ContainerMinMaxGD: a Toolbox for (Min,+)-Linear Systems

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1 Introduction

According to the theory of Network Calculus based on the (min,+) algebra (see [2] and [5]), analysis and measure of worst-case performance in communication networks can be made easily and several toolboxes such as COINC [1] or DISCO [6] offer to do it. However, the exact computations – sum, inf-convolution, subadditive closure – of such systems are often memory consuming and time costly (see [1] and [4]). That is why we developed a toolbox called ContainerMinMaxGD which handles some "container" of ultimately pseudo-periodic functions and makes approximated computations. The convexity properties of the bounds of a container provide efficient algorithms (linear and quasi-linear complexity) for sum, inf-convolution and subadditive closure.

The ContainerMinMaxGD toolbox[1] is a set of C++ classes which can be found at the following address: http://www.istia.univ-angers.fr/~euriell.lecorronc/Recherche/softwares.php.

2 ContainerMinMaxGD Toolbox

The elementary object handled by the toolbox is called a container and defined as the following intersection illustrated by the grey zone of Fig. 1:

$$[\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}} \triangleq [\ \underline{f}\ ,\ \overline{f}\ ]\ \cap\ [\ \overline{f}\ ]_{\mathcal{L}},$$

where $[\ \underline{f}\ ,\ \overline{f}\ ]$ is an interval of functions and $[\ \overline{f}\ ]_{\mathcal{L}}$ is the equivalence class of $\overline{f}$ modulo the Legendre-Fenchel transform[2] $\mathcal{L}$.

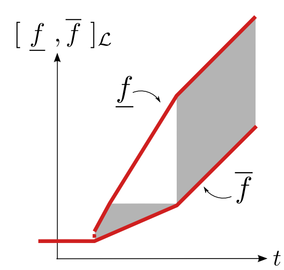

Fig. 1: Container $[\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}} \in \mathbf{F}$.

[1] It it important to note that this toolbox is an extension of the library MinMaxGD which handles increasing periodic series of the idempotent semiring $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ (see [3]).

[2] A non-injective mapping defined by $\mathcal{L}(f)(s) \triangleq \sup_t\{s.t - f(t)\}$ from the set of increasing and positive functions $\mathcal{F}$ to the set of convex functions $\mathcal{F}_{acx}$.

A function f is approximated by a container $[\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}}$ if $\underline{f} \preccurlyeq f \preccurlyeq \overline{f}$ and $[f]_{\mathcal{L}} = [\ \overline{f}\ ]_{\mathcal{L}}$. This means that f necessarily belongs to the grey zone of the figure, and by denoting $\mathcal{C}vx$ the convex hull of a function, that $\forall f \in [\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}}, \overline{f} = \mathcal{C}vx(f)$. Handling such containers amounts doing computations modulo $\mathcal{L}$. We thus obtain the equivalence class of the non-approximated result f. Therefore, even throughout the computations, the extremal points of $\overline{f}$ truly belong to the exact function f, and the asymptotic slope of $\overline{f}$ is the one of f.

Such a container belongs to the following set:

$$\mathbf{F} \triangleq \{\ [\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}} \mid \underline{f} \in \mathcal{F}_{\text{acv}},\ \overline{f} \in \mathcal{F}_{\text{acx}},\ \sigma(\underline{f}) = \sigma(\overline{f})\ \}.$$

Its bounds $\underline{f}$ and $\overline{f}$ are non-decreasing, piecewise affine and ultimately affine functions. They are in addition concave for the lower bound (set $\mathcal{F}_{\text{acv}}$), and convex for the upper bound (set $\mathcal{F}_{\text{acx}}$). Moreover, their asymptotic slopes $\sigma(\underline{f})$ and $\sigma(\overline{f})$ are equals, so are the slopes of their ultimately affine parts.

According to the computations, let us first recall that the elementary operations of the Network Calculus are:

- sum: $(f \oplus g)(t) = \min\{f(t), g(t)\}$,
- inf-convolution: $(f * g)(t) = \min_{\tau \geq 0}\ \{f(\tau) + g(t - \tau)\}$,
- subadditive closure: $f^{\star}(t) = \min_{\tau \geq 0}\ f^{\tau}(t)$ with $f^{0}(t) = e$.

On the set $\mathbf{F}$ of containers, these operations are now denoted $[\circ] \in \{\ [\oplus]\ ,\ [*]\ ,\ ^{[\star]}\ \}$ and redefined as inclusion functions such that for $\mathbf{f} = [\ \underline{f}\ ,\ \overline{f}\ ]_{\mathcal{L}} \in \mathbf{F}$, $\mathbf{g} = [\ \underline{g}\ ,\ \overline{g}\ ]_{\mathcal{L}} \in \mathbf{F}$, $\forall f \in \mathbf{f}$, and $\forall g \in \mathbf{g}$:

$$\begin{cases} \mathbf{f}[\circ]\mathbf{g} \in \mathbf{F}, \\ f \circ g \in \mathbf{f}[\circ]\mathbf{g}. \end{cases}$$

Thanks to the convexity characteristics of the bounds of a container, the computation algorithms of these inclusion functions are of linear complexity depending on the input size for the sum $[\oplus]$, the inf-convolution $[*]$ and the upper bound of the subadditive closure $^{[\star]}$, whereas the algorithm for the computation of the lower bound of $^{[\star]}$ is of quasi-linear complexity depending on the input size.

Finally, it is interesting to have an idea of the performance of this toolbox by the following method. First, an exact system A is approximated by a container $\mathbf{A}$ ($A \in \mathbf{A}$). Then, the subadditive closures of both the exact system $A^{\star}$ and the container $\mathbf{A}^{[\star]}$ are computed, and the result obtained with the exact system is approximated by another container: $A^{\star} \in \mathbf{B}$. At last, the pessimism of the toolbox is given by comparing $\mathbf{B}$ (obtained from the exact system), and $\mathbf{A}^{[\star]}$ (obtained from the approximated system). After experiments, we reach a pessimism of about 30%.

References

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