Motion planning

Florent Lamiraux

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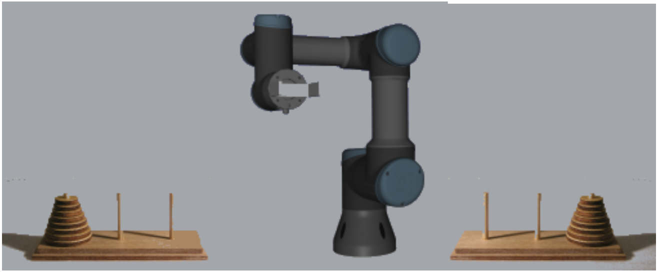
Task and Motion Planning

Definition



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- initial configurations for robots and objects
- goal configurations for robots and objects

Task and Motion planning : automatically computing a feasible trajectory between the initial and goal configurations.

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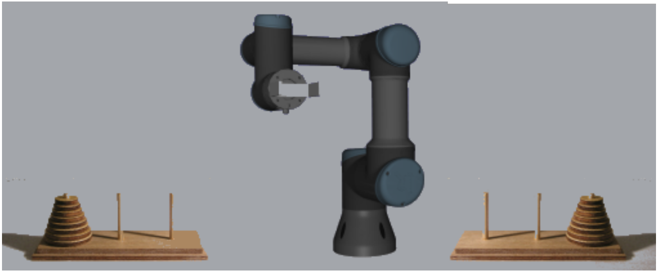

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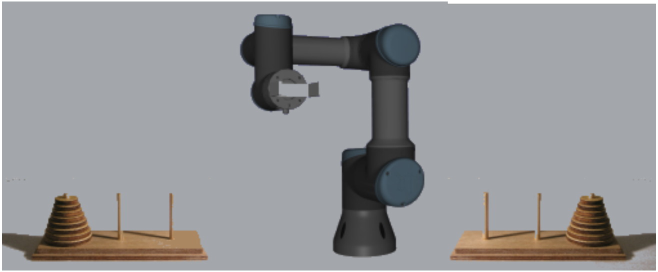

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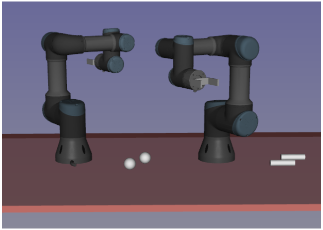
Configuration space



Configuration

$$\mathbf{q} = (\mathbf{q}_{r_1}, \mathbf{q}_{r_2}, \mathbf{q}_{c_1}, \mathbf{q}_{c_2}, \mathbf{q}_{s_1}, \mathbf{q}_{s_2})$$
$$\mathbf{q}_{r_1}, \mathbf{q}_{r_2} \in \mathbb{R}^6$$
$$\mathbf{q}_{c_1}, \mathbf{q}_{c_2}, \mathbf{q}_{s_1}, \mathbf{q}_{s_2} \in \mathbb{R}^7$$

where

$\mathbf{q}_{r_1} = (q1, \cdots, q_6)$ is the vector of joint angles,

$\mathbf{q}_{c_1} = (x, y, z, X, Y, Z, W)$

$W + Xi + Yj + Zk$is a **unit** quaternion.

The configuration space is a differential manifold.

Quaternions

Non-commutative field isomorphic to $\mathbb{R}^4$, spanned by three elements i, j, k that satisfy the following relations :

$$i^2 = j^2 = k^2 = ijk = -1$$

from which we immediately deduce

$$ij = k, \; jk = i, \; ki = j$$

Hamilton (1843)

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Unit Quaternions and rotations

