

ContainerMinMaxGD: a toolbox for $(\min, +)$ -linear systems

WoNeCa'12

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Motivations

Why an efficient toolbox ?

- Existing tools: MinMaxGD¹, COINC², DISCO³, ...
- Elementary operations of Network Calculus (\oplus , $*$ and \star) time costly and memory consuming
- Poorly managed development of transient behavior

¹[Cottenceau *et al*, 2000] Data processing tool for calculation in diod. WODES'00

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³[Schmitt and Zdarsky, 2006] The disco network calculator: a toolbox for worst case analysis. ValueTools'06

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Objective

Approximations of functions in order to obtain computation algorithms more efficient

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Outlines

1 ContainerMinMaxGD toolbox

- Container definition
- Container operations

2 Tests and application

- Pessimism of computations and gain in memory consumption
- Example of application

3 Conclusions and perspectives

Legendre-Fenchel transform^{4,5} \mathcal{L}

Non-injective application $\mathcal{L} : (\mathcal{F}, \oplus, *) \mapsto (\mathcal{F}_{acx}, \oplus, +)$

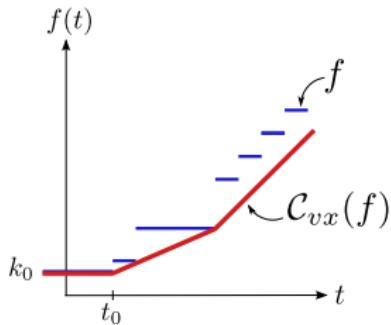
\mathcal{F}_{acx} : set of convex functions

⁴ [Fidler et Recker, 2006] Conjugate network calculus... Elsevier

⁵ [Baccelli et al, 1992, Th 3.38] Synchronization and linearity... Wiley and sons

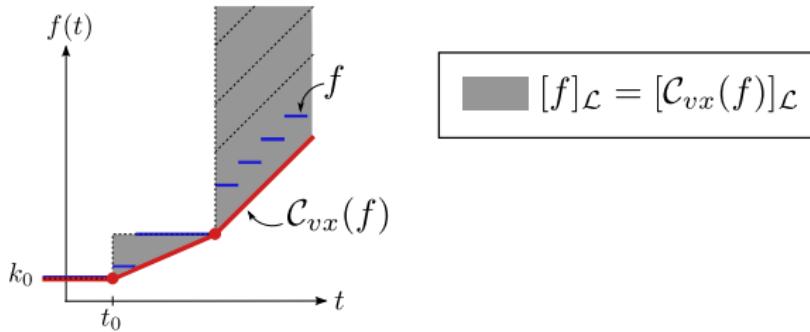
Legendre-Fenchel transform^{4,5} \mathcal{L} Non-injective application $\mathcal{L} : (\mathcal{F}, \oplus, *) \mapsto (\mathcal{F}_{acx}, \oplus, +)$

$$\mathcal{L}(f) = \mathcal{L}(g) \Leftrightarrow \mathcal{C}_{vx}(f) = \mathcal{C}_{vx}(g)$$

 \mathcal{F}_{acx} : set of convex functions $\mathcal{C}_{vx}(\cdot) \in \mathcal{F}_{acx}$: convex hull⁴ [Fidler et Recker, 2006] Conjugate network calculus... Elsevier⁵ [Baccelli et al, 1992, Th 3.38] Synchronization and linearity... Wiley and sons

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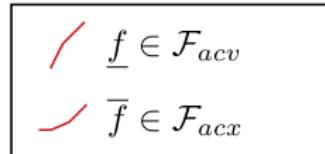
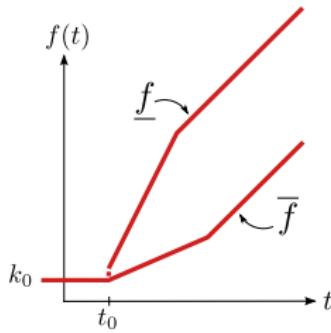
$$\mathcal{L}(f) = \mathcal{L}(g) \Leftrightarrow \mathcal{C}_{vx}(f) = \mathcal{C}_{vx}(g) \Leftrightarrow [f]_{\mathcal{L}} = [g]_{\mathcal{L}}$$

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Definition of a container $\mathbf{f} \in \mathcal{F}$

$$\mathbf{f} = [\underline{f}, \bar{f}]_{\mathcal{L}} \quad \text{with} \quad \begin{cases} \underline{f} \in \mathcal{F}_{acv} \\ \bar{f} \in \mathcal{F}_{acx} \\ \sigma(\underline{f}) = \sigma(\bar{f}) \end{cases}$$

