

MONTE-CARLO ANALYSIS OF OBJECT REENTRY IN EARTH'S ATMOSPHERE BASED ON TAGUCHI METHOD

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ABSTRACT

The risk of space debris is now perceived as primordial by governments and international space agencies. Since the last decade, international space agencies have developed tools simulate the re-entry of satellites and orbital stations in order to assess casualty risk on the ground. Nevertheless, all current tools provide deterministic solutions, though models include various parameters that are not well known. Therefore, the provided results are strongly dependent on the assumptions made.

One solution to obtain relevant and exploitable results would be to include uncertainties around those parameters in order to perform Monte-Carlo analysis. But such a study is very time consuming due to the large parameter space to explore (that necessitate hundreds of thousands simulations).

To reduce the parameter search space, we present an application of the Taguchi Method, to model spacecraft debris re-entry in Earth's atmosphere. The Taguchi Method is a statistical analysis method that permits one to determine the parameters uncertainty that have the biggest impact on the results of the numerical simulation. We show how to use this method so as to restrain the quantity of parameters to consider for a Monte-Carlo analysis. Finally, we present the new object-oriented re-entry tool Calima developed by R.Tech. This new tool features three degree of freedom model, featuring also the perturbation of initial parameters of a numerical simulation in order to perform automatized Monte-Carlo Analysis. Calima is accelerated via Graphics Processing Unit (GPU) devices which have many cores architecture and that consume less energy than classical CPUs.

1. INTRODUCTION

During several decades the problem of space debris was not of primary importance for international space agencies, sending more and more satellites in orbit around the Earth, and either letting them in orbit at the end of their life, or causing their atmospheric re-entry. The first approach presents the major inconvenient that

every inactive object must be monitored in order to avoid collision between those debris and active spacecraft or satellite. This strategy is finally expensive and not foolproof with for example the collision in 2009, between IRIDIUM satellite and an inactive Russian satellite, or in 2013 the collision with the Ecuadorian micro-satellite Pegasus and a piece of a Russian rocket tank, one month after launch. The main inconvenient of the second approach is that if the re-entry is not well mastered, non-negligible sized objects can reach the ground and cause human casualty and damage to properties.

But recently things have changed, with for example the adoption in France of the Law on Space Operations, in 2008, that imposed to French industrial the mastery of technical risks linked with space activities and among others the management of satellites end of life. Moreover, during the 6th European conference on space debris, the European Space Agency (ESA) highlighted his will to reduce the amount of space debris orbiting around Earth with the "Clean Space" program.

In this new international context, the modelling of atmospheric re-entry of spacecraft became of major interest and several space agencies and private companies have developed tools to predict the atmospheric re-entry of space debris in order to assess their casualty area.

In this paper, we give an overview in Section 2 of prediction tools used by space agency and present their limitations. In Section 3 we introduce the Taguchi method, a statistical analysis method. In section 4, we present Calima a tool developed by R.Tech in collaboration with LAAS-CNRS.

Section 5 deals with the use of Calima and the Taguchi method in order to determine the probability density function of the impact point of an atmospheric re-entering object.

2. RELATED WORK

Several space agencies have developed tools to determine atmospheric re-entry of satellites or spacecraft, to assess their total casualty area, the state of the debris reaching the ground (remaining mass, kinetic

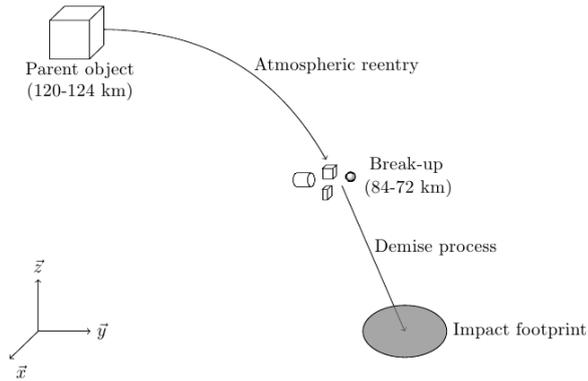


Figure 1: Sketch of spacecraft atmospheric re-entry

energy, etc. ...) and coordinates of impact point of each debris. To date, there are seven published atmospheric re-entry tools that can be divided in two categories: Object-oriented Code and the Spacecraft Oriented code.

