Stochastic Optimization Based Approach for Multifingered Grasps Synthesis

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Summary

In this paper, we propose an approach for computing suboptimal grasps of polyhedral objects. Assuming n hard-finger contact with Coulomb friction model and based on central axes of the grasp wrench, we develop a new necessary and sufficient condition for n-finger grasps to achieve force-closure property. Accordingly, we reformulate the proposed force-closure test as a new linear programming problem, which we solve using an Interior Point Method. Furthermore, we present an approach for finding appropriate stable grasps for a robotic hand on arbitrary objects. We use the Simulated Annealing technique for synthesizing suboptimal grasps of 3D objects. Through numerical simulations on arbitrary shaped objects, we show that the proposed approach is able to compute good grasps for multifingered hands within a reasonable computational time.

KEYWORDS: Force-closure; Grasp planning; Multifingered robot hand.

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Notations

- n: Number of contacts.
- m: Number of side facets linearizing a friction cone.
- $\mathbf{c}_i$: Position vector of the ith contact ($i = 1, 2, \ldots, n$).
- $\mathbf{s}_{ij}$: The jth vector of the polyhedral cone at the ith contact ($i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$).
- $\mathbf{f}_i$: Contact force applied by the ith finger at contact point $\mathbf{c}_i$.
- $\mathbf{t}_{i/\mathbf{O}}$: Moment of the force $\mathbf{f}_i$ reduced at the origin $\mathbf{o}$.
- $\mathbf{t}_{ij}$: Primitive moment reduced at the contact point $\mathbf{c}_1$ ($i = 2, \ldots, n, j = 1, 2, \ldots, m$).
- $H(\mathbf{t}_{ij})$: Convex hull of the primitive contact moments $\mathbf{t}_{ij}$ reduced at $\mathbf{c}_1$.
- $\mathbf{w}_{ij}$: Primitive contact wrench ($i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$).
- $\mathbf{w}_i$: Wrench induced on the object by the contact force $\mathbf{f}_i$.
- $\mathbf{w}_g$: Wrench applied by the hand on the grasped object.
- Δ_g: Central axis of the grasp wrench $\mathbf{w}_g$.
- Δ_g^*: Central axes that pass throught a contact point $\mathbf{c}_i$.
- $(\mathbf{o}, \mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o)$: Coordinate system attached to the grasped object.
- $(\mathbf{o}_h, \mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$: Coordinate system attached to the hand's palm.
- $\mathbf{x}_{sp}$: Vector of the geometric parameters that describe the starting posture of the hand.
- r_{ball}: Raduis of the largest ball inscribed inside the convex hull of $\mathbf{w}_{ij}$.
- Q_{LP}: Optimal objective value of the proposed linear formulation.

1 Introduction

Multifingered hands are flexible and powerful mechanisms that can provide industrial, service and humanoid robots with a high capacity to perform both fine and complex manipulation tasks. However, an appropriate grasp planner is required to grasp and manipulate objects. Generally, the grasped object has arbitrary shape and described by a geometric model, transformed from modeler software or derived from sensor data. A grasp planner computes optimal contact locations while satisfying basic mechanical properties. The *force-closure* (FC) property is specially used in grasp analysis[1,2,3,4]. We say that a given grasp achieves FC, if the fingers can apply appropriate contact forces on the object to produce wrenches in any direction and hence, they compensate any external wrench (up to a certain magnitude).

Recall that several constraints must be satisfied in grasp planning process[5]. First, the contact forces applied on the object's surface must balance any external wrenches. This is accomplished by satisfying the FC condition. Second, the generated contact points must be reachable by the hand, which is ensured by considering the kinematics' structure of the robotized hand. Third, the grasp must be planned without collisions between the different solids involved (fingers, palm, object, and the environment). Finally, the generated grasps must be optimized with respect to one or several quality criteria.

The main contributions of our paper are:

• we show, using the polyhedral approximation of friction cones, that the FC condition can be transformed into a new formulation as a Linear programming Problem (LP). The proposed FC test is concluded from the feasibility/unfeasibility of the linear program. Because of its ability to discover LP unfeasibility[30], the Interior Point Method (IPM) is adopted. The algorithm we propose is computationally efficient and, the optimal solution of the LP defines a quantitative measure of FC grasps.

