomial is a multilinear function of the uncertain parameters. These families arise when considering polytopes of polynomial matrices whose entries are independent interval polynomials. Equivalently, these families arise when considering the characteristic polynomial of an interval family of constant matrices [2]. It has been shown that checking stability of multilinear polynomial families is an NP-hard problem [4].

- Polytopes of polynomial matrices, which are linear combinations of a set of given polynomial matrices. These families are sometimes also referred to as polynomic polynomial families, since coefficients of the determinant of a polynomial matrix in the polytope are multivariable polynomials [2]. As an obvious consequence of the above, checking stability of these families is also an NP-hard problem. This is precisely the problem we study in the present paper.

These negative results for polytopes of constant or polynomial matrices have naturally led to the development of conservative, yet tractable, conditions for checking robust stability. One possibility is to strenghten the standard notions of stability and allowable uncertainty intervals [11]. Another possibility is to pursue the so-called quadratic stability approach in the context of Lyapunov theory [1, 7]. It paved the way for the development of efficient stability tests based on LMI optimization [5]. The approach we pursue in this note relies on an extension of quadratic stability results.

In order to address the problem of robust stability of polynomial matrix polytopes, we combine two very recent results, namely

- a new stability condition for polynomial matrices [9], and
- a new robust stability condition for constant matrix polytopes that significantly reduces the amount of conservatism inherent to the quadratic stability approach. The condition was first proposed in [8] when studying applications of positive realness conditions to structural and robust stability. It was later on used in [14] and [15] to check stability of constant matrix polytopes.

As an outcome, the main contribution of this paper is a new sufficient condition for robust stability of polynomial matrix polytopes. The condition is expressed as a standard convex LMI feasibility problem. Moreover, it encompasses in a natural way continuous-time and discrete-time stability.

Problem Statement

Given N square complex polynomial matrices $A_1(s), \ldots, A_N(s)$ of dimension n and degree d, we define the polytope $\mathbf{P}$ as the set of all polynomial matrices

$$A(s,\lambda) = \lambda_1 A_1(s) + \cdots + \lambda_N A_N(s)$$

when real parameters $\lambda_1, \ldots, \lambda_N$ vary within the unit simplex, i.e.

$$\begin{array}{l} 0 \leq \lambda_i \leq 1, \quad i = 1, \ldots, N \\ \lambda_1 + \cdots + \lambda_N = 1. \end{array}$$

We are interested in knowing whether $\mathbf{P}$ is stable, i.e. whether all polynomial matrices $A(s,\lambda)$ are stable.

The stability regions we consider are given by

$$\mathcal{D} = \{s \in \mathbb{C} : B_{00} + B_{01}s + B_{10}s^\star + B_{11}ss^\star = \left[\begin{array}{c} 1 \\ s \end{array}\right]^\star B \left[\begin{array}{c} 1 \\ s \end{array}\right] < 0\}$$

where the star denotes transpose conjugate and

$$B = B^\star = \left[\begin{array}{cc} B_{00} & B_{01} \\ B_{10} & B_{11} \end{array}\right]$$

is a given 2×2 complex matrix. Such stability regions, referred to as first-order stability regions, were introduced in a more general setting in [9, 15] as an extension of the so-called

LMI regions [6]. They include most notably the left half-plane (continuous-time stability) and the unit disk (discrete-time stability) for the respective particular choices

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

For simplicity and without loss of generality, we assume that there is no polynomial matrix with zeros at infinity in $\mathbf{P}$. The technical reason behind this choice can be found in [9]. Zeros at infinity can be handled via a slight modification of the stability region, as described in [9].

Main Result

Define

$$\mathcal{A}_i = \begin{bmatrix} A_i^0 & A_i^1 & \cdots & A_i^d \end{bmatrix}$$

as the constant matrix associated with the polynomial matrix

$$A_i(s) = A_i^0 + sA_i^1 + \cdots + s^d A_i^d$$

of dimension n and degree d. Let $\mathcal{N}_{\mathcal{A}_i}$ denote a basis for the right null-space of matrix $\mathcal{A}_i$, i.e.

$$\mathcal{A}_i \mathcal{N}_{\mathcal{A}_i} = 0.$$

Finally, define

$$\mathcal{R} = \begin{bmatrix} I_n & & & & 0 \\ & I_n & & & 0 \\ & & \ddots & & \vdots \\ & & & I_n & 0 \\ 0 & I_n & & & \\ 0 & & I_n & & \\ \vdots & & & \ddots & \\ 0 & & & & I_n \end{bmatrix}$$

as a projection matrix of size $2dn \times (d+1)n$ and let $\otimes$ stand for the Kronecker product. With these notations, we can state our main result.

