

Control of uncertain $(\min,+)$ -linear systems

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Abstract This paper deals with the control of uncertain $(\min,+)$ -linear systems which belong to an interval. Thanks to the residuation theory, a precompensator controller placed upstream of the studied system is given in such a way that even if the system's behavior is not perfectly known, it has the property to delay the input as much as possible while keeping the input/output behavior unchanged. This precompensator is called neutral.

1 Introduction

Discrete Event Dynamic Systems (DEDS) such as production systems, computing networks and transportation systems which are characterized by delay and synchronization phenomena can be described by linear models. Thanks to the particular algebraic structure called *idempotent semiring* (or *doid*), this translation into a linear model is possible through for instance the $(\min,+)$ -algebra. This approach, detailed in [1] and [4], has numerous analogies with the classical automatic theory and in particular, the control of these systems can be considered. For instance, some model matching problems are solved by the way of different control structures (open-loop or close-loop structures) as presented in [2], [6] and [8]. These results rely on the residuation theory and assume that the model is perfectly known.

This paper puts forward a control synthesis problem when the system is modeled with some parametric uncertainties. More precisely, the following conditions are assumed:

- the system has a $(\min,+)$ -linear input/output behavior denoted h ,

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- because of uncertainties, h is unknown but belongs to an interval $[\underline{h}, \bar{h}]$, the bounds of which are known.

Under these assumptions, a precompensator controller p for the unknown system h is computed in order to achieve two goals:

- the precompensator p is the greatest as possible, *i.e.* the one which delays the input as much as possible,
- the input/output transfer¹ is unchanged, *i.e.* $h * p = h$.

In a manufacturing context, such a controller allows the work-in process to be reduced while keeping the same process output. This enables to preserve input/output stream while decreasing internal congestions.

It is important to note that our approach is different from the one presented in [7]. Indeed, in [7], the system also belongs to an interval ($h \in [\underline{h}, \bar{h}]$) but is subject to fluctuation² within the interval limits and admits a precompensator $p \in [\underline{p}, \bar{p}]$ such that $h * p \in [\underline{h}, \bar{h}]$. In this paper, a precompensator p is computed such that the equality $h * p = h$ is satisfied, provided that h is a stationary (min,+)-linear system.

In order to introduce this work, the paper is organized as follows. The second section recalls some algebraic tools required to the DEDS study through idempotent semiring and residuation theory. In the third section, models and controls of (min,+)-linear systems are presented. Finally, in the fourth section, the neutral precompensator controller p is proposed and an example is given.

2 Algebraic preliminaries

2.1 Dioid theory

An idempotent semiring \mathcal{D} is a set endowed with two inner operations denoted \oplus and \otimes (see [1, §4.2]). The sum \oplus is associative, commutative, idempotent (*i.e.* $\forall a \in \mathcal{D}, a \oplus a = a$) and admits a neutral element denoted ε . The product \otimes is associative, distributes over the sum and accepts ε as neutral element. An idempotent semiring is said to be complete if it is closed for infinite sums and if the product distributes over infinite sums too. Moreover, the greatest element of \mathcal{D} is denoted T (for *Top*) and represents the sum of all its elements.

Due to the sum idempotency, an order relation can be associated with \mathcal{D} by the following equivalences: $\forall a, b \in \mathcal{D}, a \succeq b \iff a = a \oplus b$ and $b = a \wedge b$. Because of the lattice properties of a complete idempotent semiring, $a \oplus b$ is the least upper bound of \mathcal{D} whereas $a \wedge b$ is its greatest lower bound.

Example 1 ($\overline{\mathbb{Z}}_{min}$). The set $\overline{\mathbb{Z}}_{min} = (\mathbb{Z} \cup \{-\infty, +\infty\})$, endowed with the min operator as sum \oplus and the classical sum as product \otimes , is a complete idempotent semiring

¹ where $*$ is the convolution product.