Object-oriented Codes consider the satellite as an assembly of simple geometric shapes (box, sphere, cylinder or plate) contained in a parent object. Fig. 1 shows how the atmospheric re-entry of spacecraft or satellites is typically modelled in an object-oriented code. The satellite performs an atmospheric re-entry until it reaches a critical altitude (typically between 72 and 84 km), where it is decomposed into its elementary parts. Here fragmentation is prescribed; the break-up altitude is empirical. Then each component of the satellite is followed independently until it is demised or reaches the ground [9]. Among the seven published tools, six are object oriented: DAS (NASA) [10], ORSAT (NASA) [11], ORSAT-J (JAXA) [12], DRAMA-SESAM (ESA) [13], DEBRISK (CNES) [14] and DRAPS (CHINA) [15].

Spacecraft-oriented tools seek to model the spacecraft as close as possible to the real one. In such a tool, there is no prescribed break-up altitude, but fragmentation and ablation of the components of the spacecraft depend on the aerothermodynamics constraints during the atmospheric re-entry of the structure. In case of fragmentation, each fragment is followed independently until it demises or reaches the ground [9]. To the best of our knowledge there is only one published spacecraft-oriented tool: SCARAB [16] developed for ESA.

The main caveat of all these tools is that they are deterministic. However atmospheric re-entry is a very chaotic system. A slight modification of any initial condition can dramatically change the result of a simulation. Moreover, the deterministic approach does not allow considering the uncertainties on simulations due to the simplification of the aero and thermodynamic models used, the uncertainties of material properties at very high temperature or the uncertainties on the initial conditions of the re-entry. It is still possible to perform Monte-Carlo analysis with existing tools but due to the amount of parameters to consider this could be very time consuming. This raises two questions: is it possible

to reduce the amount of parameters to perturb and can we reduce computing time without using a supercomputer or a cluster.

3. TAGUCHI METHOD

The Taguchi method, introduced first by Genichi Taguchi in 1987 [1], is a statistical analysis method alternative to the traditional Monte-Carlo analysis to process probabilistic phenomenon. Unlike Monte-Carlo analysis which consists in performing a computation for each combination of input parameters, randomly selected on a given space parameter, and which therefore needs a huge amount of simulation to properly map the input space parameter [2], the Taguchi method permits one to perform only few computations (depending on the amount of input parameters: 27 computations for less than 13 input parameters, 36 computations for less than 23 parameters,...), by only taking “representatives” values of the input parameters. The counterpart is that unlike the Monte-Carlo analysis, which permits one to determine which set of input parameter drive to a given result, the Taguchi method only permits to determine which parameters are the most influential on the variation of a result and which have less influence, or none. So the Taguchi method is a good tool to reduce the amount of parameters to take into account for a Monte-Carlo analysis.

3.1. Taguchi Method principle

The Taguchi Method is based on the “Design of Experiment (DE) concept” developed by Ronald A. Fisher in the 1920s [2]. The aim of the design of experiment is to find the relationship between various input parameters of a model and how this model responds to them. The Taguchi method extends the design of experiment by considering uncertainties around input parameters.

To do so, the Taguchi method generates combination of “representative” values of the uncertain input parameters thanks to Orthogonal Arrays (OA). The OA are matrices, constructed by combining Latin Square [3], and are defined by three parameters:

- The number of levels, i.e. the set of values that can be taken by elements of the matrix.
- The factor number: the number of columns of the table. In the case of the Taguchi Method, each column matches an input parameter.
- The observation number: the number of rows of the table. For the Taguchi Method each row corresponds to an experiment (i.e. a combination of input parameters).