Theorem 1 *Polynomial matrix polytope* $\mathbf{P}$ *is robustly stable if there exists some matrices* $P_1, \ldots, P_N, Q$ *solving the LMI feasibility problem*

$$\begin{array}{ll} \begin{bmatrix} \mathcal{R} \\ \mathcal{A}_i \end{bmatrix}^\star \begin{bmatrix} B \otimes P_i & Q \\ Q^\star & 0 \end{bmatrix} \begin{bmatrix} \mathcal{R} \\ \mathcal{A}_i \end{bmatrix} \prec 0 & \quad i = 1, \ldots, N. \\ P_i = P_i^\star \succ 0 & \end{array} \qquad (1)$$

In the above inequalities, each matrix P_i has size $dn \times dn$ and matrix Q has size $2dn \times n$. In order to prove this theorem, we need to recall the following lemma [9] on stability of a given polynomial matrix $A_i(s)$. For simplicity and without loss of generality, it is assumed that $A_i(s)$ has no zero at infinity.

Lemma 1 *Polynomial matrix $A_i(s)$ is stable if and only if there exists a matrix P_i solving the LMI feasibility problem*

$$\begin{array}{l} \mathcal{N}^\star_{\mathcal{A}_i}\mathcal{R}^\star(B \otimes P_i)\mathcal{R}\mathcal{N}_{\mathcal{A}_i} \prec 0 \\ P_i = P_i^\star \succ 0. \end{array} \tag{2}$$

Proof of Lemma 1: The complete proof is rather involved and not reproduced here for conciseness. It can be found in [9]. The main lines of the proof can be sketched as follows. First of all, the problem of checking stability of a polynomial matrix is formulated as a quadratic optimization problem. Second, an equivalent LMI optimization problem with a non-convex rank constraint is derived using LFRs. Third, it is shown that the rank constraint is irrelevant. Fourth and last, the LMI formulation (2) is obtained via standard semi-definite programming duality arguments. □

Proof of Theorem 1: The key idea, originally proposed in [8], is to build a stability condition simultaneously linear in matrices P_i and $\mathcal{A}_i$. This is done with the help of an arbitrary matrix Q providing extra degrees of freedom. To see this, note that condition (1) can equivalently be written

$$\mathcal{R}^\star(B \otimes P_i)\mathcal{R} + \mathcal{A}_i^\star Q^\star \mathcal{R} + \mathcal{R}^\star Q \mathcal{A}_i \prec 0. \tag{3}$$

Since both matrices P_i and $\mathcal{A}_i$ appear linearly in the above inequality, we can write

$$\sum_{i=1}^{N} \lambda_i \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}_i \end{array}\right]^\star \left[\begin{array}{cc} B \otimes P_i & Q \\ Q^\star & 0 \end{array}\right] \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}_i \end{array}\right] = \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}(\lambda) \end{array}\right]^\star \left[\begin{array}{cc} B \otimes P(\lambda) & Q \\ Q^\star & 0 \end{array}\right] \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}(\lambda) \end{array}\right] \prec 0 \tag{4}$$

where

$$P(\lambda) = P^\star(\lambda) = \lambda_1 P_1 + \cdots + \lambda_N P_N \succ 0$$

and

$$\mathcal{A}(\lambda) = \lambda_1 \mathcal{A}_1 + \cdots + \lambda_N \mathcal{A}_N$$

for any real parameters $\lambda_1, \cdots, \lambda_N$ in the unit simplex. From relation (4) it follows that

$$\mathcal{N}^\star_{\mathcal{A}(\lambda)} \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}(\lambda) \end{array}\right]^\star \left[\begin{array}{cc} B \otimes P(\lambda) & Q \\ Q^\star & 0 \end{array}\right] \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}(\lambda) \end{array}\right] \mathcal{N}_{\mathcal{A}(\lambda)} = \mathcal{N}^\star_{\mathcal{A}(\lambda)}\mathcal{R}^\star(B \otimes P(\lambda))\mathcal{R}\mathcal{N}_{\mathcal{A}(\lambda)} \prec 0.$$