² h is not necessarily (min,+)-linear.

where $\varepsilon = +\infty$, $e = 0$ and $T = -\infty$. On $\overline{\mathbb{Z}}_{min}$, the greatest lower bound \wedge takes the sense of the max operator.

2.2 Residuation theory

Residuation is a general notion in lattice theory which allows to define “pseudo-inverse” of some isotone maps (see [1]). In particular, the residuation theory provides optimal solutions to inequalities such as $f(x) \preceq b$ (respectively $f(x) \succeq b$), where f is an order-preserving mapping defined over ordered sets.

A mapping f defined over ordered sets is isotone, respectively antitone, if $a \preceq b \Rightarrow f(a) \preceq f(b)$, respectively $f(a) \succeq f(b)$. Now, let $f: \mathcal{E} \rightarrow \mathcal{F}$ be an isotone mapping, where (\mathcal{E}, \preceq) and (\mathcal{F}, \preceq) are ordered sets. Mapping f is said residuated if $\forall b \in \mathcal{F}$, the greatest element denoted $f^\sharp(b)$ of subset $\{x \in \mathcal{E} \mid f(x) \preceq b\}$ exists and belongs to this subset. Mapping f^\sharp is called the residual of f . When f is residuated, f^\sharp is the unique isotone mapping such that $f \circ f^\sharp \preceq \text{Id}_{\mathcal{F}}$ and $f^\sharp \circ f \succeq \text{Id}_{\mathcal{E}}$, where $\text{Id}_{\mathcal{F}}$ (respectively $\text{Id}_{\mathcal{E}}$) is the identity mapping on \mathcal{F} (respectively on \mathcal{E}).

Example 2 (Left product). Mapping $L_a: x \mapsto a \otimes x$ defined over a complete idempotent semiring \mathcal{D} is residuated. Its residual represents the optimal solution to inequality $a \otimes x \preceq b$ and is usually denoted $L_a^\sharp: x \mapsto a \backslash x$ (left quotient).

Remark 1 (Isotony and antitony). $\forall x, y, a \in \mathcal{D}$, an ordered set, these properties are given:

$$x \preceq y \Rightarrow \begin{cases} a \backslash x \preceq a \backslash y & (x \mapsto a \backslash x \text{ is isotone}), \\ x \backslash a \succeq y \backslash a & (x \mapsto x \backslash a \text{ is antitone}). \end{cases} \quad (1)$$

3 Models and control of (min,+)-linear systems

3.1 Counter functions

Some idempotent semiring algebras enable to model DEDS which involve synchronization and delay phenomena. The behavior of such systems can be represented by discrete functions called “counter” functions. More precisely, a discrete variable $x(t)$ is associated to an event labeled x and represents the occurrence number x at time t (the numbering conventionally beginning at 0). For negative values of t , these variables are defined as constant so they can be manipulated as mappings from \mathbb{Z} to $\overline{\mathbb{Z}}_{min}$. Thanks to these counter functions, the studied DEDS can be modeled on the idempotent semiring $\overline{\mathbb{Z}}_{min}$ by the following linear state representation:

$$\begin{cases} x(t) = Ax(t-1) \oplus Bu(t), \\ y(t) = Cx(t), \end{cases} \quad (2)$$

where $A \in \overline{\mathbb{Z}}_{min}^{n \times n}$, $B \in \overline{\mathbb{Z}}_{min}^{n \times p}$ and $C \in \overline{\mathbb{Z}}_{min}^{q \times n}$ while n , p and q refer respectively to the state vector (x) size, the input vector (u) size and the output one (y).