Let $q = q_0 + q_1 i + q_2 j + q_3 k$ be a unit quaternion :

$$q_0^2 + q_3^2 + q_2^2 + q_3^2 = 1$$

$\forall x = (x_0, x_1, x_2) \in \mathbb{R}^3$, let $u = x_0 i + x_1 j + x_2 k$

$$q \,.\, u \,.\, q^* = y_0 i + y_1 j + y_2 k$$

where $q^* = q_0 - q_1 i - q_2 j - q_3 k$ is the conjugate of q.
$y = (y_0, y_1, y_2)$ is the image of x by the rotation of matrix

$$\begin{pmatrix} 1 - 2(q_2^2 + q_3^2) & 2q_2 q_1 - 2q_3 q_0 & 2q_3 q_1 + 2q_2 q_0 \\ 2q_2 q_1 + 2q_3 q_0 & 1 - 2(q_1^2 + q_3^2) & 2q_3 q_2 - 2q_1 q_0 \\ 2q_3 q_1 - 2q_2 q_0 & 2q_3 q_2 + 2q_1 q_0 & 1 - 2(q_1^2 + q_2^2) \end{pmatrix}$$

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Unit Quaternions and rotations

- Notice that q and $-q$ represent the same rotation

Definitions

- Workspace : $\mathcal{W} = \mathbb{R}^2$ or $\mathbb{R}^3$: space in which the robot evolves
- Obstacle in workspace : compact subset of $\mathcal{W}$, denoted by $\mathcal{O}$.
- Configuration space : $\mathcal{C}$.
- Position in configuration $\mathbf{q}$ of a point $M \in \mathcal{B}_i$: $\mathbf{x}_i(M, \mathbf{q})$.
- Obstacle in the configuration space :

$$\begin{aligned} \mathcal{C}_{obst} = \{\mathbf{q} \in \mathcal{C}, \quad & \exists i \in \{1, \cdots, m\},\ \exists M \in \mathcal{B}_i,\ \mathbf{x}_i(M, \mathbf{q}) \in \mathcal{O} \text{ or} \\ & \exists i, j \in \{1, \cdots, m\},\ \exists M_i \in \mathcal{B}_i,\ \exists M_j \in \mathcal{B}_j, \\ & \mathbf{x}_i(M_i, \mathbf{q}) = x_j(M_j, \mathbf{q})\} \end{aligned}$$

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Motion

- Configuration space :
  - differential manifold
- Motion :
  - continuous function from $[0, 1]$ to $\mathcal{C}$.
- Collision-free motion :
  - continuous function from $[0, 1]$ to $\mathcal{C}_{free}$.

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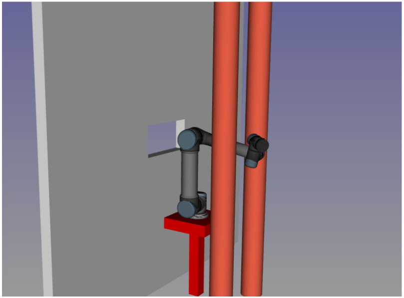
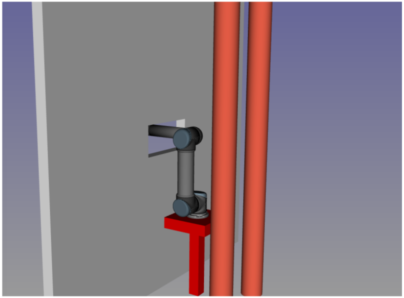
Motion planning problem



initial configuration



goal configuration

$$\mathcal{C} = \mathbb{R}^6$$

History

- Before the 1990's : mainly a mathematical problem
  - real algebraic geometry,
  - decidability : Schwartz and Sharir 1982,
    - Tarski theorem, Collins decomposition,
- from the 1990's : an algorithmic problem
  - random sampling (1993),
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Random methods

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- Principle
  - shoot random configurations
  - test whether they are in collision
  - build a graph (roadmap) the nodes of which are free configurations
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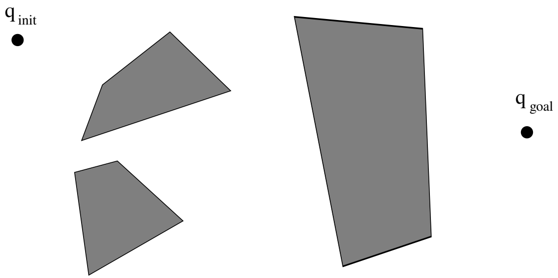