In virtue of Lemma 1, this means that polynomial matrix

$$A(s,\lambda) = \lambda_1 A_1(s) + \cdots + \lambda_N A_N(s)$$

is stable for all admissible values of parameters $\lambda_1, \ldots, \lambda_N$. In other words, polynomial matrix polytope $\mathbf{P}$ is robustly stable. □

At this point, it is important to emphasize the role played by matrix Q. As can be seen from stability condition (1), matrix Q allows simultaneous linear dependence in matrices A_i and P_i, hence in parameter λ in inequality (4). This linear dependence is destroyed when working directly with stability condition (2).

If matrix P is not allowed to vary with parameter λ, we obtain the following corollary to Theorem 1.

Corollary 1 *Polynomial matrix polytope* $\mathbf{P}$ *is quadratically stable if and only if there exists some matrices* P,Q *solving the LMI feasibility problem*

$$\begin{array}{l} \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}_i \end{array}\right]^\star \left[\begin{array}{cc} B \otimes P & Q \\ Q^\star & 0 \end{array}\right] \left[\begin{array}{c} \mathcal{R} \\ \mathcal{A}_i \end{array}\right] \prec 0, \quad i = 1,\ldots,N \\ P = P^\star \succ 0. \end{array} \tag{5}$$

Proof of Corollary 1: Necessity is obvious since if quadratic stability holds within the whole polytope $\mathbf{P}$, it must hold at each vertex $\mathcal{A}_i$. Sufficiency follows along the same lines as in the proof of Theorem 1. $\square$

Clearly, quadratic stability condition (5) implies robust stability condition (1), just choose $P_1 = \cdots = P_N = P$ as a common matrix shared by all the elements $A(s,\lambda)$ of polytope $\mathbf{P}$. Therefore, condition (1) is less conservative than condition (5) for checking robust stability of $\mathbf{P}$.

Constant Matrices

In the particular case that

$$A_i(s) = A_i - sI_n,$$

Theorem 1 reduces to Theorem 4 in [15] and becomes a sufficient condition for robust stability of a polytope of constant matrices. Indeed, the zeros of pencil matrix $A_i(s)$ coincide with the eigenvalues of constant matrix A_i.

For example, in the case of continuous-time stability, condition (1) becomes

$$\left[\begin{array}{cc} I_n & 0 \\ 0 & I_n \\ A_i & -I_n \end{array}\right]^\star \left[\begin{array}{ccc} 0 & P_i & F \\ P_i & 0 & G \\ F^\star & G^\star & 0 \end{array}\right] \left[\begin{array}{cc} I_n & 0 \\ 0 & I_n \\ A_i & -I_n \end{array}\right] = \left[\begin{array}{cc} FA_i + A_i^\star F^\star & P_i + A_i^\star G^\star - F \\ P_i + GA_i - F^\star & -G - G^\star \end{array}\right] \prec 0$$

where matrix

$$Q = \left[\begin{array}{c} F \\ G \end{array}\right]$$

captures the additional degrees of freedom and allows for simultaneous linear dependence in matrices P_i and A_i. A basis for the right null-space of matrix

$$\mathcal{A}_i = \left[\begin{array}{cc} A_i & -I_n \end{array}\right]$$

is then obviously

$$\mathcal{N}_{\mathcal{A}_i} = \begin{bmatrix} I_n \\ A_i \end{bmatrix}$$

and condition (2) becomes

$$\begin{bmatrix} I_n \\ A_i \end{bmatrix}^\star \begin{bmatrix} I_n & 0 \\ 0 & I_n \end{bmatrix}^\star \begin{bmatrix} 0 & P_i \\ P_i & 0 \end{bmatrix} \begin{bmatrix} I_n & 0 \\ 0 & I_n \end{bmatrix} \begin{bmatrix} I_n \\ A_i \end{bmatrix} = A_i^\star P_i + P_i A_i \prec 0$$

which is the matrix inequality arising in the standard Lyapunov Theorem. As pointed out in [15], the matrix $P(\lambda)$ appearing in condition (4) is then a parameter-dependent Lyapunov matrix for the polytopic linear system

$$\dot{x} = (\lambda_1 A_1 + \cdots + \lambda_N A_N)x.$$

Numerical Experiments

Scalar Case

Numerical experiments were first carried out to evaluate the amount of conservatism of sufficient robust stability conditions (1) and (5) for scalar polynomials, i.e. $n = 1$. In this case, we can easily and exactly check robust stability of polynomial polytopes with the help of the Edge Theorem [2, 3].