• we address the grasp optimization problem. Based on the proposed FC test, we present a fast grasp planning algorithm and use the Simulated Annealing (SA) technique for synthesizing suboptimal grasps of polyhedral objects. The proposed approach computes good grasps in very acceptable time and can be applied to various objects and robot hands.

• we implement the proposed grasp planner under the GraspIt! public simulator[6] and, we present some numerical results that show the effectiveness of the proposed approach.

The paper is organized as follow: In Section II, we review the most relevant works in grasp analysis and grasp synthesis of 3D objects. Section III presents an overview of the relevant pieces of grasping and central axis theories and, illustrates the relationship between central axes and FC property. In Section IV, we put forward a new necessary and sufficient condition for n-finger equilibrium and FC grasps and present the proposed FC algorithm. Section V introduces the proposed grasp planner. Section VI discusses the implementation of the proposed algorithms. Section VII concludes the paper.

2 Previous Works

Many research efforts have been directed towards testing the FC property of a given grasp. Salisbury and Roth[7] characterized the FC property by the following geometric condition: *"the primitive contact wrenches of contact forces positively span the entire wrench space"*. This condition is equivalent to saying that the origin of wrench space lies strictly inside the convex hull of the primitive contact wrenches[8]. For spatial grasps, Ponce *et al.*[9] illustrated that 4-finger FC grasps fall into three classes: concurrent, pencil and regulus grasps, and developed techniques for computing them. Jia-Wei Li *et al.*[10] extended their work in[11] and proposed a geometric algorithm for computing 3-finger FC grasps. Liu *et al.*[12] assumed that $n-1$ fingers do not achieve FC and they proposed an algorithm for computing all grasp points on the object for the nth finger to achieve FC with these $n-1$ fingers. Liu[13] formalized qualitative test of 3D FC grasps as an LP problem based on the duality between convex hulls and convex polytopes. Recently, Han *et al.*[14] pointed out that friction constraints have the form of Linear Matrix Inequalities (LMIs) and formulated the grasping force optimization problem as a convex optimization problem involving LMIs. For grasp quality, Kirkpatrick *et al.*[15] defined the quantitative measure as the radius of the largest sphere inscribed inside the convex hull of contact wrenches. This measure has been proposed in several forms, but it is best described by Ferrari and Canny[16]. In this paper, we develop a FC algorithm and giving rigorous theoretical demonstrations. Through numerical simulations, we confirm the real-time efficiency of the proposed algorithm when compared with the qualitative ray-shooting algorithm[13]. The advantage of the proposed FC test is its capability to give a good quality measure of the FC grasp without computing the sphere in six-dimensional wrench space[15], which efficiently reduces the computational cost.

Computing optimal FC grasps has been investigated for over two decades. Ponce and Faverjon[17] proposed a method for computing 4-finger grasps on polyhedral objects. Kirkpatrick *et al.*[15], Ferrari and Canny[16], Mishra *et al.*[8], Mirtich and Canny[18] and Mantriota *et al.*[19] addressed the problem of computing and planning optimal grasps. Zhu *et al.*[20] introduced the Q-distance and used the radius of the maximum volume Q-ball inscribed in the convex hull of wrenches as a measure of FC quality. In[21], Liu *et al.* developed an algorithm for searching FC grasp on a discretized object. Based on polyhedral approximation of friction cones, another algorithm was later proposed by Zhu *et al.*[22]. This iterative algorithm performs better than that based on Q-distance. These mentioned approaches do not consider the kinematic structure of the multifingered hand in grasps synthesis.

