Control of uncertain (max,+)-linear systems in order to decrease uncertainty.

WODES'10

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September 1, 2010

WODES 2010

Motivations

Uncertain (max,+)-linear systems

- Input/output behavior h unknown
- Framed by an interval
  [fastest behavior , slowest behavior] $\rightarrow [\ \underline{h}\ ,\ \overline{h}\ ]$
- Uncertainty (time and event) over the output $y \in [\ \underline{h}u\ ,\ \overline{h}u\ ]$

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Control synthesis problem

Decrease the uncertainty at the output of the system

- Upstream controller p
- Fixed point of isotone mapping

Outlines

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Theory of (max,+)-linear systems [1]

- Discrete Event Dynamic Systems (DEDS) characterized by delay and synchronization phenomena

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- Application areas: manufacturing systems, computing networks [2], transportation systems [3]

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Idempotent semiring

Set $\mathcal{D}$ endowed with two inner operations

- $\oplus \rightarrow$ associative, commutative, idempotent ($a \oplus a = a$)
  neutral element ε
- $\otimes \rightarrow$ associative, distributes over the sum
  neutral element e

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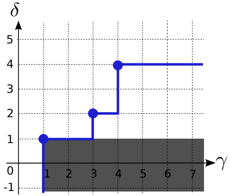

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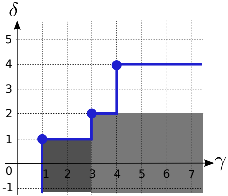

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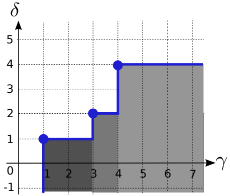

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DEDS linear modelling on $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

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Input/output relation

$$y = hu = CA^{\star}Bu$$

where h is the transfer function of the system

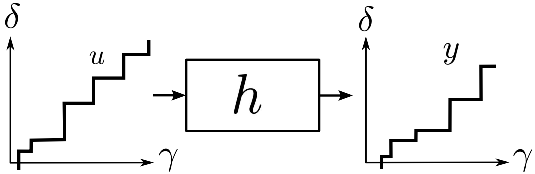
h is periodic and causal

Exact system

u, *y* and *h* are known

---

u and *y* → trajectories event / time



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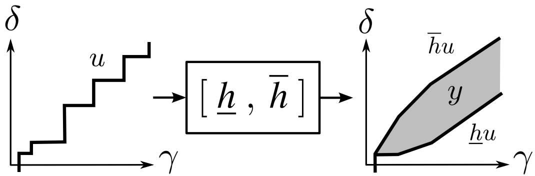
Uncertain system

- u exact
- $h \in [\ \underline{h}\ ,\ \overline{h}\ ]$
- $y \in [\ \underline{h}u\ ,\ \overline{h}u\ ]$

$[$ fastest , slowest $] \rightarrow [\ \underline{h}\ ,\ \overline{h}\ ]$



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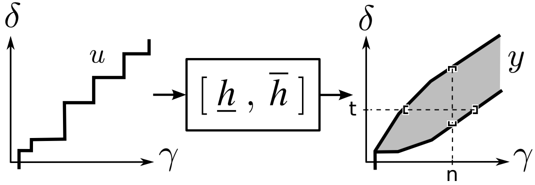

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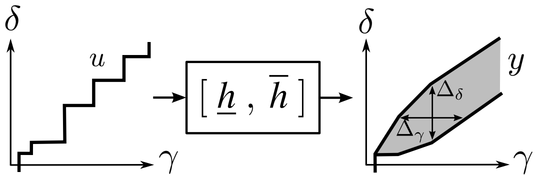

Maximal uncertainty

- event domain: Δ_γ
- time domain: Δ_δ

Computation of this uncertainty [4]

Maximal uncertainty over y for all u: $\overline{h} \diamond\!\!\!\backslash \underline{h}$

Residuation theory: $a \diamond\!\!\!\backslash b$ is the optimal solution to inequality $a \otimes x \preceq b$
$\diamond\!\!\!\backslash \longrightarrow$ left quotient