$\sigma(\underline{f})$ and $\sigma(\bar{f})$: asymptotic slopes of \underline{f} and \bar{f}

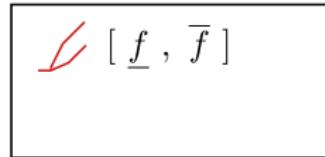
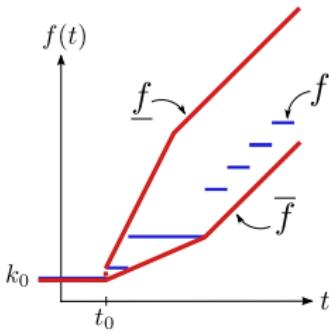


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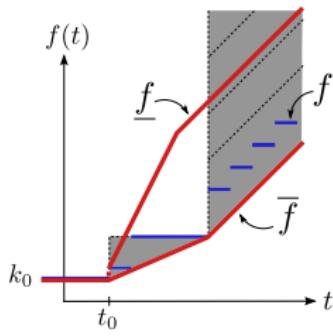
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$[\underline{f}, \bar{f}]$
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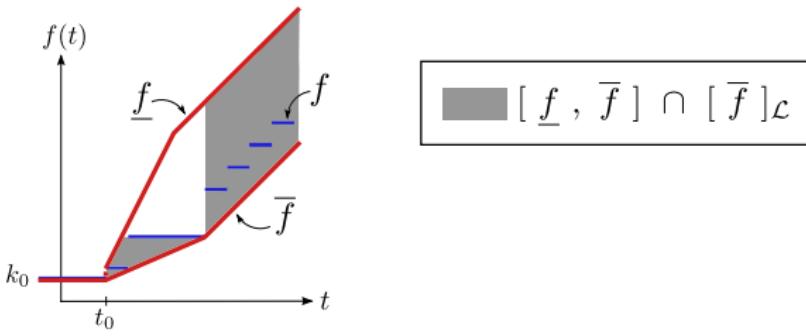
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Operations defined on set \mathbf{F} of containers

Operations

$$\diamond \in \{ \oplus, *, ^* \}$$

Operations defined on set \mathbf{F} of containers

Operations

$$[\circ] \in \{ [\oplus], [*], [+] \}$$

defined such that

$$\text{for } \mathbf{f} = [\underline{f}, \bar{f}]_{\mathcal{L}} \in \mathbf{F}, \mathbf{g} = [\underline{g}, \bar{g}]_{\mathcal{L}} \in \mathbf{F}$$

$$\left\{ \begin{array}{l} \mathbf{f}[\circ]\mathbf{g} \in \mathbf{F} \end{array} \right.$$

Operations defined on set \mathbf{F} of containers

Operations: **inclusion functions**

$$[\circ] \in \{ [\oplus], [*], [+] \}$$

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for $\mathbf{f} = [\underline{f}, \bar{f}]_{\mathcal{L}} \in \mathbf{F}$, $\mathbf{g} = [\underline{g}, \bar{g}]_{\mathcal{L}} \in \mathbf{F}$
and $\forall f \in \mathbf{f}, \forall g \in \mathbf{g}$

$$\begin{cases} \mathbf{f}[\circ]\mathbf{g} \in \mathbf{F} \\ f \circ g \in \mathbf{f}[\circ]\mathbf{g} \end{cases}$$

Definitions of inclusion functions

$$\mathbf{f}[\ast]\mathbf{g} \triangleq [\underline{f} * \underline{g}, \bar{f} * \bar{g}]_{\mathcal{L}} \quad \text{Complexity in } \mathcal{O}(n)$$

$$\mathbf{f}[\oplus]\mathbf{g} \triangleq [\mathcal{C}_{cv}(\underline{f} \oplus \underline{g}), \mathcal{C}_{vx}(\bar{f} \oplus \bar{g})]_{\mathcal{L}} \quad \text{Complexity in } \mathcal{O}(n)$$