Due to the large search space resulting from all possible hand configurations, only few work in the literature aimed at planning grasps without ignoring the kinematic structure and the geometric configuration of the robot hand. Borst *et al.*[23] proposed a local method where they formulated the problem as a set of unconstrained optimization problems where the contact and kinematic constraints and the joint limits are introduced as penalty terms in the cost function. Anther local method was proposed by Rosell *et al.*[24], they presented an optimization method to iteratively compute joint movements that reduce the distance between the fingertips and the contact points. Rosales *et al.*[5] presented a method to identify all possible hand configurations reaching a given set of grasping points. Recently, Miller *et al.*[6] developed a public simulation environment, called "GraspIt!" (used in our implementation). They also proposed a grasp planner that consists of two parts:[25] Firstly, it begins by generating a set of starting grasp locations based on a simplified object model (such as spheres, cylinders, cones and boxes). In the second step, the grasp planner computes the six-dimensional convex hull of the primitive contact wrenches. If the origin is not within this convex hull, then the given grasp does not have FC. Otherwise, the grasp achieves FC and, its quality is the radius r_{ball} of the largest ball inscribed inside the convex hull. Using the "GraspIt!" simulator, Goldfeder *et al.*[26] presented a grasp planner that can consider an arbitrary object by decomposing it into superquadrics.

In contrast to some local methods[23,24], our paper proposes a general approach for analyzing and synthesizing FC grasps. We use SA technique to optimize the starting grasp locations, which define the initial configurations of the hand with respect to (w.r.t) the object coordinate frame. The SA technique allows one to avoid local optima traps since it explores globally the object's surface, and generates good grasps. Moreover, using the proposed approach, the grasp planner generates feasible grasps without solving the kinematics of the mechanical hand. At the same time, it does require any transformation of the object's model, which significantly reduce the computational cost of the grasp planning process.

3 Background

This section includes basic grasping terminologies and introduces the theory of the grasp wrench central-axis.

3.1 Contact Model and Grasp Wrench

Consider that n hard fingers are grasping a rigid object in a 3D workspace. A hard finger located at contact points $\mathbf{c}_i$ applies a force $\mathbf{f}_i$. A commonly used model of friction in robotic grasping and manipulation is Coulomb's law. Under this model, $\mathbf{f}_i$ is constrained to lie within the friction cone whose center is the internal normal to the surface at $\mathbf{c}_i$ with half-angle α (Fig. 1-a). The static friction coefficient $\mu = \tan(\alpha)$ depends on materials that are in contact. So, the grasp forces $\mathbf{f}_i$ must satisfy the following constraints

$$\sqrt{f_{ix}^2 + f_{iy}^2} \leqslant \mu f_{iz} \quad , \quad (i = 1 \cdots n) \tag{1}$$

Where (f_{ix}, f_{iy}, f_{iz}) denote the components of the grasp force $\mathbf{f}_i$ w.r.t. the ith coordinate frame $(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$, $\mathbf{z}_i$ is the internal normal to the surface object at contact point $\mathbf{c}_i$.

The non-linear friction constraints given by (1) can be relaxed using polyhedral approximation. Each cone is linearised by an m-sided polyhedral convex cone (Fig.1-b). Under this approximation, grasp force $\mathbf{f}_i$, expressed in the object coordinate frame, is given by

$$\mathbf{f}_i = \sum_{j=1}^{m} a_{ij} \mathbf{v}_{ij} \ ; \ \mathbf{v}_{ij} = T_i \mathbf{s}_{ij} \ , \ a_{ij} \geqslant 0 \tag{2}$$

The matrix T_i specifies the location of the ith coordinate frame w.r.t. the object coordinate frame. $\mathbf{s}_{ij}$ denotes the jth edge vector of the polyhedral convex cone expressed in the ith coordinate frame and satisfies $\mathbf{s}_{ij} \cdot \mathbf{z}_i = 1$. The sum $\sum_{j=1}^{m} a_{ij}$ specifies the amplitude of the normal component of the contact force $\mathbf{f}_i$.

A hard finger at $\mathbf{c}_i$ applies the moment $\mathbf{t}_{i/\mathbf{O}} = \mathbf{c}_i \times \mathbf{f}_i$ w.r.t. the origin $\mathbf{o}$. The force and the corresponding moment are stacked into a six-dimensional vector called *wrench*. The wrench induced on the object by the grasp force $\mathbf{f}_i$, denoted $\mathbf{w}_i$, applied at the origin $\mathbf{o}$, is given by

$$\mathbf{w}_i = \left[\mathbf{f}_i \ , \ \mathbf{t}_{i/\mathbf{O}}\right]^T = \sum_{j=1}^{m} a_{ij} \mathbf{w}_{ij} \tag{3}$$