Taguchi has introduced many standard orthogonal arrays [1], as L_9 presented in Tab. 1 that is also called $L_{9,3}$, with 3 levels, a factor number equal to 4 and 9 observations.

Thus, L_9 permits one to define 9 experiments, each being a unique combination of the levels of the four

Experiments	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Table 1: Standard Orthogonal Array L_9

parameters. The key point of the standard Taguchi Method is to define three “representative” values for a given parameter that will correspond to the three levels in the OA. There exist variations of this method considering combination of input parameters with three “representative” values and others with two [8].

Basically, Taguchi Method considers a normal uncertainty around each considered input parameter [6] defined via a mean value μ and a variance σ . It is commonly accepted to consider the three following values as “representative”: μ , and $\mu \pm \Delta$, Δ being a tolerance around the mean value and taken equal to $\sqrt{1.5}\sigma$ (see [2], [3], [4] and [5]). It is common practice to match levels and representative values as follows:

- level 1 => $\mu - \Delta$,
- level 2 => μ ,
- level 3 => $\mu + \Delta$.

3.2. Analysis of Variance

Since the Taguchi Method only samples few values of the whole input space parameters, the result analysis must include an analysis of the confidence that can be placed in the results. That is the aim of a standard technique called Analysis of Variance (ANOVA). This technique permits one to determine the variability of the results, how this variability is influenced by values of the input parameters and what level of confidence can be assigned to the previous results.

ANOVA necessitates to compute many quantities and organizes them in a standard tabular format. We will not give the details in this paper all those parameters, but here is the list for informative purpose: the variance V , the sum of square SS , the pure sum of square SS' , degrees of freedom (D.o.F.) f , the error e , the variance ratio F , the percent contribution P , the correction factor CF , the number of experiments n and the total degree of freedom f_T (for more information interested readers should refer to chapter 5.3.4 of [7]). In order to determine which parameter has the biggest influence on a result and the level of confidence on these results the following parameters have to be considered: the Percent Contribution and the Variance Ratio.

Percent Contribution P permits one to determine how much the variation of a given result is caused by the effects of an input parameter. Thus, the input parameter with the highest Percent Contribution for a given result,

is the input parameter that has the biggest influence on this result. The percent contribution is obtained via Eq. 1

$$P = 100 * SS' / SS_t \quad (1)$$

where SS_t is the total square sum.

The Variance Ratio F , permits one to determine the confidence level if an input parameter really contributes to the variation of a given result. The F value computed is then compared with the values from the F-tables for a given level of significance. If the computed F value is less than the value from the F-table, then the parameter does not contribute to the variation of the result [7]. The variance ratio is obtained via Eq. 2.

$$F = V / V_e \quad (2)$$

where V_e denotes the variance of the error.

3.3. Monte Carlo method

The Monte Carlo method consists of using a random number generator to simulate a large number of combinations of parameters within tolerances. The results of every generated combination of input parameters is computed, after which the mean and the variance of the results are computed. The Monte Carlo method also permits one to explore the space of results in order to find the combination of input parameters that provides a desired result [3].

For obtaining accurate results that are not perturbed with statistical bias due to the generation of the random numbers, Monte Carlo method necessitates to perform a large number of simulation (thousands or hundreds of thousands), all the more important that there is input parameters, and such a method could be very time consuming, especially if each simulation requires a large amount of computing time.

4. CALIMA

Due to their deterministic approach, and because of the uncertainties in the models they used, all the existing tools use very conservative models that can be very limiting for industrials and threaten the durability of space activities. We decided to opt for a complete different approach by developing a brand new tool using less conservative models but able to perform statistical analysis : Calima.

Calima is a three Degree of Freedom (D.o.F.) object-oriented atmospheric re-entry simulation tool, developed by R.Tech, in collaboration with the C.D.A. team of LAAS-CNRS. So far, Calima is a basic tool with imposed aero-coefficients and no thermal model (no ablation).