In the SISO³ case ($p = 1$ and $q = 1$), the state equation leads to the following input/output relation:

$$y(t) = \bigoplus_{\tau \geq 0} CA^\tau Bu(t - \tau). \quad (3)$$

Moreover, setting $h(\tau) = CA^\tau B$, and defining the *inf-convolution* (or *(min,+)-convolution*) as follows (see [9] and [5]), $\forall f, g \in \overline{\mathbb{Z}}_{min}$:

$$(f * g)(t) \triangleq \bigoplus_{\tau \geq 0} [f(\tau) \otimes g(t - \tau)] = \min_{\tau \geq 0} [f(\tau) + g(t - \tau)],$$

relation (3) can be rewritten as $y(t) = (h * u)(t)$, which is actually the transfer relation of the considered system, with $h(t)$ the transfer function⁴.

According to [1, Theorem 5.39] and [3], a *(min,+)*-linear system defined as (2) is necessarily such that $h(t)$ is periodic and causal *i.e.*:

$$\exists T_0, N, T \in \mathbb{N} \mid \forall t \geq T_0, h(t + T) = N \otimes h(t) \quad [\text{periodicity}], \quad (4)$$

$$\begin{cases} h(t) = h(0) & \text{for } t < 0 \\ h(t) \geq 0 & \text{for } t \geq 0 \end{cases} \quad [\text{causality}]. \quad (5)$$

Let us note that the set of nondecreasing⁵ mappings from \mathbb{Z} to $\overline{\mathbb{Z}}_{min}$ endowed with the two inner operations \oplus as pointwise addition and $*$ as inf-convolution is also an idempotent semiring denoted $(\overline{\mathbb{Z}}_{min}^{\mathbb{Z}}, \oplus, *)$ where ε and e are defined by:

$$\forall t, \varepsilon : \varepsilon(t) \mapsto +\infty \quad \text{and} \quad e : e(t) \mapsto \begin{cases} 0 & \text{for } t < 0, \\ +\infty & \text{for } t \geq 0. \end{cases} \quad (6)$$

In the MIMO⁶ case, the input/output relation becomes $Y(t) = (H * U)(t)$, where $U \in (\overline{\mathbb{Z}}_{min}^{\mathbb{Z}})^p$, $Y \in (\overline{\mathbb{Z}}_{min}^{\mathbb{Z}})^q$ and $H \in (\overline{\mathbb{Z}}_{min}^{\mathbb{Z}})^{q \times p}$ is such that H_{ij} is periodic. The inf-convolution $*$ is then naturally extended to matrices as $Y_j(t) = (\bigoplus_{i=0}^p (H_{ij} * U_i))(t)$.

3.2 Precompensator control

A specific *(min,+)*-linear controller, called precompensator p , can be placed upstream of process h so that $u(t) = (p * v)(t)$ and where v is the controller input. In $(\overline{\mathbb{Z}}_{min}^{\mathbb{Z}}, \oplus, *)$, the output of the controlled system becomes $y(t) = (h * p * v)(t)$.

³ Single Input Single Output.

⁴ Let us note that $h(t)$ corresponds to the impulse response of the system, *i.e.* the output due to the particular input: if $t < 0$, $u(t) = 0$ and if $t \geq 0$, $u(t) = +\infty$.

⁵ Nondecreasing in the natural order *i.e.* for $t_1 > t_2 \Rightarrow h(t_1) \geq h(t_2)$.

⁶ Multiple Inputs Multiple Outputs.

With this configuration, the controller p aims at slowing down the system input. Moreover, the residuation theory shows (see [8]) that there exists an optimal neutral precompensator given by:

$$\hat{p}(t) = \sup\{p(t) \mid (h * p)(t) = h(t)\} = (h \setminus h)(t),$$

where the mapping $x \mapsto a \setminus x$ is in that case the residual of the inf-convolution product. This optimal controller is said to be neutral since it lets the input/output behavior unchanged. Nevertheless, it delays the process input u as much as possible in order to avoid useless accumulations into h . The computation of \hat{p} requires thus the use of the residual of the inf-convolution product (see [9]):

$$\hat{p}(t) = (h \setminus h)(t) = \bigwedge_{\tau \in \mathbb{Z}} [h(\tau - t) \setminus h(\tau)] = \max_{\tau \in \mathbb{Z}} [h(\tau) - h(\tau - t)]. \quad (7)$$

Remark 2 (Periodicity). If function $h(t)$ is periodic, $(h \setminus h)(t)$ is periodic too.