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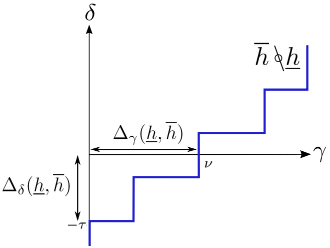

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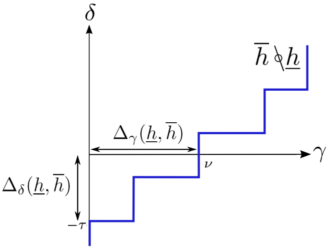

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- $\gamma^{0}\delta^{-\tau} \in \overline{h} \setminus\!\!\!\circ\, \underline{h} \quad \rightarrow \quad \tau = \Delta_{\delta}(\underline{h}, \overline{h})$

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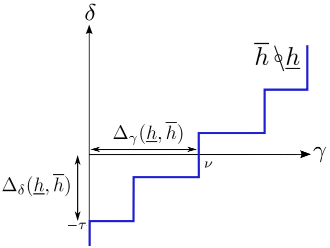

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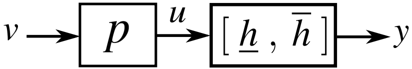
Outlines

1. Uncertain (max,+)-linear systems
   - Introduction
   - Linear modelling
2. Control synthesis problem
   - Uncertain controlled system
   - Reduction of the uncertainty
   - Application
3. Conclusions

Influence of a controller over an uncertain system

- Upstream controller p ($u = pv$)



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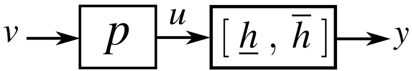
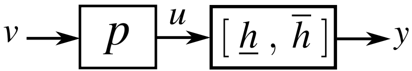


Influence of a controller over an uncertain system

- Upstream controller p ($u = pv$)
- Exact (max,+)-linear system (filter)
- Possible decreasing of the uncertainty

$$\begin{aligned} \Delta_\gamma(\underline{h}, \overline{h}) &\geq \Delta_\gamma(\underline{h}p, \overline{h}p) \\ \Delta_\delta(\underline{h}, \overline{h}) &\geq \Delta_\delta(\underline{h}p, \overline{h}p) \end{aligned}$$



Goals

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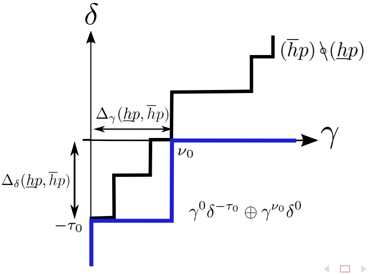

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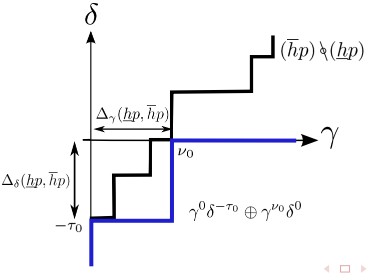
- Greatest controller $\hat{p}$ (just in time criterion)



Knaster-Tarski theorem

Let f be an isotone mapping defined over $\mathcal{D}$

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If f admits a fixed point $x \in \mathcal{F}_f$

$\rightarrow$ convergence toward the greatest fixed point $\hat{y} = x_m \preceq val$

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Proposition 2. Removal of the uncertainty ($\nu_0 = \tau_0 = 0$)

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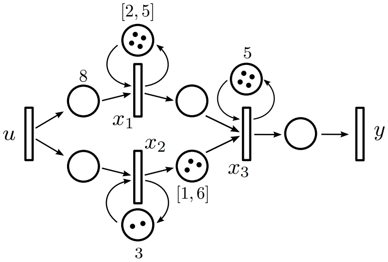
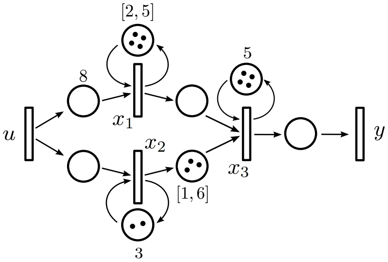
$$p = p \wedge \overline{h} \rightslice (\underline{h}p) \wedge \mathrm{Pr}_{caus}(p) \wedge val$$