The aim of Calima is to perform statistical analysis of atmospheric re-entry debris in order to determine the casualty area. Currently, there are 11 input parameters that can be perturbed in Calima: the initial mass, altitude, latitude, longitude, azimuth, speed, angle of

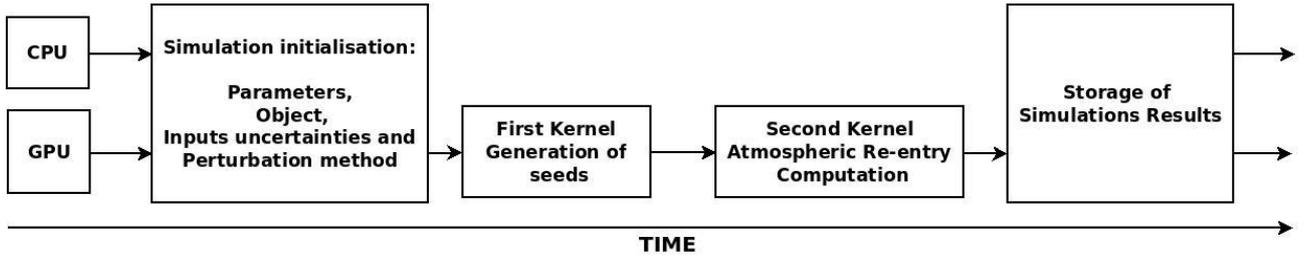


Figure 2: Steps of the GPU implementation of Calima

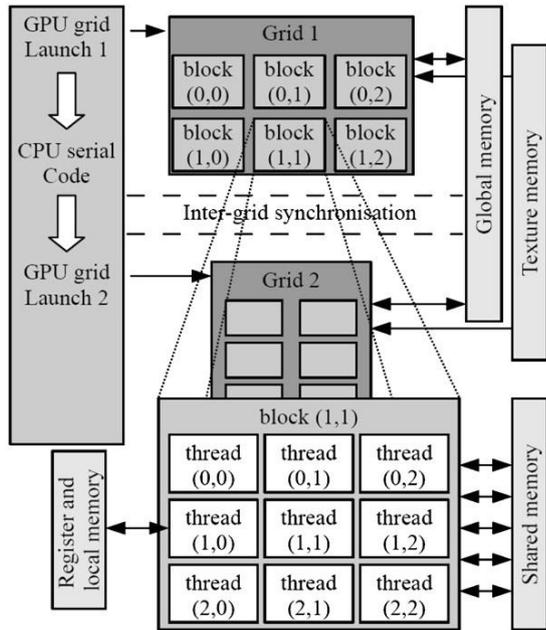


Figure 3: Thread and memory hierarchy in GPUs

attack, angle of yaw, angle of roll and Flight Path Angle (F.P.A.), using different perturbation methods : random, Gaussian, fixed steps or imposed values. Once the uncertainties on the parameters, and the perturbation method are filled in Calima, the tool will automatically, ensures the coverage of the input parameters space. Also, in order to prepare a Monte Carlo analysis Calima permits one to perform an automated Taguchi analysis, which will indicate to the user which are the input parameters most influencial.

Furthermore, Calima permits one to reduce the required computing time of Monte-Carlo analysis by taking advantage of Graphics Processing Unit (GPU) to parallelize computations.

Indeed, the GPUs are highly parallel, multithreaded, many-core architectures. They are better known for image processing. Nevertheless, NVIDIA introduced in 2006 CUDA (Compute Unified Device Architecture), a technology that enables users to take benefit of GPU cards to address parallel applications, multi-parametric problems and complex optimisation problems [17], [18], [19] [20]. As shown in Fig. 3, a parallel code on GPU (hereafter named the device), is interleaved with a serial code executed on the CPU (hereafter named the host).

The parallel threads are grouped into blocks which are organized in a grid. The grid is launched via a single CUDA program, the so-called kernel. The GPU implementation of Calima is performed using CUDA 6.0.