Remark 3 (Argument of the maximum). Let us note that if h is periodic, there exists at least a τ_0 (not necessarily unique) such that $\max_{\tau \in \mathbb{Z}} [h(\tau) - h(\tau - t)] = h(\tau_0) - h(\tau_0 - t)$ and defined by:

$$\tau_0 \in \arg \max_{\tau \in \mathbb{Z}} [h(\tau) - h(\tau - t)]. \quad (8)$$

4 Neutral precompensator for unknown systems

Usually, this optimal neutral precompensator is given for (min,+)-linear system the transfer function h of which is perfectly known. This section deals with the problem of finding such a precompensator when h presents some parametric uncertainties and belongs to an interval $[\underline{h}, \bar{h}]$. In such a case, we will see that $\hat{p} = e \oplus \bar{h} \setminus \underline{h}$ is the greatest precompensator which is *neutral* for all systems, i.e. $\forall h \in [\underline{h}, \bar{h}], h * \hat{p} = h$.

4.1 SISO case

Proposition 1. Let $[\underline{h}, \bar{h}]$ be an interval with $\underline{h}, \bar{h} \in \overline{\mathbb{Z}}_{min}$:

$$\forall h_i \in [\underline{h}, \bar{h}], h_i \setminus h_i \succeq \bar{h} \setminus \underline{h}.$$

Proof. According to the left quotient isotony and antitony properties (1), $h_i \setminus h_i \succeq \bar{h} \setminus h_i \succeq \bar{h} \setminus \underline{h}$. \square

Proposition 2. Let $[\underline{h}, \bar{h}]$ be an interval with $\underline{h}, \bar{h} \in \overline{\mathbb{Z}}_{min}$, two periodic and causal functions (see (4) and (5)):

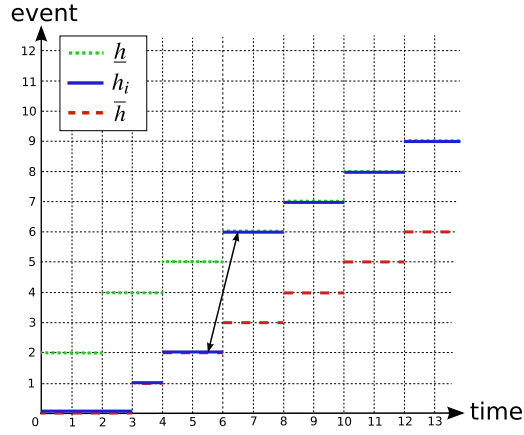
$$\forall t_i > 0, \exists h_i \in [\underline{h}, \bar{h}] \text{ such that } (h_i \backslash h_i)(t_i) = (\bar{h} \backslash \underline{h})(t_i).$$

Proof. Thanks to the residual of the inf-convolution product definition (see (7) and (8)): $(\bar{h} \backslash \underline{h})(t_i) = \bigwedge_{\tau \in \mathbb{Z}} \bar{h}(\tau - t_i) \backslash \underline{h}(\tau) = \bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i)$ with $\tau_i \in \arg \max_{\tau \in \mathbb{Z}} [\underline{h}(\tau) - \bar{h}(\tau - t_i)]$. This τ_i leads to the h_i following definition⁷:

$$h_i(t) \triangleq \begin{cases} \bar{h}(t), & \text{for } t < \tau_i, \\ \underline{h}(t), & \text{for } t \geq \tau_i. \end{cases} \quad (9)$$

On Fig. 1 is illustrated an example of the h_i function for which $t_i = 1$ and $\tau_i = 6$. In that case $(h_i \backslash h_i)(1) = (\bar{h} \backslash \underline{h})(1) = \underline{h}(6) - \bar{h}(5) = 4$. As specified in remark 3, τ_i is not unique for $t_i = 1$ and belongs to the set $\{3, 4, 6, 8, 10, \dots\}$.