Uncertain Single Input Single Output system



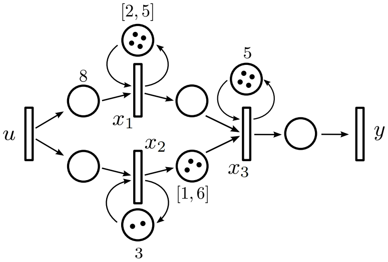
Uncertain Single Input Single Output system

- Time variations



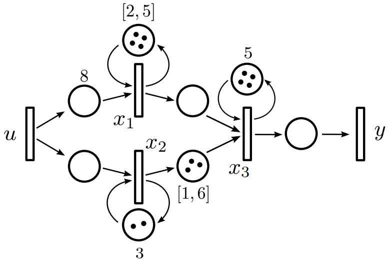
Uncertain Single Input Single Output system

- Time variations
- Fastest behavior: minimum delays



Uncertain Single Input Single Output system

- Time variations
- Fastest behavior: minimum delays
- Slowest behavior: maximum delays



Uncertain Single Input Single Output system

- Time variations
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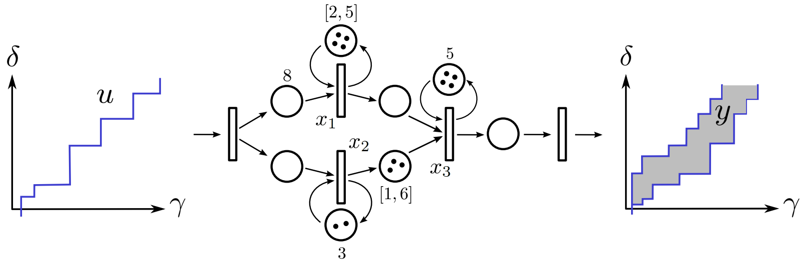

Uncertain Single Input Single Output system

- Time variations
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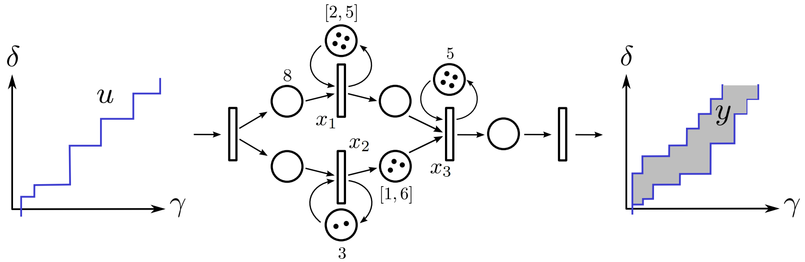

Maximal uncertainty

System alone

$$\Delta_\gamma(\underline{h},\overline{h}) = 4 \quad \text{and} \quad \Delta_\delta(\underline{h},\overline{h}) = 5$$

Uncertain Single Input Single Output system

- Time variations
- Fastest behavior: minimum delays
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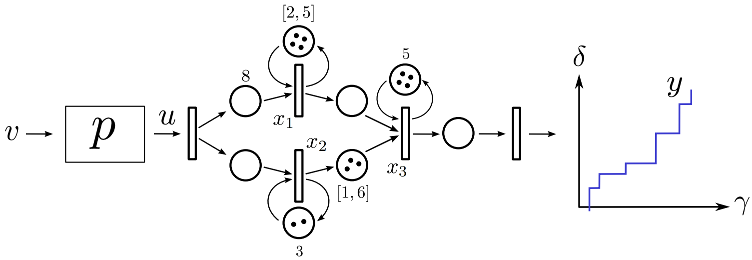

Maximal uncertainty

Controlled system

$$\Delta_\gamma(\underline{h}p, \overline{h}p) = 0 \quad \text{and} \quad \Delta_\delta(\underline{h}p, \overline{h}p) = 0$$

Outlines

What have we done?

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$$\hat{p} = \bigoplus \{p \mid (\overline{h}p) \diamond\!\!\!\backslash (\underline{h}p) \succeq \gamma^0 \delta^{-\tau_0} \oplus \gamma^{\nu_0} \delta^0, p \text{ causal}, p \preceq val\}$$

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- Decreasing of the uncertainty at the output of the controlled system

Thank you for your attention ...

Questions ?

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