We randomly generated 100 polytopes of polynomials for different values of degree d and number of vertices N. Each vertex of each polytope was chosen as a random stable monic continuous-time polynomial generated with macro `prand` of the Polynomial Toolbox for MATLAB [16]. Of course, stability of each vertex does not imply robust stability of the whole polytope. In Table 1 we reported the number of polytopes that were found robustly stable

- with quadratic stability condition (5) (columns labeled QS)
- with our new stability condition (1) (columns labeled NS)
- with the Edge Theorem (columns labeled RS).

We used the LMI Control Toolbox for MATLAB [13] for solving LMI feasibility problems (1) and (5). From the figures in Table 1, we can see that our new stability condition scarcely appeared conservative for low values of d. This is in sharp contrast with the quadratic stability condition.

d	N	QS	NS	RS	d	N	QS	NS	RS	d	N	QS	NS	RS
2	2	86	100	100	4	2	51	92	92	6	2	38	73	79
2	3	68	100	100	4	3	15	73	73	6	3	7	54	58
2	4	52	100	100	4	4	5	73	73	6	4	3	38	46
2	5	43	100	100	4	5	3	61	61	6	5	0	21	36
2	6	29	100	100	4	6	3	52	53	6	6	0	4	20
2	7	27	100	100	4	7	0	49	52	6	7	0	3	17
3	2	55	98	98	5	2	34	83	83	7	2	20	70	77
3	3	31	96	96	5	3	11	72	72	7	3	4	34	49
3	4	20	95	95	5	4	3	56	57	7	4	0	22	34
3	5	7	85	85	5	5	1	42	44	7	5	0	9	22
3	6	6	81	81	5	6	0	29	31	7	6	0	0	10
3	7	4	78	78	5	7	0	22	26	7	7	0	0	3

Table 1 - Stable polytopes with different stability criteria.

As an illustrative example of the conservatism of quadratic stability, consider the continuous-time polynomial polytope of degree $d = 3$ whose $N = 3$ vertices are given by

$$\begin{aligned} A_1(s) &= 28.3820 + 34.7667s + 8.3273s^2 + s^3 \\ A_2(s) &= 0.2985 + 1.6491s + 2.6567s^2 + s^3 \\ A_3(s) &= 4.0421 + 9.3039s + 5.5741s^2 + s^3. \end{aligned}$$

For this choice, LMI problem (5) was found infeasible. This means that the quadratic stability test does not help to conclude about robust stability of the polynomial polytope. However, the LMI problem (1) corresponding to our new stability condition was found feasible, which means that the polytope is actually robustly stable. This fact is confirmed by the Edge Theorem and the root locus of the edges of the polytopic family, represented in Figure 1.