Fig. 1 Example of a h_i function where $t_i = 1$ and $\tau_i = 6$. The arrow represents the distance (= 4) between \underline{h} and \bar{h} for these values.



Then, equation (7) shows that: $(h_i \backslash h_i)(t_i) = \bigwedge_{\tau \in \mathbb{Z}} h_i(\tau - t_i) \backslash h_i(\tau)$. This latter expression can be factorized, since $\forall \tau, \underline{h}(\tau) \preceq \bar{h}(\tau)$ and according to (1) we obtain:

$$\begin{aligned} \bigwedge_{\tau < \tau_i} \bar{h}(\tau - t_i) \backslash \bar{h}(\tau) &\succeq \bigwedge_{\tau < \tau_i} \bar{h}(\tau - t_i) \backslash \underline{h}(\tau) \succeq \bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i), \\ \bigwedge_{\tau_i + t_i \leq \tau} \underline{h}(\tau - t_i) \backslash \underline{h}(\tau) &\succeq \bigwedge_{\tau_i + t_i \leq \tau} \bar{h}(\tau - t_i) \backslash \underline{h}(\tau) \succeq \bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i). \end{aligned}$$

Moreover:

$$\left(\bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i) \right) \wedge \left(\bigwedge_{\tau_i < \tau < \tau_i + t_i} \bar{h}(\tau - t_i) \backslash \underline{h}(\tau) \right) = \bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i).$$

Finally, by defining h_i as in (9), $(h_i \backslash h_i)(t_i) = \bar{h}(\tau_i - t_i) \backslash \underline{h}(\tau_i) = (\bar{h} \backslash \underline{h})(t_i)$. □

⁷ It is important to recall that the order \preceq of $\overline{\mathbb{Z}}_{min}^{\mathbb{Z}}$ is the opposite to the natural order \geq of functions. Moreover, as illustrated in Fig. 1, $h_i(t)$ is still a nondecreasing function.

Remark 4. Let us note that for $t_i = 0$, $(h \backslash h)(0) = 0$ whereas $(\bar{h} \backslash \underline{h})(0) \leq 0$ (≥ 0). For instance, on Fig. 1 $(\bar{h} \backslash \underline{h})(0) = \underline{h}(2) - \bar{h}(2) = 4$.

These preliminary results lead to the following proposition.

Proposition 3. Let $[\underline{h}, \bar{h}]$ be an interval with $\underline{h}, \bar{h} \in \overline{\mathbb{Z}}_{\min}^{\mathbb{Z}}$, two periodic and causal functions (see (4) and (5)):

$$e \oplus \bar{h} \backslash \underline{h} = \bigwedge_{h \in [\underline{h}, \bar{h}]} h \backslash h.$$

Proof. According to proposition 2, $\forall t_i > 0, \exists h_i \in [\underline{h}, \bar{h}]$ such that $(h_i \backslash h_i)(t_i) = (\bar{h} \backslash \underline{h})(t_i)$. So, thanks to proposition 1, $\forall t > 0$, a subset of systems $\mathcal{H} \subset [\underline{h}, \bar{h}]$ exists such that $\bigwedge_{h_i \in \mathcal{H}} (h_i \backslash h_i)(t) = (\bar{h} \backslash \underline{h})(t)$. Moreover, for $t = 0$ and according to remark 4, $\forall t \geq 0, \bigwedge_{h_i \in \mathcal{H}} (h_i \backslash h_i)(t) = e \oplus (\bar{h} \backslash \underline{h})(t)$. To conclude, $\forall h \in [\underline{h}, \bar{h}]$, $h \backslash h \succeq \bigwedge_{h_i \in \mathcal{H}} (h_i \backslash h_i)(t)$ and finally $e \oplus \bar{h} \backslash \underline{h} = \bigwedge_{h \in [\underline{h}, \bar{h}]} h \backslash h$. \square

Proposition 3 must be interpreted as follows: the precompensator $\hat{p} = e \oplus \bar{h} \backslash \underline{h}$ is the greatest precompensator which is neutral for all systems $h \in [\underline{h}, \bar{h}]$ i.e. $h * \hat{p} = h$.