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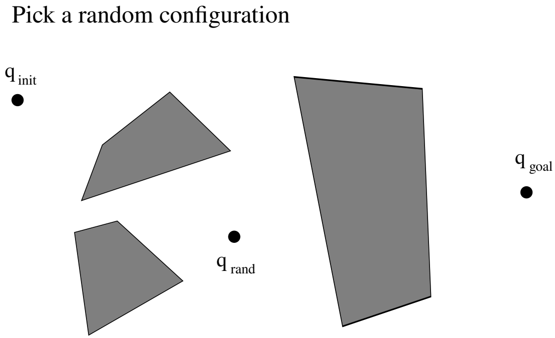

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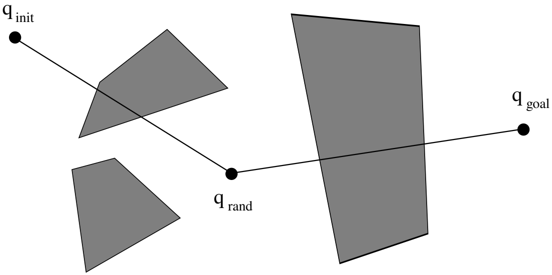
Try to connect it to the nearest nodes of each connected component



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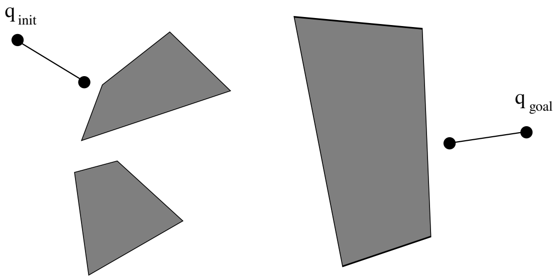
Keep collision-free parts of paths



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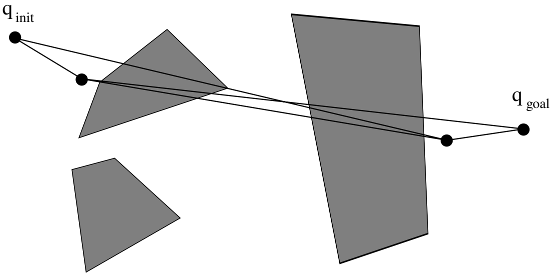
Try to connect new nodes to nearest nodes of other connected components



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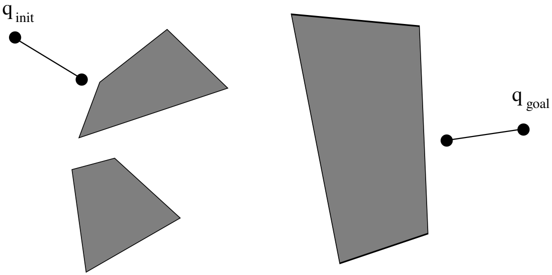

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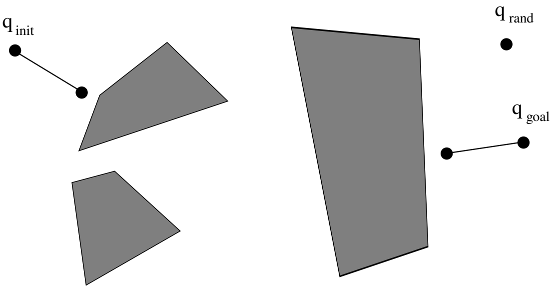

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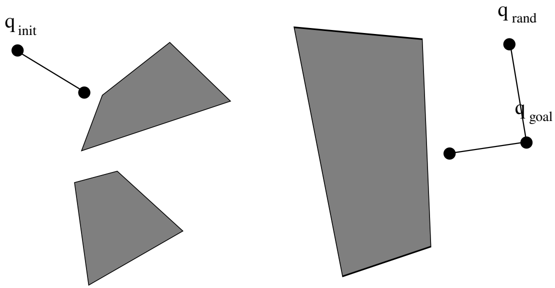