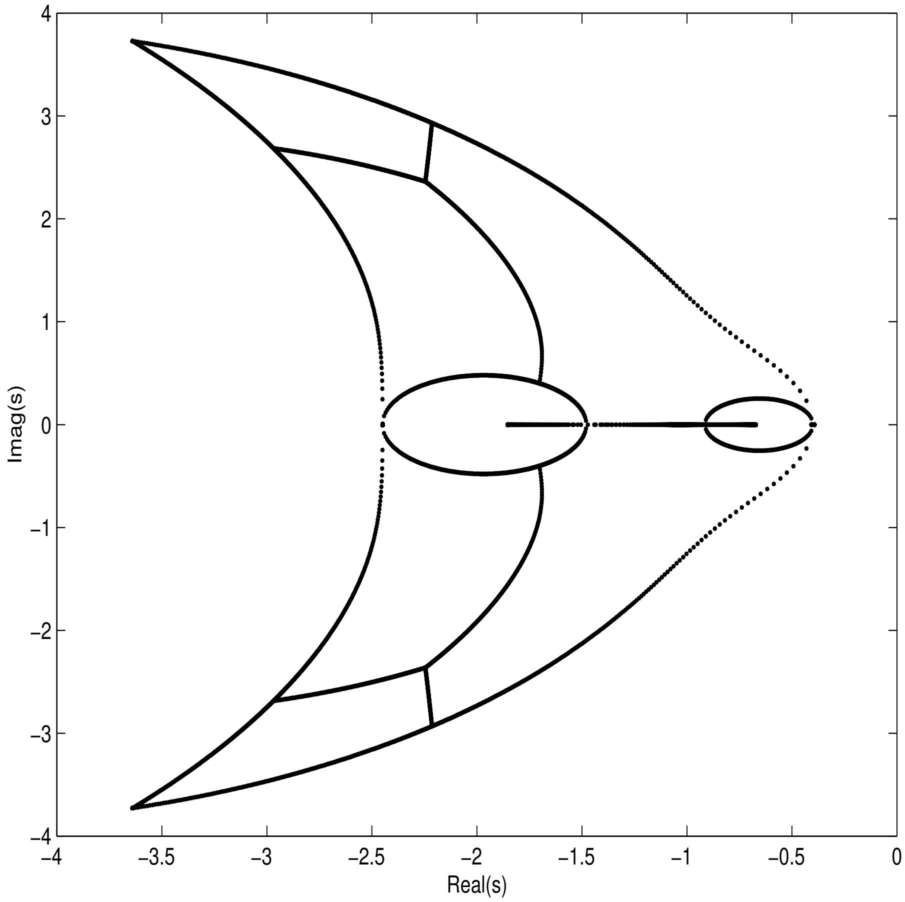

Figure 1 - Root locus of the edges of the polynomial polytope.

Matrix Case

In the matrix case ($n > 1$) there is no tractable robust stability conditions for polynomial matrix polytopes. Therefore, we only compared quadratic stability condition (5) and our new stability condition (1).

We randomly generated 100 polytopes of polynomial matrices for different values of dimension n, degree d and number of vertices N. Each vertex of each polytope was chosen as a random stable polynomial matrix generated as follows

1. Build random polynomial matrix $B(s)$ of dimension n and degree d with macro `prand` of the Polynomial Toolbox for MATLAB.

2. Extract the stable spectral factor $A(s)$ of dimension n and degree d solution to the para-Hermitian equation $A^\star(s)A(s) = B^\star(s)B(s)$. This step if performed by spectral factorization macro `spf` of the Polynomial Toolbox for MATLAB.

In Tables 2 and 3 we reported the number of polytopes that were found robustly stable with quadratic stability condition (5) (columns labeled QS) and our new stability condition (1) (columns labeled NS), in continuous-time and discrete-time settings, respectively. We can see that our technique significantly reduces the conservatism inherent to the quadratic stability approach.

n	d	N	QS	NS
2	2	2	60	95
2	2	3	34	87
2	3	2	43	97
2	3	3	24	83
3	2	2	44	86
3	2	3	11	70
3	3	2	33	91
3	3	3	3	61

Table 2 - Stable continuous-time polynomial matrix polytopes.

n	d	N	QS	NS
2	2	2	40	96
2	2	3	12	92
2	3	2	21	98
2	3	3	3	81
3	2	2	14	93
3	2	3	0	73
3	3	2	9	84
3	3	3	0	78

Table 3 - Stable discrete-time polynomial matrix polytopes.

As an illustrative example of the conservatism of quadratic stability, consider the discrete-time polynomial matrix polytope of dimension $n = 3$ and degree $d = 3$ whose $N = 2$ vertices are given by

$$\begin{bmatrix} -0.1+1.2s+0.25s^2+1.5s^3 & -0.11+0.022s+1.2s^2+1.3s^3 & -0.37-0.79s-0.73s^2+0.27s^3 \\ 0.28-0.74s+s^2 & 0.3-1.3s+0.47s^2+2.3s^3 & 0.76+1.5s-1.6s^2+1.1s^3 \\ -0.47-0.11s+1.2s^2 & 0.05+0.099s+0.28s^2 & 0.0035-0.38s-0.22s^2+2.5s^3 \end{bmatrix}$$

and

$$\begin{bmatrix} -0.096-1.8s-1.3s^2+2s^3 & -0.67+0.0078s+1.3s^2-0.37s^3 & -0.027+0.94s-0.45s^2-2s^3 \\ -0.45+1.4s-0.98s^2 & 0.34-0.75s+0.035s^2+1.9s^3 & -0.38-2s-0.55s^2-1.1s^3 \\ 0.71+1.7s+0.6s^2 & 0.15-0.11s-0.13s^2 & -0.51-s+0.19s^2+s^3 \end{bmatrix}.$$