4.2 MIMO extension

Proposition 3 given for all uncertain SISO systems belonging to an interval can be extended to MIMO systems. For an uncertain (min,+)-linear system $H \in (\overline{\mathbb{Z}}_{\min}^{\mathbb{Z}})^{q \times p}$ in an interval $[\underline{H}, \bar{H}]$, the greatest neutral precompensator is now defined by $\hat{P} = I \oplus \bar{H} \backslash \underline{H}$, where I is the identity matrix of $(\overline{\mathbb{Z}}_{\min}^{\mathbb{Z}})^{p \times p}$.

Proposition 4. Let $h_a, h_b \in \overline{\mathbb{Z}}_{\min}^{\mathbb{Z}}$:

$$\bar{h}_b \backslash h_a = \bigwedge_{\substack{h_a \in [\underline{h}_a, \bar{h}_a] \\ h_b \in [\underline{h}_b, \bar{h}_b]}} h_b \backslash h_a.$$

Proof. Thanks to (1). \square

Proposition 5. Let $[\underline{H}, \bar{H}]$ be a matrix interval with $\underline{H}, \bar{H} \in (\overline{\mathbb{Z}}_{\min}^{\mathbb{Z}})^{q \times p}$ and which represents the behavior of an uncertain p -input q -output system:

$$I \oplus \bar{H} \backslash \underline{H} = \bigwedge_{H \in [\underline{H}, \bar{H}]} H \backslash H.$$

Proof. Thanks to [1, Equation (4.82)], $\forall H \in [\underline{H}, \bar{H}]$, $(H \backslash H)_{ij} = \bigwedge_{k=1}^n H_{ki} \backslash H_{kj}$. On the one hand, if $i = j$, $(H \backslash H)_{ii} = \bigwedge_{k=1}^n H_{ki} \backslash H_{ki}$ with $H_{ki} \in [\underline{H}_{ki}, \bar{H}_{ki}]$. Thanks to proposition 3, $(\bigwedge_{H \in [\underline{H}, \bar{H}]} H \backslash H)_{ii} = \bigwedge_{k=1}^n (e \oplus \bar{H}_{ki} \backslash \underline{H}_{ki})_{ii}$. On the other hand,

for $i \neq j$, $(H \setminus H)_{ij} = \bigwedge_{k=1}^n H_{ki} \setminus H_{kj}$ with $H_{ki} \in [\underline{H}_{ki}, \overline{H}_{ki}]$, $H_{kj} \in [\underline{H}_{kj}, \overline{H}_{kj}]$ and thanks to proposition 4, $(\bigwedge_{H \in [\underline{H}, \overline{H}]} H \setminus H)_{ij} = \bigwedge_{k=1}^n (H_{ki} \setminus H_{kj})_{ij} = \overline{H}_{ki} \setminus \underline{H}_{kj}$. Finally, $\forall i, j, (\bigwedge_{H \in [\underline{H}, \overline{H}]} H \setminus H)_{ij} = (I \oplus \overline{H} \setminus \underline{H})_{ij}$. \square

4.3 Example of neutral precompensator for MIMO systems

A MIMO system with $H \in (\overline{\mathbb{Z}}_{min}^{\mathbb{Z}})^{1 \times 2}$ (two inputs, one output) the transfer function of which belongs to an interval $([\underline{H}_{11}, \overline{H}_{11}] [\underline{H}_{12}, \overline{H}_{12}])$ is considered. The bounds are the periodic and causal functions given in table 1.