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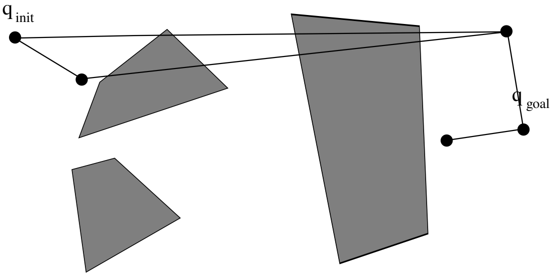

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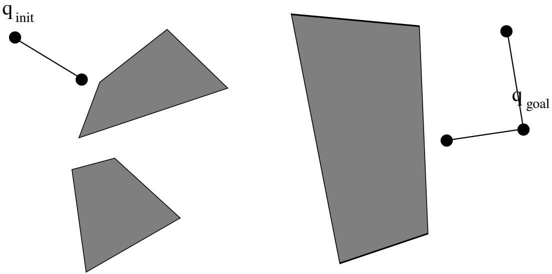

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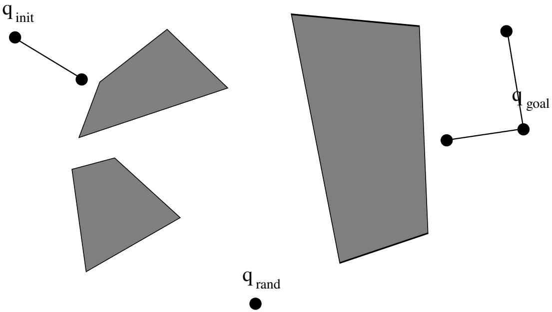

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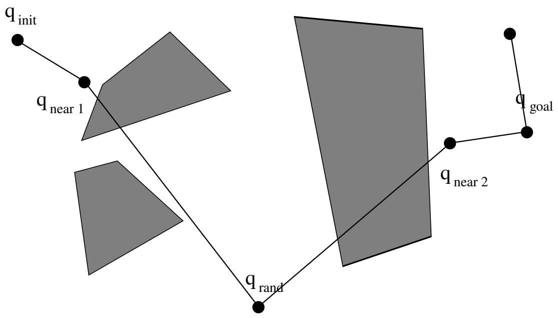

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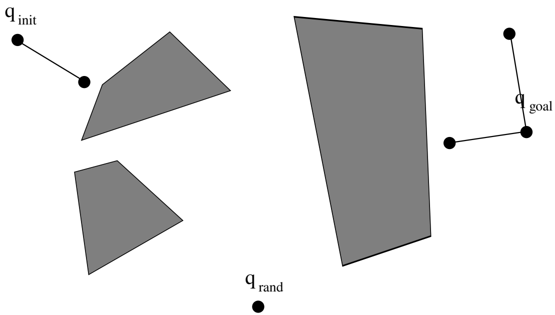

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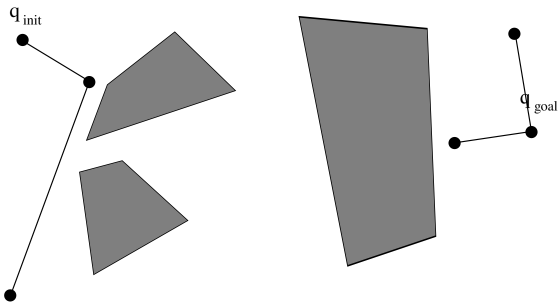

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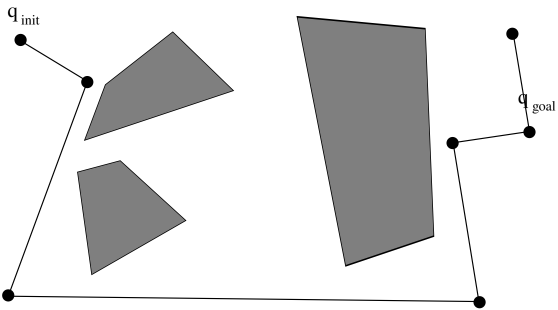