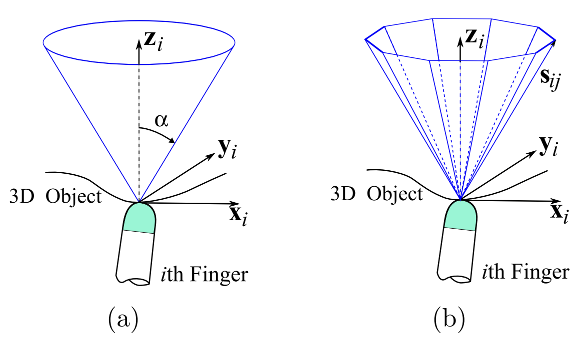


Figure 1: Interpretation of the friction law: (a) Spatial-friction cone, (b) Quadratic cone approximated by an m-sided polyhedral cone.

Where $\mathbf{w}_{ij}$ denotes the primitive contact wrenches of the ith finger. They are given, w.r.t. the object coordinate frame, by

$$\mathbf{w}_{ij} = [\mathbf{v}_{ij}\ ,\ \mathbf{c}_i \times \mathbf{v}_{ij}]^T \tag{4}$$

The net wrench applied by the hand on the grasped object is the sum of all primitive contact wrenches. It is given by

$$\mathbf{w}_g = \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}\mathbf{w}_{ij} = \left[\mathbf{f}_g\ ,\ \mathbf{t}_{g/\mathbf{O}}\right]^T \tag{5}$$

The whole external wrench applied on the object is the sum of the applied grasp wrench by the robotic hand $\mathbf{w}_g$ and the required wrench to achieve the desired task $\mathbf{w}_t$ (perturbations are included). It is given by

$$\mathbf{w}_{ext} = \mathbf{w}_g + \mathbf{w}_t = \left[\mathbf{f}_g\ ,\ \mathbf{t}_{g/\mathbf{O}}\right]^T + \ \left[\mathbf{f}_t\ ,\ \mathbf{t}_{t/\mathbf{O}}\right]^T \tag{6}$$

3.2 Central Axes of the Grasp Wrench

Poinsot's central-axis theorem states that every system of wrenches is equivalent to a single force plus a single moment acting on the same line[27]. Therefore, assuming that the resultant grasp forces is not zero ($\mathbf{f}_g \neq 0$), the central axis Δ_g of the grasp wrench $\mathbf{w}_g$ is defined as follows:

$$\Delta_g = \left\{(\mathbf{f}_g \times \mathbf{t}_{g/\mathbf{O}} / \parallel \mathbf{f}_g \parallel^2) + \lambda \mathbf{f}_g\ \ : \lambda \in \mathbb{R}\right\} \tag{7}$$

Δ_g is a directed line in the $\mathbf{f}_g$ direction that passes through point $\mathbf{I} = \mathbf{f}_g \times \mathbf{t}_{g/\mathbf{O}} / \parallel \mathbf{f}_g \parallel^2$. The moment about Δ_g is:

$$\mathbf{t}_{g/\mathbf{I}} = \mathbf{f}_g \cdot \mathbf{t}_{g/\mathbf{O}} \cdot \mathbf{f}_g / \parallel \mathbf{f}_g \parallel^2 \tag{8}$$

Using an example, we illustrate the relationship between the FC condition and the central axes of the grasp wrench. We vary randomly the amplitudes and the orientations of fingertip forces $\mathbf{f}_i$ inside the corresponding friction cones using equation (2). Grasp wrench central axes are computed from (7). In section 4, we will demonstrate that a necessary FC condition is satisfied if the grasp wrenches can generate central axes that positively span $\mathbb{R}^3$ w.r.t. any arbitrary point. The following examples show the central axes that pass through the origin $\mathbf{o}$.

