1 Introduction

Since the seventies and the seminal work of Kučera, a sheer volume of literature has been devoted to the polynomial (or algebraic) approach to system control, lending new insights into several fundamental analysis and design issues. In broad terms, the line of attack of polynomial methods consists of establishing the equivalence between solving control problems and associated matrix polynomial equations. It has been shown that almost all control problems at hand are amenable to this kind of treatment, and significant results have been achieved that illustrate the power and elegance of the approach. As a matter of fact, matrix equations over the ring of polynomials play a key role in the realm of control theory.

During the last two decades, major breakthroughs in numerical linear algebra and the always increasing availability of more powerful computational tools have mostly been exploited by control engineers pursing the state-space approach to system control. Perhaps due to historical reasons, numerically reliable and efficient algorithms were mainly developed in the scope of the state-space approach, in contrast with the polynomial approach, which was rather theoretically oriented or aimed at educational purposes. Actually, only very recently it has been noticed that powerful and tractable numerical routines may also be devised to deal with control problems involving polynomial matrices. This is far from being surprising since both state-space and polynomial approaches are known to be closely related.

In light of this, the first objective of this thesis is to draw a state of the art of numerical techniques dealing with polynomial matrices. First, we underline the importance of polynomial matrices in control theory and propose a fairly complete survey of matrix polynomial equations arising in control problems. We shortly describe already existing algorithms without insisting so much on theoretical aspects. Different approaches are rather illustrated on practical problems arising from control theory. Numerical properties are investigated for each of these approaches and the lack of sufficiently general, reliable and efficient algorithms is underlined.

To fulfill the need for such reliable algorithms is the second objective of the thesis, and constitutes the bulk of our work. Our research has been proceeding along two thrusts: Sylvester matrices and interpolation, two popular and well-understood polynomial techniques. Our algorithms are mostly designed for overcoming numerical unstability, a problem which has plagued for long years control engineers using old fashioned numerical methods for solving matrix polynomial equations. However, numerical experiments tend to illustrate that our algorithms are also competitive with respect to computational time.

2 Relevance of polynomial matrices in control

First, we set the ground of the polynomial approach to systems control by proposing a comprehensive classification of matrix polynomial equations stemming from control problems. Some of these equations are also encountered in signal processing and filtering. We

do not only confine our attention to basic control tasks such as deadbeat controller design or LQG regulation but also mention more involved problems such as $\mathcal{H}_\infty$ optimization, MIMO model matching or non-interacting control, to mention just a few. We essentially distinguish between linear and quadratic matrix polynomial equations because it seems that there is no practical need for more general forms, at least in the realm of systems control. In this thesis, we deliberately restrict ourselves to linear equations, that is to say, to matrix equations where unknown polynomial coefficients enter linearly.

By proper manipulations, it is shown how key steps in the resolution of the most part of control problems are embodied into the resolution of the general linear matrix polynomial equation

$$A(z)X(z) = B(z) \qquad (1)$$

where $A(z)$, $B(z)$ are given polynomial matrices in the indeterminate z (in either continuous- or discrete-time setting) and $X(z)$ is a polynomial matrix to be found. General form (1) allows for accurate definitions of homogeneous solution, polynomial null-space and minimum degree solution. Most notably, it unifies all the concepts previously defined on particular instances of equation (1).

We also mention a different kind of linear equation that cannot be reduced to the above form. It arises when solving quadratic matrix polynomial equations (e.g. J-spectral factorization) via linearization schemes and reads

$$A^\star(z)X(z) + X^\star(z)A(z) = B(z) \qquad (2)$$

where the star denotes the transpose conjugate. At first glance, unified notions of homogeneous solution, polynomial null-space and minimum degree solution do not seem to carry out easily for equation (2).

2.1 Operations on polynomial matrices

When facing control problems through the polynomial approach, several preliminary steps have usually to be performed. They aim at transforming the polynomial matrix describing the system under study into a standard or canonical form. The first category includes the following operations.