For this choice, LMI problem (5) was found infeasible. Hence quadratic stability does not help to conclude about robust stability of the polytope. In contrast, LMI problem (1) was found feasible, which means that the polytope is actually robustly stable. For illustration, resolution of LMI stability condition (1) required 2.71 seconds and 1258 flops with MATLAB 5.2 running on a Sun Ultra 5 workstation. The root locus of the polytope, obtained by a straightforward sweeping, is represented in Figure 2.

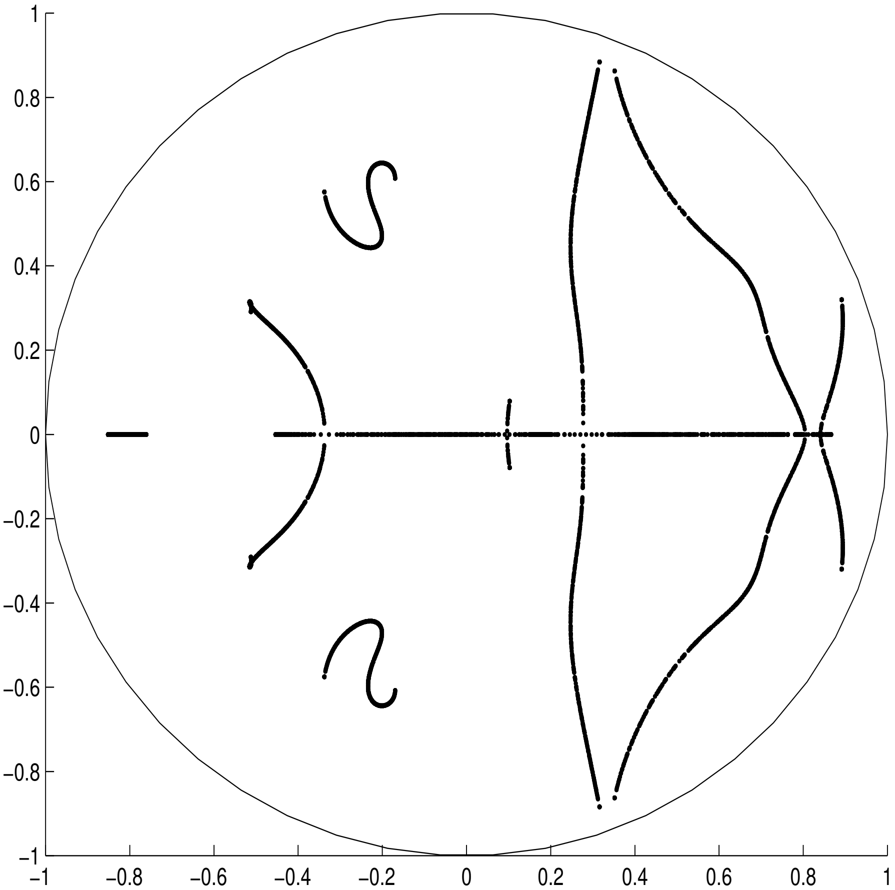

Figure 2 - Root locus of the polynomial matrix polytope.