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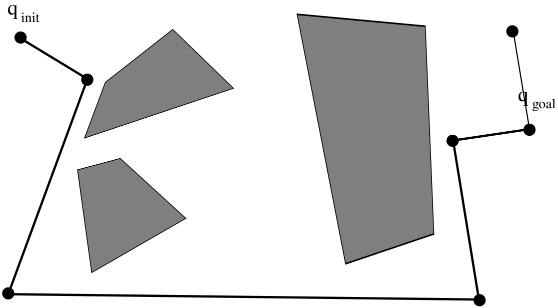

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- Pros :
  - no explicit computation of the free configuration space,
  - easy to implement,
  - robust.
- Cons :
  - no completeness property, only probabilistic completeness,
  - difficult to find narrow passages.
- Requested operators :
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- is the motion of
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Numerical constraints

Constraints to which manipulation motions are subject can be expressed numerically.

- Numerical constraints :

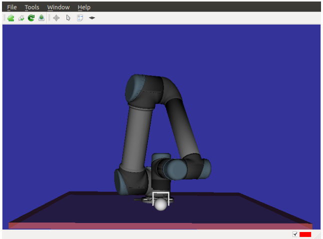
$$f(\mathbf{q}) = 0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m) \end{array}$$

  - `setConstantRightHandSide(True)`
- Parameterizable numerical constraints :

$$f(\mathbf{q}) = f_0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m) \\ f_0 \in \mathbb{R}^m \end{array}$$

  - `setConstantRightHandSide(False)`

Example : robot manipulating a ball



$$\mathcal{C} = [-\pi, \pi]^6 \times \mathbb{R}^3 \quad (1)$$

$$\mathbf{q} = (q_0, \cdots, q_5, x_b, y_b, z_b) \quad (2)$$

Two *states* :

- `placement` : the ball is lying on the table,
- `grasp` : the ball is held by the end-effector.

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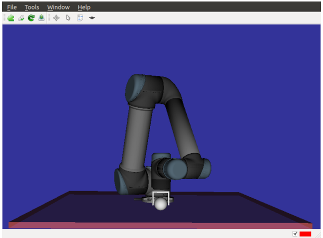

Each state is defined by a numerical constraint

- placement

$$z_b = 0$$

- grasp

$$\mathbf{x}_{gripper}(q_0, \cdots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Each state is a sub-manifold of the configuration space

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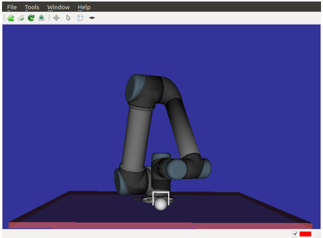

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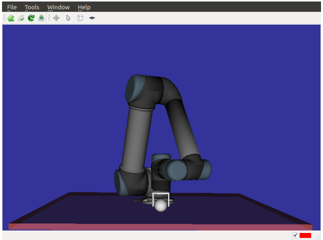
Motion constraints

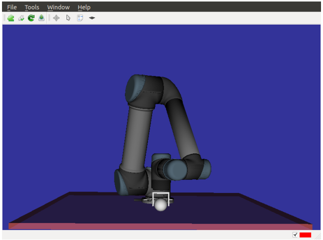


Two *types of motion* :

- `transit` : the ball is lying and **fixed** on the table,
- `transfer` : the ball moves with the end-effector.