- State-space realization to coprime matrix fraction description (MFD) [9, p.123].
- Non-coprime left MFD to coprime right MFD or vice-versa [6, §6.3.1] [1, p.607].
- Reduction of a polynomial matrix into row-reduced or column-reduced form [6, §6.3.2] [1, p.599].
- Extraction of a greatest common divisor of two (or more) polynomial matrices [6, §6.3.1] [1, p.592].
- Transformation of a polynomial matrix into triangular form [6, §6.3.1].

All these operations correspond to solving an homogeneous equation (1) with zero right handside matrix. Roughly speaking, they are equivalent to the extraction of the polynomial null-space of $A(z)$.

2.2 Unilateral polynomial matrix equations

Once the input polynomial matrices are in the correct form, some control problems amounts to solving a unilateral equation of the type

$$\hat{A}(z)\hat{X}(z) + \hat{B}(z)\hat{Y}(z) = \hat{C}(z) \tag{3}$$

where $\hat{A}(z)$, $\hat{B}(z)$, $\hat{C}(z)$ are given and $\hat{X}(z)$, $\hat{Y}(z)$ are to be found. This equation is referred to as a Diophatine equation and arises in the following control problems.

- Closed-loop dynamics assignment of a multivariable linear system, by state or output feedback [9, p.123, p.232].
- Deadbeat regulation and tracking [7], considered as a special case of dynamic assignment where $\hat{C}(z)$ is the identity matrix [9, §3.2].
- Extraction of a factor by polynomial division [6, §6.3.2] [1, p.604].

It must be noted that usually, one has some degree requirements on the solutions $\hat{X}(z)$, $\hat{Y}(z)$ to equation (3). For instance, static state feedback, static state observver or static output injection can be devised when looking for constant solutions $\hat{X}(z)$, $\hat{Y}(z)$[10] [9, §3.1, §4.1]. Deadbeat regulation and tracking is generated when seeking a solution of least degree in $\hat{X}(z)$, see [7]. A parametrization of solutions to (3) of prespecified degrees may also prove useful [11]. Note that, upon writing

$$A(z) = \begin{bmatrix} \hat{A}(z) & \hat{B}(z) \end{bmatrix}, \quad X(z) = \begin{bmatrix} \hat{X}(z) \\ \hat{Y}(z) \end{bmatrix}, \quad B(z) = \hat{C}(z)$$

then Diophantine equation (3) is nothing but a special form of linear matrix polynomial equation (1).

2.3 Bilateral polynomial matrix equations

Some other control problems, usually more involved than those considered above, yield two coupled bilateral polynomial equations

$$\begin{array}{rcl} \hat{A}(z)\hat{X}(z) + \hat{Z}(z)\hat{B}(z) & = & \hat{C}(z) \\ \hat{A}(z)\hat{Y}(z) + \hat{Z}(z)\hat{D}(z) & = & \hat{E}(z) \end{array} \tag{4}$$

where $\hat{A}(z)$, $\hat{B}(z)$, $\hat{C}(z)$, $\hat{D}(z)$, $\hat{E}(z)$ are given and three unknowns $\hat{X}(z)$, $\hat{Y}(z)$, $\hat{Z}(z)$ must be determined. The above equations arise in the following control problems.

- Linear Quadratic Gaussian (LQG) regulation or prediction problems with state or output feedback [9, §3.3, §4.3, §5.3].
- Stationary LQG control problem for descriptor systems [8].
- Standard $\mathcal{H}_2$ control problem [3].
- LQG regulation based on output feedback and disturbance measurement feedforward. Then two independent couples of polynomial equations (4) are derived which yield the feedback and the feedforward parts of the regulator [14, 2].