Table 1 Bounds of H

t	0	1	2	3	4	$t \geq 5$
\underline{H}_{11}	0	3				$t \geq 2, \underline{H}_{11}(t) = 3 \otimes \underline{H}_{11}(t-2)$
\overline{H}_{11}	0	0	0	1	2	$t \geq 5, \overline{H}_{11}(t) = 3 \otimes \overline{H}_{11}(t-2)$
\underline{H}_{12}	0	2				$t \geq 2, \underline{H}_{12}(t) = 2 \otimes \underline{H}_{12}(t-2)$
\overline{H}_{12}	0	0				$t \geq 2, \overline{H}_{12}(t) = 1 \otimes \overline{H}_{12}(t-1)$

As previously said, (min,+)-linear systems are always characterized by periodic functions ([1, Theorem 5.39]) and according to remark 2, residuals of the inf-convolution are periodic functions too. Thus, for this system and thanks to proposition 5, the computation of the neutral precompensator $\hat{P} \in (\overline{\mathbb{Z}}_{min}^{\mathbb{Z}})^{2 \times 2}$ given by $\hat{P} = I \oplus \overline{H} \setminus \underline{H}$, is described in table 2. Let us note that for this example, $\hat{P}_{21} = \varepsilon$ with ε defined by (6).

Table 2 Neutral precompensator \hat{P}

t	0	1	2	$t \geq 3$
\hat{P}_{11}	0	5	7	$t \geq 3, \hat{P}_{11}(t) = 3 \otimes P_{11}(t-2)$
\hat{P}_{12}	2			$t \geq 1, \hat{P}_{12}(t) = 1 \otimes P_{12}(t-1)$
\hat{P}_{22}	0	3		$t \geq 2, \hat{P}_{22}(t) = 1 \otimes P_{22}(t-1)$

5 Conclusion

This paper has introduced the control of unknown (min,+)-linear systems belonging to an interval the bounds of which are known. A neutral precompensator controller

placed upstream of these systems has been given without changing the input/output behavior while delaying the process input as much as possible. This precompensator is enabled both for SISO and MIMO systems and an example has been given in order to illustrate these propositions.

References

1. Baccelli F, Cohen G, Olsder GJ, Quadrat JP (1992) Synchronisation and linearity: an algebra for discrete event systems. Wiley and sons.
2. Cottenceau B, Hardouin L, Boimond JL, Ferrier, JL (2001) Model reference control for timed event graphs in dioids. *Automatica*, Elsevier 37(9):1451–1458.
3. Gaubert S (1992) PhD Thesis. Théorie des systèmes linéaires dans les dioïdes. Ecole Nationale Supérieure des Mines de Paris.
4. Heidergott B, Olsder GJ, Woude J (2006) Max plus at work, modeling and analysis of synchronized systems: a course on max-plus algebra and its applications. Princeton University Press.
5. Le Boudec JY, Thiran P (2001) Network calculus: a theory of deterministic queuing systems for the internet. Springer.
6. Lhommeau M, Hardouin L, Cottenceau B (2003) Optimal control for (max,+)-linear systems in the presence of disturbances. International Symposium on Positive Systems: Theory and Applications, Roma. POSTA 03.
7. Lhommeau M, Hardouin L, Ferrier JL, Ouerghi I (2005) Interval analysis in dioid: application to robust open-loop control for timed event graphs. 44th IEEE Conference on Decision and Control and European Control Conference 7744–7749, Seville. CDC-ECC 05.
8. Maia CA, Hardouin L, Santos-Mendes R, Cottenceau B (2003) Optimal closed-loop control of timed event graphs in dioids. *IEEE Transactions on Automatic Control* 48(12):2284–2287.
9. Max Plus (1991) Second order theory of min-linear systems and its application to discrete event systems. Proceedings of the 30th IEEE Conference on Decision and Control, Brighton. CDC 91.