Example : robot manipulating a ball

Motion constraints



- `transit`

$$\left.\begin{array}{rcl} x_b & = & x_0 \\ y_b & = & y_0 \end{array}\right\} \text{ parameterizable}$$

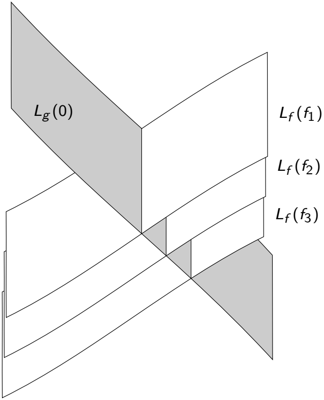
$$\left. z_b = 0 \right\} \text{ simple}$$

- `transfer`

$$\mathbf{x}_{gripper}(q_0, \cdots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Foliation

Motion constraints define a foliation of the admissible configuration space (`grasp` ∪ `placement`).



- f : position of the ball

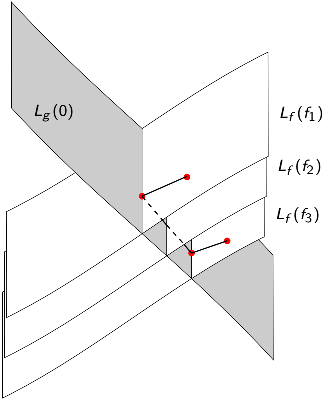
$$L_f(f_1) = \{\mathbf{q} \in \mathcal{C}, f(\mathbf{q}) = f_1\}$$

- g : grasp of the ball

$$L_g(0) = \{\mathbf{q} \in \mathcal{C}, g(\mathbf{q}) = 0\}$$

Foliation

Motion constraints define a foliation of the admissible configuration space (`grasp` $\cup$ `placement`).



Solution to a manipulation planning problem is a concatenation of *transit* and *transfer* paths.

General case

In a manipulation problem,

- the state of the system is subject to
  - numerical constraints
- trajectories of the system are subject to
  - numerical constraints
  - parameterizable numerical constraints.

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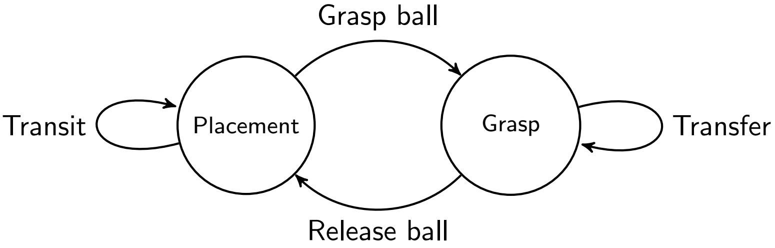
Constraint graph

A manipulation planning problem can be represented by a *manipulation graph*.

- **Nodes** or *states* are numerical constraints.
- **Edges** or *transitions* are parameterizable numerical constraints.



Projecting configuration on constraint

Newton-Raphson algorithm

- $\mathbf{q}_0$ configuration,
- $f(\mathbf{q}) = 0$ non-linear constraint,
- ϵ numerical tolerance

Projection $(\mathbf{q}_0, f)$:

$\mathbf{q} = \mathbf{q}_0$; $\alpha = 0.95$

for i from 1 to max_iter :

$$\mathbf{q} = \mathbf{q} - \alpha \left(\frac{\partial f}{\partial \mathbf{q}}(\mathbf{q}) \right)^{+} f(\mathbf{q})$$

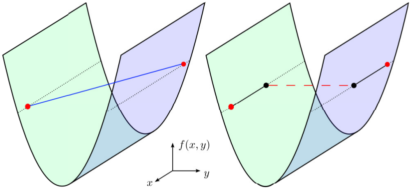
if $\|f(\mathbf{q})\| < \epsilon$: return $\mathbf{q}$

return failure

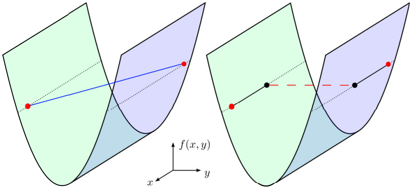
Projecting path on constraint

- *path* : mapping from $[0, 1]$ to $\mathcal{C}$
- $f(\mathbf{q}) = 0$ non-linear constraint,

