Control of uncertain (min,+)-linear systems.

POSTA'09

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Motivations

Uncertain (min,+)-linear systems

(min,+)-linear input/output behavior h

- h is unknown $\Rightarrow h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

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Control synthesis problem

Precompensator controller p

- to delay the input as much as possible
- to keep the input/output transfer unchanged $\Rightarrow h * p = h$

Outlines

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1. Modeling and control of (min,+)-linear systems
   - Introduction
   - Algebraic preliminaries
   - DEDS linear modeling
   - Controller structure
2. Neutral precompensator for uncertain systems
   - Introduction
   - SISO case
   - Example
   - MIMO case
3. Conclusions and prospects
   - What have we done ?
   - Prospects

Theory of (min,+)-linear system [1]

- Discrete Event Dynamic Systems (DEDS) characterized by delay and synchronization phenomena

[1] F. Baccelli *and al.* : Synchronisation and Linearity. Wiley and sons, 1992.

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- Linear modeling $\Rightarrow$ idempotent semiring

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- Control matching problems [2] $\Rightarrow$ residuation theory

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Application areas

- Manufacturing systems
- Computing networks [3]
- Transportation systems [4]

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Idempotent semiring

Set $\mathcal{D}$ endowed with two inner operations

- $\oplus$: associative, commutative, idempotent ($a \oplus a = a$)
  neutral element ε
- $\otimes$: associative, distributes over the sum
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Order relation

$$a \succeq b \quad \Leftrightarrow \quad \begin{cases} a = a \oplus b \\ b = a \wedge b \end{cases}$$

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Example : idempotent semiring $\overline{\mathbb{Z}}_{min}$

$$\overline{\mathbb{Z}}_{min} = (\mathbb{Z} \cup \{-\infty, +\infty\}, min, +)$$

Residuation theory

Optimal solutions to inequalities

- f is residuated
- $\forall b, f(x) \preceq b$ admits a greatest solution $f^{\sharp}(b)$
- $f^{\sharp}$ is called the "*residual of f*"

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Example : left product and left quotient

- $L_a : x \mapsto a \otimes x$ is residuated
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- $L_a^\sharp(b) = a \backslash\!\!\!\circ\, b$ is the optimal solution to inequality $a \otimes x \preceq b$
- Isotony and antitony properties

$$x \preceq y \quad \Rightarrow \quad \begin{cases} a \backslash\!\!\!\circ\, x \preceq a \backslash\!\!\!\circ\, y & (x \mapsto a \backslash\!\!\!\circ\, x \quad \text{is isotone}) \\ x \backslash\!\!\!\circ\, a \succeq y \backslash\!\!\!\circ\, a & (x \mapsto x \backslash\!\!\!\circ\, a \quad \text{is antitone}) \end{cases}$$

Counter functions

$x(t)$ is the number of the last event x at time t

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Linear modeling on $\overline{\mathbb{Z}}_{min}$

- State representation

$$\begin{cases} x(t) = Ax(t-1) \oplus Bu(t) \\ y(t) = Cx(t) \end{cases}$$

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$$\begin{cases} x(t) = Ax(t-1) \oplus Bu(t) \\ y(t) = Cx(t) \end{cases}$$

- Input/output relation ($*$ = inf-convolution)

$$\begin{aligned} y(t) &= \bigoplus_{\tau \geq 0} CA^{\tau} Bu(t-\tau) \\ &= (h * u)(t) \\ &= \min_{\tau \geq 0} [h(\tau) + u(t-\tau)] \end{aligned}$$

with h the transfer function

Theorem : $h(t)$ characteristics

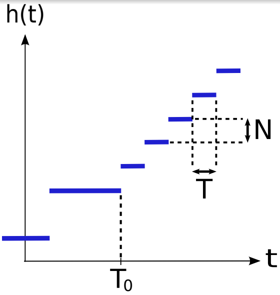
- Periodicity

$$\exists T_0, N, T \in \mathbb{N} \mid \forall t \geq T_0,\ h(t+T) = N \otimes h(t)$$

with $\sigma(h) = \frac{N}{T}$ the asymptotic slope of h

- Causality

$$\begin{cases} h(t) = h(0) & \text{for } t < 0 \\ h(t) \geq 0 & \text{for } t \geq 0 \end{cases}$$