Finally, we illustrate the fact that our condition can also be used to assess stability of interval polynomial matrices. Consider the continuous-time interval polynomial matrix

$$\left[\begin{array}{cc} [7.7-2.3s+4.3s^2, 3.7+2.7s+4.3s^2] & [-3.1-6s-2.2s^2, -4.1-7s-2.2s^2] \\ 3.6+6.4s+4.3s^2 & [3.2+11s+8.2s^2, 16+12s+8.2s^2] \end{array}\right].$$

Associated to the above interval polynomial matrix is a polynomial matrix polytope with $2^3 = 8$ vertices. LMI problem (5) was found infeasible. Hence quadratic stability does not help to conclude about robust stability of the interval polynomial matrix. In contrast, LMI problem (1) was found feasible, which means that the interval polynomial matrix is actually robustly stable. For illustration, resolution of LMI stability condition (1) required 0.71 seconds and 2749 flops with MATLAB 5.2 running on a Sun Ultra 5 workstation. The root locus of the interval polynomial matrix, obtained by a brute-force three-dimensional sweeping, is represented in Figure 3.

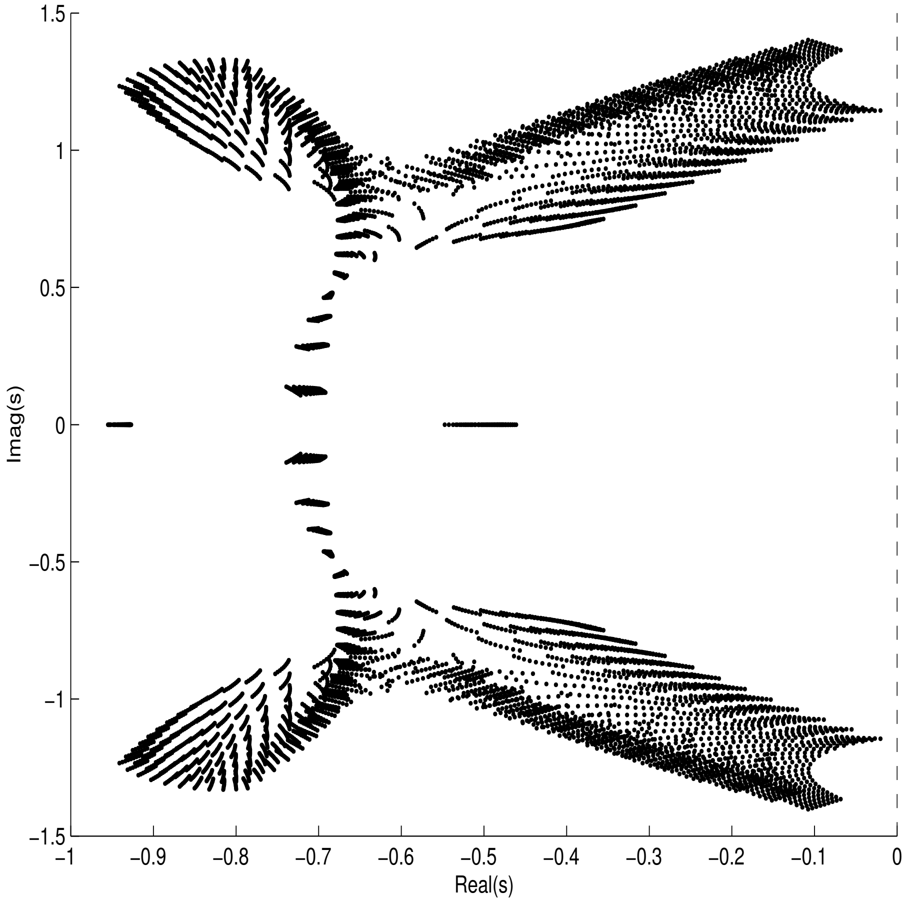

Figure 3 - Root locus of the interval polynomial matrix.

Conclusion

We have proposed a sufficient LMI condition for checking robust stability of a polytope of polynomial matrices. Numerical experiments reveal that the additional degrees of freedom provided by the technique described in [8, 14, 15] can advantageously be combined with the general methodology developed in [9] so as to reduce the amount of conservatism inherent to the quadratic stability approach.

For the sake of clarity, we only focused on first-order stability regions such as the left half-plane or the unit disk. We may however extend without difficulty these results to the more general stability regions described in [9], including arbitrary half-planes, disks or disk complements, ellipsoids, parabolas or unions thereof.

An extension of the results of [9] to the case of polynomial matrices affected by norm-bounded or block-structured uncertainty is currently under way. Similarly, the technique described in [8, 14, 15] should soon be extended to linear fractional uncertain models. A combination of these results would certainly provide additional insights into the challenging problem of robust stability analysis.

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