Similarly to the Diophantine equation (3), degree constraints must be imposed on the solutions $\hat{X}(z)$, $\hat{Y}(z)$, $\hat{Z}(z)$ depending on the nature of the control design. For instance, static feedback implies that $\hat{X}(z)$ and $\hat{Y}(z)$ must be constant. Note also that there is usually some degree constraints on the solution $\hat{Z}(z)$, see [5]. Finally, it is important to underline the fact that coefficients in equations (4) are obtained by solving a spectral factorization problem, a special type of quadratic matrix polynomial equation.

Let $\otimes$ stand for the Kronecker product, vec$A$ for the column vector obtained by stacking the columns of matrix A and $\mathbf{I}$ for the identity matrix. These operators are such that for arbitrary matrices F, G, H of compatible dimensions, vec$FGH = (H' \otimes F)$vec$G$, see [12]. Upon denoting

$$A(z) = \begin{bmatrix} \mathbf{I} \otimes \hat{A}(z) & \mathbf{0} & \hat{B}'(z) \otimes \mathbf{I} \\ \mathbf{0} & \mathbf{I} \otimes \hat{A}(z) & \hat{D}'(z) \otimes \mathbf{I} \end{bmatrix}, \quad X(z) = \begin{bmatrix} \mathrm{vec}\hat{X}(z) \\ \mathrm{vec}\hat{Y}(z) \\ \mathrm{vec}\hat{Z}(z) \end{bmatrix}, \quad B(z) = \begin{bmatrix} \mathrm{vec}\hat{C}(z) \\ \mathrm{vec}\hat{E}(z) \end{bmatrix}, \tag{5}$$

it can be shown that coupled bilateral equations (4) may be considered as a special form of linear matrix polynomial equation (1).

2.4 Symmetric polynomial matrix equations

In control problems, there is another type of linear matrix polynomial equations that, surprisingly and in constrast to the equations described in the previous sections, cannot be reduced to equation (1). It reads

$$A^*(z)X(z) + X^*(z)A(z) = B(z) \tag{6}$$

where the star denotes the conjugate transpose.

Polynomial matrix spectral factorization, a standard technique applied in many $\mathcal{H}_2$ control and filtering problems –such as LQ optimal regulation, Kalman filtering, mininum variance LQG control – is tradtionally handled by a Newton-Raphson iterative scheme consisting of solving symmetric polynomial matrix equation (6) in every step [4]. Next to its control application, equation (6) is often encountered in the field of signal processing. There it is namely used in the polynomial approach to computation of an ARMA covariance function [15].

2.5 Other polynomial matrix equations

Finally, me mention other types of linear polynomial matrix equations that are integral part of controller synthesis for linear systems. They stem from the general equation

$$\hat{A}(z)\hat{X}(z)\hat{B}(z) + \hat{C}(z)\hat{Y}(z)\hat{D}(z) = \hat{E}(z) \tag{7}$$

where $\hat{A}(z)$, $\hat{B}(z)$, $\hat{C}(z)$, $\hat{D}(z)$, $\hat{E}(z)$ are given rational matrices and $\hat{X}(z)$, $\hat{Y}(z)$ are rational matrices to be found. Note that, by appropriate manipulations including least common multiple extraction and use of Kronecker product, rational matrix equation (7) can be written as a linear matrix polynomial equation of the type (1). In [13], several instances of equation (7) are enumerated.

- $\hat{A}(z)\hat{X}(z)\hat{B}(z) = \hat{C}(z)$, in disturbance decoupling problems [13, p.79], non-interacting control problems [13, p.163] and model-matching [13, p.97].
- $\hat{A}(z)\hat{X}(z)+\hat{Y}(z)\hat{C}(z) = \hat{D}$ (skew primeness equation) in regulator problems (asymptotic tracking of a constant signal or asymptotic rejection of a disturbance signal) [13, p.102] and non-interacting control problems [13, p.158].
- General form (7) in regulator problems with measurement feedback [13, p.108].