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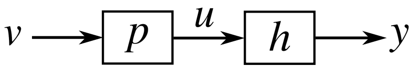
Idempotent semiring considered

Set of non-decreasing mappings from $\mathbb{Z}$ to $\overline{\mathbb{Z}}_{min}$

$$(\overline{\mathbb{Z}}_{min}^{\mathbb{Z}}, \oplus, *)$$

Precompensator controller

- Precompensator p placed upstream of process h
- Transfer relation $\Rightarrow h * p$
- Input/output relation $\Rightarrow y = (h * p) * v$



Optimal neutral precompensator controller $\hat{p}$

- To slow down the system input as much as possible : p is maximized
- To keep the input/output behavior unchanged : $h * p = h$

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Practical aim

Manufacturing systems or computing networks

- To improve internal streams (internal stock $\searrow$)
- To avoid useless accumulations (queuing size $\searrow$)

Computation of $\hat{p}$ ($\phi\!\!\!\backslash$ = residual of the inf-convolution) [5]

$$
\begin{aligned}
\hat{p}(t) &= (h \phi\!\!\!\backslash h)(t) \\
&= \bigwedge_{\tau \in \mathbb{Z}} [h(\tau - t) \phi\!\!\!\backslash h(\tau)] \\
&= \max_{\tau \in \mathbb{Z}} [h(\tau) - h(\tau - t)]
\end{aligned}
$$

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$\hat{p}$ characteristic

- Periodicity

$$h \text{ periodic} \Rightarrow \hat{p} \text{ periodic}$$

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Outlines

Optimal neutral precompensator controller $\hat{p}$

When h is known

$$\hat{p} = \oplus\{p \mid h * p = h\} \quad \Leftrightarrow \quad \hat{p} = h \mathbin{\backslash\!\!\circ} h$$

Optimal neutral precompensator controller $\hat{p}$

When $h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

$$\hat{p} \text{ is neutral } \forall h \in [\ \underline{h}\ ,\ \overline{h}\ ] \quad \Leftrightarrow \quad \hat{p} = \bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \,\circ\!\!\!\!\diagdown\, h$$

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- Optimal neutral $\hat{p}$ $(\forall h_i, i \in \{1, \ldots\})$

$$\begin{aligned} \hat{p} &= \bigoplus\{p \mid h_1 * p = h_1 \text{ and } h_2 * p = h_2 \text{ and } \ldots\} \\ &= \bigoplus\{p \mid p \preceq h_1 ⧑ h_1, p \preceq h_2 ⧑ h_2, \ldots\} \\ &= h_1 ⧑ h_1 \wedge h_2 ⧑ h_2 \wedge \ldots \end{aligned}$$

Optimal neutral precompensator controller $\hat{p}$

When $h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

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- Computation problem $\Rightarrow x \,\backslash\!\!\!\circ\, x$ is not monotonic

Proposition 1 : SISO case

Let $[\ \underline{h}\ ,\ \overline{h}\ ]$ be an interval with $\underline{h}, \overline{h} \in \overline{\mathbb{Z}}_{min}^{\mathbb{Z}}$, two periodic and causal functions

$$\hat{p} = \bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \lozenge\!\!\!\backslash h = e \oplus \overline{h} \lozenge\!\!\!\backslash \underline{h}$$

Lemma 1

$e \oplus \overline{h} \lozenge\!\!\backslash \underline{h}$ is neutral $\forall h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

$$\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \lozenge\!\!\backslash h \succeq e \oplus \overline{h} \lozenge\!\!\backslash \underline{h}$$

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$e \oplus \overline{h} \lessdot\backslash \underline{h}$ is neutral $\forall h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

$$\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \lessdot\backslash h \succeq e \oplus \overline{h} \lessdot\backslash \underline{h}$$

Sketch of proof

- $h \lessdot\backslash h \succeq \overline{h} \lessdot\backslash \underline{h}$
  because $x \mapsto a \lessdot\backslash x$ isotone and $x \mapsto x \lessdot\backslash a$ antitone
- $\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \lessdot\backslash h \succeq e$
- $\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \lessdot\backslash h \succeq \overline{h} \lessdot\backslash \underline{h}$