The implementation of Calima is divided in three synchronous steps, described Fig.2. First, the CPU part of the code loads the initial conditions, and uncertainties on the unified memory between the CPU and the GPU. Then, each parallel thread on the GPU performs a complete re-entry simulation from the initial point until the object reaches the ground, using a unique set of initial conditions derived from the initial conditions and uncertainties provided by the user. Finally, when threads have finished, the host recovers the results from the unified memory.

5. COMPUTATIONAL TESTS

In this chapter we display and analyse the results relative to the application of the Taguchi method. The goal is the determination of the impact area of an object and the range of kinetic energy at the impact, using the Calima code. As the current paper focusses on the methodology the presented results are not workable. Perturbed parameters and physical models implemented in Calima will be extended in future versions.

5.1. Computational test

We are considering a sphere with a diameter of 1.0 m and we define the initial state of this sphere with the parameters provided in Tab. 2. We assume Gaussian uncertainties around each of these parameters, with the mean values and standard deviation defined in Tab. 2.

Parameter	Mean μ	Standard deviation σ
Mass (kg)	247.224	4.12
Altitude (km)	120.0	0.25
Latitude ($^{\circ}$)	0.0	0.05
Longitude ($^{\circ}$)	0.0	0.1
Azimuth ($^{\circ}$)	45.0	0.15
Speed (m.s ⁻¹)	7272.582	48.51
Flight Path Angle	-2.612	0.01

Table 2: Initial conditions and uncertainties

We use the Orthogonal Array L_{27_3} , and we assign the

Parameter	D.o.F.: f	Sum of Square: SS	Variance: V	Variance Ratio: F	Pure Sum of Square: SS'	Percent Contribution: P (%)
Mass	2	2.18e+02	1.09e+02	8.43e+04	2.18e+02	92.38
Radius	2	9.41e-02	4.70e-02	3.63e+01	9.15e-02	0.04
Latitude	2	7.41e-04	3.70e-04	2.86e-01	-1.85e-03	0.0001
Longitude	2	2.96e-03	1.48e-03	1.14e+00	3.70e-04	0.0002
Azimuth	2	1.08e+00	5.38e-01	4.15e+02	1.07e+00	0.45
Speed	2	1.68e+01	8.41e+00	6.49e+03	1.68e+01	7.11
F.P.A.	2	7.41e-04	3.70e-04	2.86e-01	-1.85e-03	0.0008
Error	12	1.56e-02	1.30e-03	1.00	-2.59e-03	0.02

Table 3: Analysis of Variance table for Impact energy using initial conditions of Tab.2

Parameter	D.o.F.: f	Sum of Square: SS	Variance: V	Variance Ratio: F	Pure Sum of Square: SS'	Percent Contribution: P (%)
Mass	2	6.70e-05	3.50e-05	4.67e+03	6.70e-05	0.000008
Radius	2	3.72e-04	1.86e-04	2.48e+04	3.72e-04	0.00004
Latitude	2	4.76e-05	2.38e-05	3.18e+03	4.75e-05	0.000005
Longitude	2	8.86e+02	4.43e+02	5.92e+10	8.86e+02	99.9
Azimuth	2	1.87e-05	9.37e-06	1.25e+03	1.87e-05	0.000002
Speed	2	1.17e-02	5.87e-03	7.85e+05	1.17e-02	0.0015
F.P.A.	2	9.19e-09	4.60e-09	6.14e-01	-5.78e-09	-6.52e-10
Error	12	8.98e-08	7.49e-09	1.00	-1.50e-08	-1.68e-09

Table 4: Analysis of Variance table for Longitude using initial conditions of Tab.2