3 Existing numerical methods for polynomial matrices

In this chapter, we spell out already existing numerical methods that handle polynomial matrices and linear matrix polynomial equations. They may be cast into several categories as follows. For the sake of clarity, we illustrate each method with a very simple and typical numerical example.

- Elementary polynomial operations, an extension to polynomials of the standard Euclidean division algorithm.
- Polynomial reduction methods, a sophisticated version of elementary polynomial operations.
- State-space realization methods, where state-space techniques are used as a bypass to solve polynomial equations.
- Mixed elementary polynomial equations and state-space realization methods, that combine advantages of the two approaches.
- Sylvester matrices, based upon equating coefficients of the powers of the indeterminate.
- Interpolation, based upon evaluating values taken by polynomial matrices at given points of the complex plane.

- Matrix pencil algorithms, relying upon Kronecker invariants.
- Symbolic methods, using exact computations over rational numbers.

In the thesis, the rationale behind each approach is briefly sketched. The most widespread method is undoubtely elementary polynomial operations. This is most likely due to historical reasons and also because it remains the simplest and more intuitive way of solving scalar polynomial equations.

Respective advantages and shortcomings of each method are then enumerated.

- Elementary polynomial operations and polynomial reduction techniques may sometimes return spurious results. Usually they are fairly accurate and require a low amount of memory, but unfortunately they may behave erratically.
- State-space realization methods rely on stable state-space techniques, but numerical difficulties may be encountered when passing from a representation to the other.
- Sylvester matrices and interpolation have been used satisfactorily for solving several instances of matrix polynomial equations, but no general results have been carried out so far. Moreover, both methods usually require a substantial amount of memory and this has precluded their systematic use some fifteen years ago.
- Matrix pencil algorithms seem to be promising, but have only been developed for very specific problems as yet. No unified approach seems to be available.
- Symbolic methods turn out to be of no practical use, except for problems of very little dimensions. This is due to intermediate expression swell, a combinatorial explosion of the amount of memory that is required to perform computations.

The above statements are based on results published in the open literature, but also on practical experiments. They bear out how unsatisfactory prevailing techniques can be, as far as computational efficiency and numerical reliability are concerned. They clearly point out that there is still a need of efficient and reliable algorithms for performing tasks related to polynomial matrices.

4 New numerical methods for polynomial matrices

Following these two survey chapters, the bulk of this thesis of then dedicated to describing numerical methods to deal with polynomial matrices and linear matrix polynomial equations. We keep with our main impetus, which is the development of sufficiently general methods that fulfill efficiency and reliability requirements. All the methods presented in this chapter are believed to be new and consitute the main contribution of this thesis.

So far, our research has been proceeding along two thrusts: Sylvester matrices and interpolation. As may be expected, we shall draw on many classical results since these two techniques have already been exploited for a substantial amount of time by linear algebra theoreticians. However, the novelty in our approach is in that we use these tools in a systematic fashion that simultaneously ensures numerical reliability and computational efficiency. The key observation is that both Sylvester matrix and interpolation formulations of linear matrix polynomial equations lead to the resolution of a linear system of equations. As a result, algorithms based on these techniques only hinge upon well-worked and widely available numerical linear algebra tools, in turn usually based on stable orthogonal transformations [P1].

The outline of this chapter is as follows. We describe in considerable detail how three key matrix polynomial problems have successfully been approached through interpolation and Sylvester matrix techniques:

- Rank evaluation (Section 4.1) [P5].
- Triangularization (Section 4.2) [P3, P4].
- Symmetric bilateral equation (Section 4.3) [P6, P7, P8].

For the sake of applicability of our results, in each section we illustrate on simple numerical examples how our Sylvester matrix and interpolation algorithms can be applied so as to dispose of three typical problems involving polynomial matrices.

- Conditions of existence of a solution to equation (1) by rank evaluation.
- Extraction of the greatest common divisor of two polynomial matrices by reduction into triangular form.
- Spectral factorization by resolution of symmetric equation (6).