Lemma 2

$$\forall t > 0,\ (\overline{h} \,\mathring{\backslash}\, \underline{h})(t) = \Big(\bigwedge_{h \in [\,\underline{h}\,,\,\overline{h}\,]} h \,\mathring{\backslash}\, h \Big)(t)$$

Lemma 2

$$\forall t > 0,\ (\overline{h} \diamond\!\!\!\backslash \underline{h})(t) = \left(\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \diamond\!\!\!\backslash h \right)(t)$$

Sketch of proof

- $h \diamond\!\!\!\backslash h \succeq \overline{h} \diamond\!\!\!\backslash \underline{h}$
- $\forall t > 0,\ \exists h \in [\ \underline{h}\ ,\ \overline{h}\ ]$ such that $(h \diamond\!\!\!\backslash h)(t) = (\overline{h} \diamond\!\!\!\backslash \underline{h})(t)$

Graphic meaning

Lemma 2

$$\forall t > 0,\ (\overline{h} \diamond\!\!\backslash \underline{h})(t) = (\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \diamond\!\!\backslash h)(t)$$

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Lemma 3

$$\text{for } t = 0, \forall h \in [\ \underline{h}\ ,\ \overline{h}\ ],\ (h \diamond\!\!\backslash h)(0) = 0\ \Rightarrow\ \hat{p}(0) = 0$$

Lemma 4

$$\forall t \geq 0,\ (e \oplus \overline{h} \circ\!\!\!\!\backslash \underline{h})(t) = \Big(\bigwedge_{h \in [\,\underline{h}\,,\,\overline{h}\,]} h \circ\!\!\!\!\backslash h \Big)(t)$$

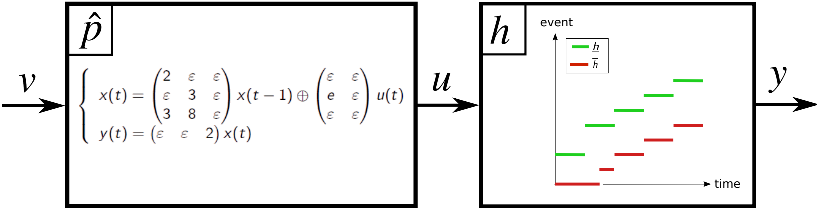
Lemma 4

$$\forall t \geq 0,\ (e \oplus \overline{h} \phi \underline{h})(t) = (\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \phi h)(t)$$

Sketch of proof

$$\hat{p}(t) = \begin{cases} (\overline{h} \phi \underline{h})(t) & \text{for } t > 0 \quad \text{(Lemma 2)} \\ 0 & \text{for } t = 0 \quad \text{(Lemma 3)} \end{cases}$$

$$\forall h \in [\, \underline{h}\, ,\ \overline{h}\,], \quad h * \hat{p} * v = h * v$$



Proposition 2 : MIMO case

Let $[\ \underline{H}\ ,\ \overline{H}\ ]$ be an interval with $\underline{H}, \overline{H} \in (\overline{\mathbb{Z}}^{\mathbb{Z}}_{min}\ )^{q \times p}$, two matrices with periodic and causal elements

$$\hat{P} = \bigwedge_{H \in [\ \underline{H}\ ,\ \overline{H}\ ]} H \lozenge\!\!\!\backslash H = Id \oplus \overline{H} \lozenge\!\!\!\backslash \underline{H}$$

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Control synthesis problem

- Unknown (min,+)-linear systems $h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

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- Computation of an optimal neutral precompensator controller

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$$\hat{p} = \bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \diamond\!\!\!\backslash h = e \oplus \overline{h} \diamond\!\!\!\backslash \underline{h}$$

$\Rightarrow$ delays the process input as much as possible
$\Rightarrow$ keeps the input/output relation unchanged

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$$\hat{p} = \bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \circ\!\!\!\!\setminus h = e \oplus \overline{h} \circ\!\!\!\!\setminus \underline{h}$$

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- SISO and MIMO systems

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- SISO and MIMO systems

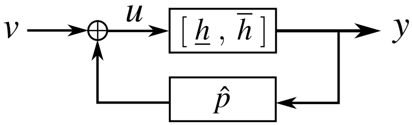
Result limitation

If $\sigma(\underline{h}) > \sigma(\overline{h})$ (with σ the asymptotic slope)

$$\overline{h} \backslash\!\!\!\circ \underline{h} = \varepsilon \quad \Rightarrow \quad e \oplus \overline{h} \backslash\!\!\!\circ \underline{h} = e$$

Extension to other controllers

- Feedback controller



Thank you for your attention ...