Parameter	D.o.F.: f	Sum of Square: SS	Variance: V	Variance Ratio: F	Pure Sum of Square: SS'	Percent Contribution: P (%)
Mass	2	2.37e-10	1.18e-10	9.32e-02	-2.30e-09	-1.04e-09
Radius	2	4.10e-11	2.05e-11	1.61e-02	-2.50e-09	-1.13e-09
Latitude	2	2.21e+02	1.11e+02	8.73e+10	2.21e+02	99.9
Longitude	2	1.70e-10	8.51e-11	6.71e-02	-2.37e-09	-1.07e-09
Azimuth	2	7.41e-14	3.70e-14	2.92e-05	-2.54e-09	-1.15e-09
Speed	2	7.41e-14	3.70e-14	2.92e-05	-2.54e-09	-1.15e-09
F.P.A.	2	2.96e-13	1.48e-13	1.17e-04	-2.54e-09	-1.15e-09
Error	12	1.52e-08	1.27e-09	1.00	-2.54e-09	-1.14e-09

Table 5: Analysis of Variance table for Latitude

Parameter	D.o.F.: f	Sum of Square: SS	Variance: V	Variance Ratio: F	Pure Sum of Square: SS'	Percent Contribution: P (%)
Mass	2	8.54	4.27	3.30e+03	26.1	26.09
Radius	2	0.11	5.44e-02	4.20e+01	0.11	0.32
Latitude	2	2.22e-03	1.11e-03	0.86	-3.70e-04	-1.13e-03
Longitude	2	4.66e-10	2.33e-10	1.80e-07	-2.59e-03	-7.92e-03
Azimuth	2	6.85	3.42	2.64e+03	6.84	20.91
Speed	2	1.72e+01	8.6	6.64e+03	17.20	52.57
F.P.A.	2	2.22e-03	1.11e-03	0.86	-3.70e-04	-1.13e-03
Error	12	0.02	1.30e-03	1.00	-2.59e-03	-7.92e-03

Table 6: Analysis of Variance table for Impact Energy using initial conditions of Tab.2

mass to the first column of the OA, the altitude to the second and so on. The three representative values for each of this input parameter are defined as explained in In this manner we define 27 experiments, each being a combination of the “representative” values of the seven input parameters. These combinations are automatically performed by Calima.

5.2. Analysis of Variance

Once the 27 cases are computed by Calima, the software performs the Analysis of Variance of the results. Tab. presents the ANOVA table for the variation of the kinetic energy. We can see immediately that the parameter having the biggest impact on the variation of the impact energy is the mass, with a 92.38% contribution. We note that the variance ratio is equal to $8.43 \cdot 10^4$, meaning that the level of confidence is higher than 99.5%. We can also note that the next largest contributor is the initial speed with only a 7% contribution, with a confidence level of 99.5% too. The other point to notice is that the initial latitude, longitude and F.P.A. have negligible influence on the variation of the impact energy, with contributions around $10^{-4}\%$. While performing the same analysis for the longitude and the latitude (Tab. 4 and 5) we note that the parameters that have the most impact on their variation are the initial latitude and longitude. In the same manner we note that the initial mass, azimuth and flight path angle have almost no impact on the variation of impact coordinates with contributions around $10^{-6}\%$ for the initials mass and the azimuth, and around $10^{-10}\%$ for the initial flight path angle.

All those results are strongly dependent on the “representative” values selected to perform the Taguchi analysis, i.e. on the estimation of the uncertainties on the initial conditions. Indeed the results presented here are only valid, for the uncertainties we assumed on the initial conditions, see Tab. 2. For example, if instead of considering the standard deviation for the input values of mass and initial speed, we consider a standard deviation of the uncertainty around the mass of 1kg, and a standard deviation for the uncertainty about the initial speed of $149 \text{ m}\cdot\text{s}^{-1}$ and the same uncertainties for the other input parameters, then the ANOVA table of the impact energy (Tab. 6) shows that now the input parameter that influence the most the variation of the impact energy is the initial speed, while the mass has almost the same contribution as the azimuth now. This highlights the importance to properly determine the uncertainties around the input parameters.