In addition to that, we provide throughout the chapter some intensive computational results that illustrate the power of our approach with comparison to already existing methods. Numerical experiments emphasize that our resolution methods have shown the most promise so far.

The new algorithms presented in this paper have been implemented and tested experimentally in the scope of the Polynomial Toolbox for MATLAB[1] [P2, P10, P12]. A precommercial version of the Polynomial Toolbox is available free of charge through the Toolbox Web Page at URL `http://www.math.utwente.nl/polbox`.

4.1 Rank evaluation

Rank evaluation over the ring of polynomials is a fundamental component of a large number of algorithms dealing with polynomial matrices. It is generally used as a preliminary

[1] MATLAB is a trademark of The MathWorks, Inc.

step when solving linear polynomial equations. For instance, it may be used for testing relative primeness of polynomial matrices.

In this section, we propose two simple and new algorithms for polynomial matrix rank evaluation. They only hinge on real matrix rank evaluation, hence relying upon powerful and well tried numerical tools such as the singular value decomposition.

Sylvester matrix algorithm If n stands for the number of columns of a polynomial matrix $A(z)$ and $K(z)$ is a polynomial matrix whose q columns constitute a basis for the polynomial right null-space of $A(z)$, i.e.

$$A(z)K(z) = \mathbf{0}, \tag{8}$$

then clearly the rank of $A(z)$ is the number $n-q$. Thus, the rank of $A(z)$ can be evaluated upon solving matrix polynomial equation (8). The Sylvester matrix approach to solving this equation consists in selecting the degree of $K(z)$, equating the powers of z in (8) and solving an equivalent linear system of equations. The only issue is the estimation of the degree of $K(z)$, which is clearly not known in advance. To overcome the difficultly, we propose an upper bound on the degree of $K(z)$. Combined with an upper bound on the degree of the polynomial left null-space of $A(z)$ and based upon the block Toeplitz structure of Sylvester matrices, we derive an explicit formula for the rank of $A(z)$ as the difference of the ranks of two constant Sylvester matrices. An application of our result in proposed in terms of a necessary condition of existence of a solution to equation (1).

Interpolation algorithm The indeterminate z is assigned a set of complex values $z_0, z_1, z_2, \ldots$ and ranks of constant matrices $A(z_i)$ are successively evaluated. The number of interpolation points that will be required directly follows from well-known structural and topological properties of the zeros of a polynomial matrix.

4.2 Triangularization

A common theme when performing computations involving polynomials is transforming a given polynomial matrix into a simpler equivalent form while preserving key properties of the original matrix. The triangularization over the ring of polynomials is a fundamental construction in this regard.

Reduction of polynomial matrices into triangular form has crucial relevance in linear system theory. Applications include solving Diophantine equations, extracting the greatest common divisor of polynomial matrices, computing the determinant of a polynomial matrix [P9, P11], as well as obtaining a triangular basis for modules over polynomial domains.

Sylvester matrix algorithm Triangularization of a polynomial matrix $A(z)$ over the ring of polynomials is equivalent to the problem of finding a unimodular polynomial matrix $U(z)$ and a lower-triangular polynomial matrix $T(z)$ such that

$$A(z)U(z) = T(z).$$

In order to devise an algorithm for finding $U(z)$ and $T(z)$, it is first shown that triangularizing $A(z)$ over the field of rational functions amounts to triangularizing a row permuted Sylvester matrix of $A(z)$, provided one knows the degree of the reduction matrix. Since this degree is usually not known beforehand, we show that it is sufficient to have an upper-bound. The next step is then to convert the problem of triangularization over the field of rational functions to triangularization over the ring of polynomials. This is achieved through the algebraic notion of degree minimization over ideals.