Applying Newton Raphson at each point may result in a discontinuous path



Discontinuous Projection



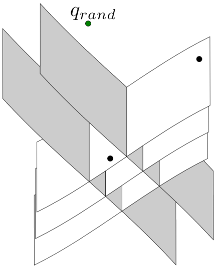
$$\mathcal{C} = \mathbb{R}^2, f(x,y) = y^2 - 1$$

$$\frac{\partial f}{\partial \mathbf{q}} = \begin{pmatrix} 0 & 2y \end{pmatrix}, \ \frac{\partial f}{\partial \mathbf{q}}^+ = \begin{pmatrix} 0 \\ \frac{1}{2y} \end{pmatrix} \text{ ou } \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$y_{i+1} = y_i + \frac{1 - y_i^2}{2y_i}$$

Algorithm

Manipulation RRT



Manipulation RRT

$\mathbf{q}_{rand}$ = shoot_random_config()

for each connected component :

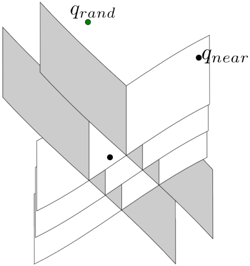
$\mathbf{q}_{near}$ = nearest_neighbor($\mathbf{q}_{rand}$, *roadmap*)
T = select_transition($\mathbf{q}_{near}$)
$\mathbf{q}_{proj}$ = generate_target_config($\mathbf{q}_{near}$, $\mathbf{q}_{rand}$, T)
$\mathbf{q}_{new}$ = extend($\mathbf{q}_{near}$, $\mathbf{q}_{proj}$, T)
roadmap.insert_node($\mathbf{q}_{new}$)
roadmap.insert_edge(T, $\mathbf{q}_{near}$, $\mathbf{q}_{new}$)
new_nodes.append ($\mathbf{q}_{new}$)

for $\mathbf{q} \in (\mathbf{q}_{new}^1, ..., \mathbf{q}_{new}^{n_{cc}})$:

connect ($\mathbf{q}$, roadmap)

Algorithm

Manipulation RRT



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Algorithm

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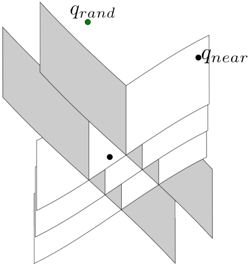

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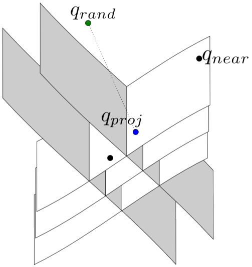
Algorithm

Manipulation RRT



Manipulation RRT

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for each connected component :

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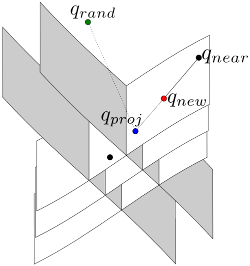
Algorithm
Manipulation RRT



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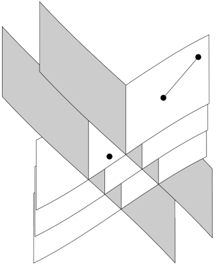
Algorithm

Manipulation RRT



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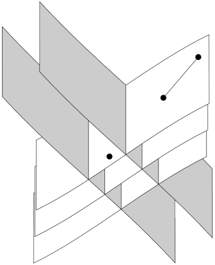
connect (q, roadmap)

Algorithm

Manipulation RRT



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Algorithm

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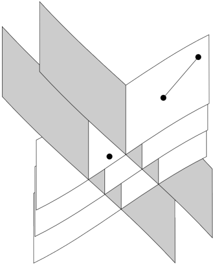

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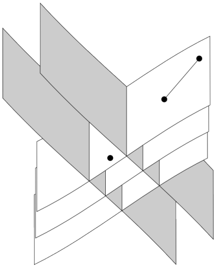