5.3. Monte-Carlo analysis

Thanks to the Taguchi Method, we have established that, for our case, we could only consider the initial altitude, latitude, longitude and speed to perform a Monte-Carlo analysis in order to determine the impact

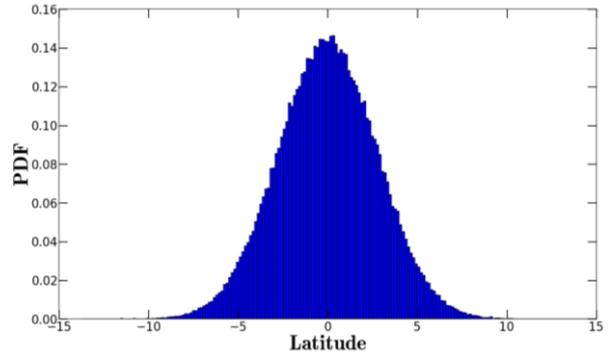


Figure 4: Probability Density Function of the latitude of the impact point

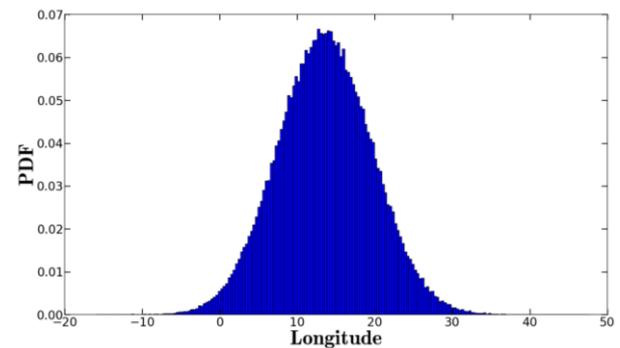


Figure 5: Probability Density Function of the Longitude of the impact point

area of our object, considering the same initial conditions and uncertainties as described in Tab. 2. The nominal simulation performed with Calima, using these parameters, impact the ground at 0.0° latitude and 13.56° longitude, with impact energy of 245.98 kJ.

In order to avoid statistical bias in the distribution of the results, we performed 200 000 simulations, in Calima. While such an amount of simulations would take around 300 hours of sequential computations using a classic re-entry simulation tool such like Debrisk on a classic CPU (Intel Xeon E5640), Calima performed all these computations in 88 seconds on a Tesla K40 GPU accelerator, with 2880 CUDA cores at 0.745GHz and 12 GB memory, four order of magnitude time faster. It is also interesting to note that, on the same CPU, Calima computes sequentially all those computations in 37 000s. Fig. 4 and 5 present the probability distribution of the impact point in latitude and in longitude. As we can see, the small uncertainties provided in the initial conditions lead to a great dispersion of the possible impact point. Thus the 3 sigma area defined is about 10° wide in latitude for 30° wide in longitude, for an ellipse of 50 257 km.

6. CONCLUSION AND FUTURE WORKS

In this paper we have shown the interest of using the

Taguchi Method for simulation of object re-entry in Earth's atmosphere. With few computations, this method permits one to determine which are the parameters that have the biggest impact on the variation of results of a re-entry model. One of the main advantages of this method is that the Taguchi method can be used with already existing tools.

We have also introduced Calima, the new object-oriented atmospheric re-entry simulation tool, in development by R.Tech, in collaboration with LAAS-CNRS. This new tool has the unique feature to intrinsically take into account the probabilistic nature of simulation parameters and initial condition, with automated exploration of the input space parameter, for Monte-Carlo analysis. Calima allows natively to perform a Taguchi analysis. Calima also includes the possibility to dramatically reduce the computing time by taking advantage of GPUs to massively parallelize the computation.

In future work, we will increase the amount of parameters that can be perturbed. We will also improve physical models in Calima, by including the possibility to define shapes of the object, computation of the aerocoefficient and taking into account ablation for examples. In long term evolution, we want to make a transition from object-oriented code to spacecraft-oriented code, and it is also envisaged to include the possibility to compute simultaneously several objects and their interactions (wake, etc...).

7. ACKNOWLEDGEMENT

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