Interpolation algorithm The interpolation algorithm is in many aspects similar to the Sylvester matrix algorithm. Indeed, the close relationship existing between a matrix built from interpolated values and the Sylvester matrix is used to prove that triangularizing $A(z)$ over the field of rational fractions amounts to triangularizing an interpolated matrix. Several apsects concerning interpolation are dealt with, such as the choice and the number of interpolation points. The obtention of a triangular form over the ring of polynomials then follows from a simplified version of the two-step standard algorithm for solving Vandermonde systems of equations.

Since based on efficient tools of numerical linear algebra such as the QR decomposition by Householder transformations, our algorithms are believed to be the first numerically reliable triangularization methods ever described. The advantage of using these new polynomial triangularization techniques over using the standard Euclidean division algorithm may be compared to the advantage of using orthogonal reductions over using the standard Gaussian elimination algorithm when solving linear systems of equations.

4.3 Symmetric equation

As pointed out above, the symmetric matrix polynomial equation (6) most importantly arises in the iterative linearization scheme for solving polynomial matrix spectral factorization. Next to its control applications, the equation is also often encountered in the field of signal processing. There it is namely used in the polynomial approach to computation of an ARMA covariance function.

Interpolation algorithm By taking advantage of symmetry properties, it is first shown that para-Hermitian matrix $B(z)$ in (6) can be represented by a restricted number of interpolation points. Using this fact, an interpolation formulation of equation (6) is proposed that results in a characterization of all solutions to (6) of expected column degrees. Symmetry properties are handled to significantly reduce the original problem dimension.

Sylvester matrix algorithm Some matrix manipulations are then needed to obtain a reduced Sylvester matrix formulation of the bilateral symmetric equation. In addition to provide an alternative method for solving (6), it throws light on relations with polynomial stability theory and with Hurwitz matrices. Moreover, the use of interpolation theory mixed with rank properties of the reduced Sylvester matrix will lead under some assumptions on $A(z)$ to conditions for existence of solutions to (6).

As far as the authors are aware, this appears to be the first numerically reliable solution technique for (6) ever published As another advantage, no a priori assumptions on $A(z)$ and $B(z)$ are necessary for the algorithm. Hence critical cases such as for instance the presence of purely imaginary values as roots of $A(z)$ are handled without taking special precautions. If, all the same, matrix $A(z)$ is known to be stable (this is always the case when applied for spectral factorization), the method gets even simpler as now a solution is guaranteed to exist.

Our method, essentially developed for continuous-time systems, is readily extended to the discrete-time case. In the thesis, this is only illustrated for the Sylvester matrix approach but applies for interpolation as well.

5 Conclusion

The thesis first features a comprehensive survey of linear matrix polynomial equations arising in control and existing numerical methods to solve them. The lack of sufficiently general, reliable and efficient algorithms is underlined. The remaining part of our work is then devoted to bridge the gap between existing algorithms dealing with polynomial matrices and efficient numerical linear algebra routines involving stable operations. It results in tractable new algorithms.

A distinguishing feature of our approach is the employment of Sylvester matrices and interpolation, two popular and well-understood polynomial techniques. Our algorithms are designed for overcoming numerical instability, a problem which has plagued for long years control engineers using old fashioned numerical methods for solving matrix polynomial equations.

Some possible extensions and future research directions are mentioned. Recent works on resolution of Toeplitz linear systems of equations hint of a possible method for solving matrix polynomial equations in a numerically reliable way while simultaneously taking advantage of the problem structure. It may result in a reduced computational cost. The matrix pencil approach also seems not to have sufficiently been exploited. It should be highly instructive to transcribe typical polynomial matrix problems in terms of matrix pencil problems. Moreover, in addition to providing useful numerical tools for the control community, the conception of new algorithms also usually lends insight into fundamental problems

We hope that the achievements described in this thesis will open up new avenues for the development of efficient routines dealing with polynomial matrices. Actually, there is still a dramatic need of numerically reliable algorithms for performing really basic tasks such as, for instance, the reduction of a polynomial matrix into Smith or diagonal form.

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