$\mathcal{C}_{cv}(\cdot) \in \mathcal{F}_{acv}$: concave approximation

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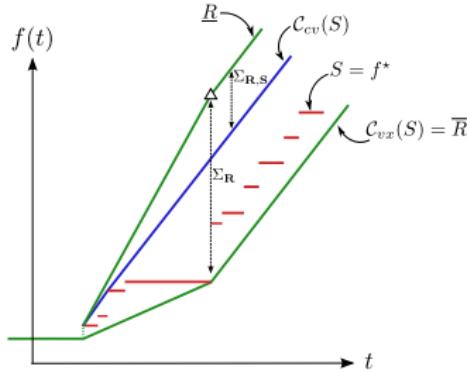
$$\begin{aligned}\mathbf{f}[*]\mathbf{g} &\triangleq [\underline{f} * \underline{g}, \bar{f} * \bar{g}]_{\mathcal{L}} && \text{Complexity in } \mathcal{O}(n) \\ \mathbf{f}[\oplus]\mathbf{g} &\triangleq [\mathcal{C}_{cv}(\underline{f} \oplus \underline{g}), \mathcal{C}_{vx}(\bar{f} \oplus \bar{g})]_{\mathcal{L}} && \text{Complexity in } \mathcal{O}(n) \\ \mathbf{f}^{[\star]} &\triangleq [\underline{\mathbf{f}}^{[\star]}, \mathcal{C}_{vx}(\bar{f}^{[\star]})]_{\mathcal{L}} && \text{Complexity in } \mathcal{O}(n \log n)\end{aligned}$$

$\mathcal{C}_{cv}(\cdot) \in \mathcal{F}_{acv}$: concave approximation

$$\begin{aligned}\text{with } \underline{\mathbf{f}}^{[\star]} &\triangleq \bigoplus_{i=0}^n \mathcal{C}_{cv}(\Delta_{t_i}^{k_i \star}) \\ &\quad \oplus \mathcal{C}_{cv} \left(e \oplus \Delta_{\tau_f}^{\kappa_f} * (\mathcal{C}_{cv}(\Delta_{\tau_f}^{\kappa_f \star}) \oplus \Gamma_f) \right)\end{aligned}$$

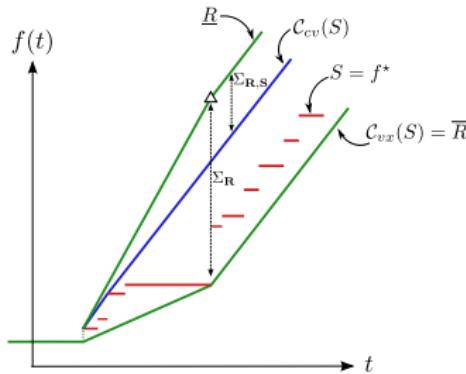
Context

- Exact system $S = f^*$ and its container $\mathbf{S} = [\mathcal{C}_{cv}(S), \mathcal{C}_{vx}(S)]_{\mathcal{L}} \in \mathbf{F}$
- Approximated system $\mathbf{R} = f^{[\star]} \in \mathbf{F}$



Context

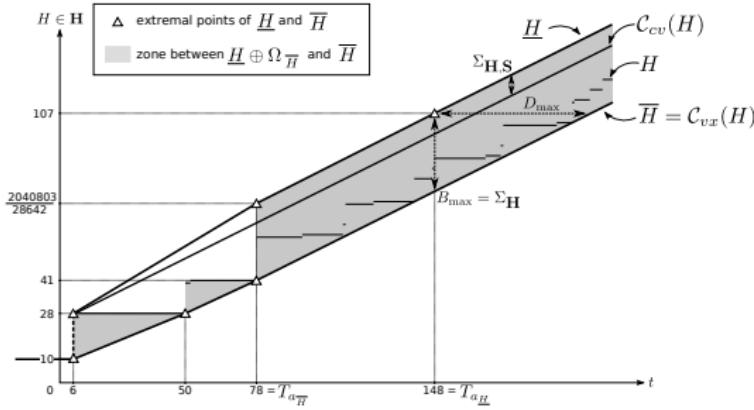
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Test on random square matrices (from 2×2 to 60×60)

- Pessimism of computations: $\Sigma_{R,S}/\Sigma_R = 27\%$
- Ratio of memory consumption saved: $1 - N_R/N_S = 71\%$

Context

- Deterministic $(\min, +)$ -linear system : $H = C * A^* * B$
- Computation of the container: $\mathbf{H} = [C[*]A^{[*]}[*]B]_{\mathcal{L}} \in \mathbf{F}$
with $[\underline{H}]_{\mathcal{L}} = [H]_{\mathcal{L}}$
- Pessimism of computations: 25,8%
- Ratio of memory consumption saved: 91, 25%



What have we done?

Toolbox with computations that are

- Approximated (inclusion functions)
- Efficient (linear and quasi-linear complexities)

What can be done?

- To complete the inclusion functions $[+]$, $[-]$, $[\setminus]$

Thank you for your attention...