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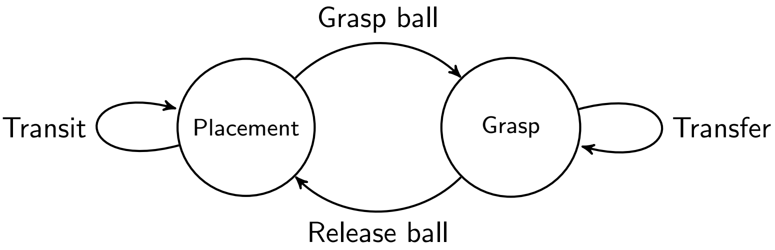
Select transition

$T = \text{select\_transition}(\mathbf{q}_{near})$

Outward transitions of each state are given a probability distribution. The transition from a state to another state is chosen by random sampling.



Generate target configuration

$\mathbf{q}_{proj} = \text{generate\_target\_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, T)$

Once transition T has been selected, $\mathbf{q}_{rand}$ is *projected* onto the destination state S_{dest} in a configuration reachable by $\mathbf{q}_{near}$.

$$f_T(\mathbf{q}_{proj}) = f_T(\mathbf{q}_{near})$$
$$f_{S_{dest}}(\mathbf{q}_{proj}) = 0$$

Extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near},\ \mathbf{q}_{proj},\ T)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on transition constraint :

- if projection successful and projected path collision free

$$\mathbf{q}_{new} \leftarrow \mathbf{q}_{proj}$$

- otherwise $(\mathbf{q}_{near}, \mathbf{q}_{new}) \leftarrow$ largest path interval tested as collision-free with successful projection.

$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}),\ f_T(\mathbf{q}) = f_T(\mathbf{q}_{near})$$

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$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}),\ f_T(\mathbf{q}) = f_T(\mathbf{q}_{near})$$

Connect

connect ($\mathbf{q}$, roadmap)

for each connected component *cc* not containing $\mathbf{q}$:

for all *n* closest config $\mathbf{q}_1$ to $\mathbf{q}$ in *cc* :

- connect ($\mathbf{q}$, $\mathbf{q}_1$)

Connect

```
connect (q_0, q_1) :
    S_0 = state (q_0)
    S_1 = state (q_1)
    T = transition (S_0, S_1)
    if T and f_T(q_0) == f_T(q_1) :
        if p = projected_path (T, q_0, q_1) collision-free :
            roadmap.insert_edge (T, q_0,q_1)
    return
```

Relative positions as numerical constraints

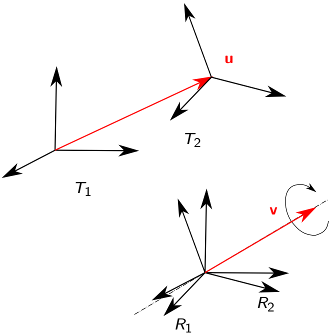
- $T_1 = T_{(R_1,t_1)} \in SE(3)$,
  $T_2 = T_{(R_2,t_2)} \in SE(3)$.
- $T_{2/1} = T_1^{-1} \circ T_2$ can be represented by a vector of dimension 6 :

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

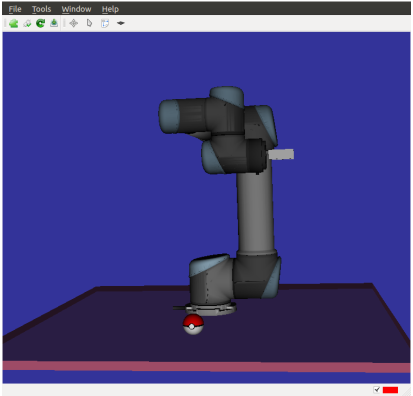
  where

$$\mathbf{u} = R_1^T(t_2 - t_1)$$

  $R_1^T R_2$ matrix of the rotation around axis $\mathbf{v}/\|\mathbf{v}\|$ and of angles $\|\mathbf{v}\|$.



A few words about the BE



- `script/grasp_ball.py`