

Flow Control with (Min,+) Algebra

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1 Introduction

According to the theory of Network Calculus based on the (min,+) algebra (see [3] and [4]), analysis and measure of worst-case performance in communication networks can be made easily. In this context, this paper deals with traffic regulation and performance guarantee of a network *i.e.* with flow control. More precisely, the optimal window size of a window flow controller is given by considering the following configuration: The data stream (from the source to the destination) and the acknowledgments stream (from the destination to the source) are assumed to be different and the service provided by the network is assumed to be known in an uncertain way, more precisely it is assumed to be in an interval.

2 Network Calculus

In the theory of Network Calculus, a communication network is seen as a black-box denoted S , with an input flow u constrained by an arrival curve α , and an output flow y . Moreover, the service provided by S is constrained by a lower curve β and an upper curve $\bar{\beta}$. These constraints combined with the following operations of the (min,+) algebra (see [2]) provide bounds on worst-case performance measures. Let f and g be two non-decreasing functions from \mathbb{R} to the dioid $\overline{\mathbb{R}}_{min} = (\mathbb{R} \cup \{-\infty, +\infty\})$, such that $f(t) = 0$ and $g(t) = 0$ for $t \leq 0$, these operations are:

- pointwise minimum : $(f \oplus g)(t) = \min[f(t), g(t)]$,
- pointwise maximum : $(f \wedge g)(t) = \max[f(t), g(t)]$,
- inf-convolution : $(f * g)(t) = \min_{\tau \geq 0} \{f(\tau) + g(t - \tau)\}$,
- deconvolution : $(f \not\wedge g)(t) = \max_{\tau \geq 0} \{f(\tau) - g(\tau - t)\}$,
- subadditive closure : $f^*(t) = \min_{\tau \geq 0} f^\tau(t)$ with $f^0(t) = e$.

3 Window flow control

First, a difference is made between the data stream represented by network S_1 , and the acknowledgments stream represented by network S_2 . Indeed, the

acknowledgments stream requires considerably less bandwidth than the data itself (see [1]), so the computation of the window size will have benefit of this profit of bandwidth.

Second, the service provided by the network is assumed to be included in interval, *i.e.* into $[\underline{\beta}_1, \bar{\beta}_1]$ for S_1 and $[\underline{\beta}_2, \bar{\beta}_2]$ for S_2 . In that way, the size of the window can be computed as well as for the worst case than for the best case of traffic without damaging the service provided.

Finally, let γ_w be the representative function of the window size w ($\gamma_w(t) = w$ for $t < 0$ and $+\infty$ for $t \geq 0$).

The service curve of the whole system is included in the interval:

$$[\underline{\beta}_1(\gamma_w \underline{\beta}_2 \underline{\beta}_1)^*, \bar{\beta}_1(\gamma_w \bar{\beta}_2 \bar{\beta}_1)^*].$$

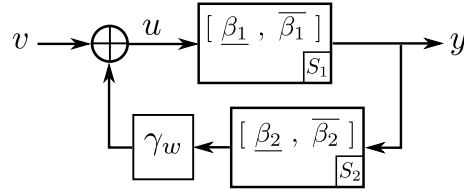


Fig. 1: Configuration of the window flow control system.

The chosen point of view is to compute a minimal window size such that the global network behavior, *i.e.* the controlled one, is the same as the open-loop network behavior, *i.e.* the one of S_1 only. This objective can be stated as follows:

$$\hat{\gamma}_w = \bigoplus \{ \gamma_w \mid \underline{\beta}_1(\gamma_w \underline{\beta}_2 \underline{\beta}_1)^* = \underline{\beta}_1 \text{ and } \bar{\beta}_1(\gamma_w \bar{\beta}_2 \bar{\beta}_1)^* = \bar{\beta}_1 \}. \quad (1)$$

Proposition 1. *In order to obtain a behavior of the closed-loop system unchanged in comparison to the one of the open-loop (see equation (1)), the optimal window size \hat{w} represented by function $\hat{\gamma}_w$ is given below:*

$$\hat{\gamma}_w = (\underline{\beta}_1 \searrow \underline{\beta}_1 \phi(\underline{\beta}_2 \underline{\beta}_1)) \wedge (\bar{\beta}_1 \searrow \bar{\beta}_1 \phi(\bar{\beta}_2 \bar{\beta}_1)).$$

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