Questions ?

Graphic meaning

Computation of $\overline{h} \diamond\!\!\backslash \underline{h}$

Maximal variation between $\underline{h}$ and $\overline{h}$ in a window of width t

$$(\overline{h} \diamond\!\!\backslash \underline{h})(t) = \max_{\tau \in \mathbb{Z}} [\underline{h}(\tau) - \overline{h}(\tau - t)]$$

Graphic meaning

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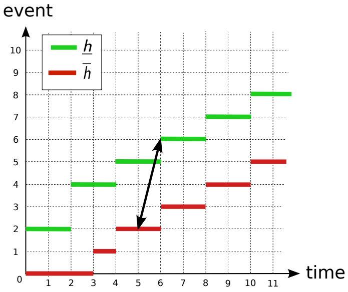


$$t = 1, \ \tau = 6$$

$$(\overline{h} \circ\!\!\!\!\setminus \underline{h})(1) = \underline{h}(6) - \overline{h}(5) = 4$$

Back

Graphic meaning

Computation of $\overline{h} \mathbin{\lozenge\!\backslash} \underline{h}$

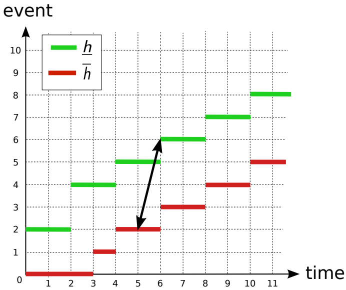
Maximal variation between $\underline{h}$ and $\overline{h}$ in a window of width t

$$(\overline{h} \mathbin{\lozenge\!\backslash} \underline{h})(t) = \max_{\tau \in \mathbb{Z}} [\underline{h}(\tau) - \overline{h}(\tau - t)]$$

Link with $h \mathbin{\lozenge\!\backslash} h$, $\forall h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

For a given t, an increasing function h exists

$$(h \mathbin{\lozenge\!\backslash} h)(t) = (\overline{h} \mathbin{\lozenge\!\backslash} \underline{h})(t)$$



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Computation of $\overline{h} \obslash \underline{h}$

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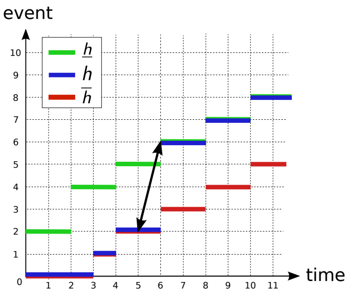

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Back

Graphic meaning

Computation of $\overline{h} \diamond\!\!\!\!\backslash\, \underline{h}$

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Link with $h \diamond\!\!\!\!\backslash\, h$, $\forall h \in [\ \underline{h}\ ,\ \overline{h}\ ]$

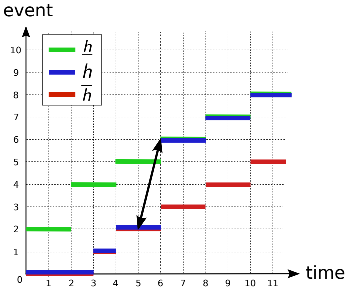
For a given t, an increasing function h exists

$$(h \diamond\!\!\!\!\backslash\, h)(t) = (\overline{h} \diamond\!\!\!\!\backslash\, \underline{h})(t)$$

Thus

Collection of functions h

$$(\overline{h} \diamond\!\!\!\!\backslash\, \underline{h})(t) = (\bigwedge_{h \in [\ \underline{h}\ ,\ \overline{h}\ ]} h \diamond\!\!\!\!\backslash\, h)(t)$$



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