3.2.1 Examples

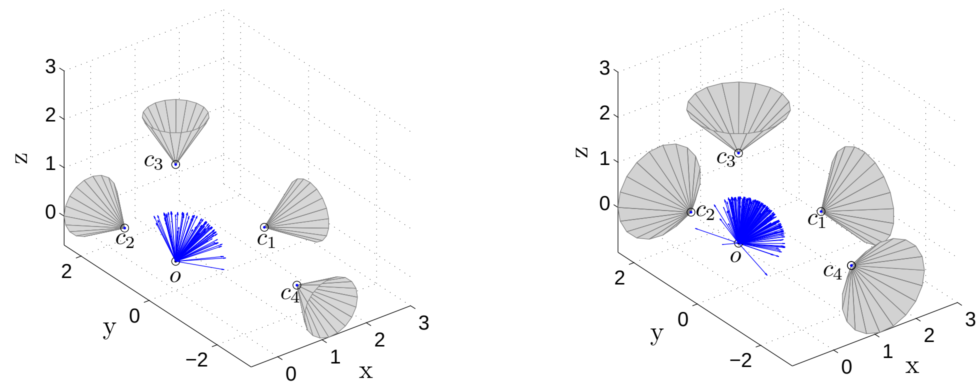
We use the examples presented in[28], which are 4-fingered grasps of a polyhedral object. The contact points $\mathbf{c}_i$ and the normal vectors $\mathbf{z}_i$ are given by:

$$\mathbf{c}_1 = \left(\ 2,\ \ 0,\ \ 0\ \right);\ \mathbf{c}_2 = \left(\ 0,\ \ 1.5,\ \ 0\ \right);\ \mathbf{c}_3 = \left(\ 0,\ \ 0,\ \ 2\ \right);\ \mathbf{c}_4 = \left(\ 1.2,\ \ -2,\ \ 0\ \right)$$

$$\mathbf{z}_1 = \left(\ -1,\ \ 0,\ \ 0\ \right);\ \mathbf{z}_2 = \left(\ 0,\ \ -1,\ \ 0\ \right);\ \mathbf{z}_3 = \left(\ 0,\ \ 0,\ \ -1\ \right);\ \mathbf{z}_4 = \left(\ 0,\ \ 1,\ \ 0\ \right)$$

This grasp is not FC when $\mu = 0.3$. For the central axes, Fig. 2-a shows that they cannot entirely positively span $\mathbb{R}^3$ at the origin $\mathbf{o}$ and consequently, the corresponding grasp wrenches cannot generate all possible central axes. When the friction coefficient reaches the value $\mu = 0.5$, the given grasp becomes FC. In Fig. 2-b, we can see that the central axis positively spans $\mathbb{R}^3$ at $\mathbf{o}$.

Many simulations have been done on several planar and spatial grasps. We have noticed that if a grasp is force-closed, its wrench can generate any arbitrary central axis. Accordingly, we will derive, in the next Section, a FC test based on the grasp wrench central axes independently of the fingers number.



(a) The grasp is not FC for $\mu = 0.3$. The central axis cannot span positively $\mathbb{R}^3$ at $\mathbf{o}$.

(b) The grasp achieves FC if $\mu >= 0.5$. The central axis positively spans $\mathbb{R}^3$ at $\mathbf{o}$.

Figure 2: The central axes that pass through the origin $\mathbf{o}$.

4 Force-Closure Condition

In the present section, we rigorously prove the proposed equilibrium and FC conditions[33]. Therefore, we formulate the FC condition as a new LP and we put forward the proposed FC algorithm.

4.1 Necessary and Sufficient Equilibrium Condition

In equilibrium study, we consider only the grasp forces applied by the n fingers of the hand. We split these forces into two categories: force $\mathbf{f}_i$ which is applied by the ith finger at the contact point $\mathbf{c}_i$ and forces $\mathbf{f}_r$ applied by the other $n-1$ fingers. Thus, the object is under the action of two wrenches $\mathbf{w}_i = (\mathbf{f}_i, \mathbf{t}_{i/\mathbf{c}_i})^T$ and $\mathbf{w}_r = (\mathbf{f}_r, \mathbf{t}_{r/\mathbf{c}_i})^T$ w.r.t the point $\mathbf{c}_i$. Torques $\mathbf{t}_{i/\mathbf{c}_i}$ produced by $\mathbf{f}_i$ w.r.t. $\mathbf{c}_i$ are zero because $\mathbf{f}_i$ pass through point $\mathbf{c}_i$. So, the equilibrium condition is:

$$\mathbf{w}_g = \mathbf{w}_i + \mathbf{w}_r = 0 \Longrightarrow \begin{cases} \mathbf{f}_r = -\mathbf{f}_i \\ \mathbf{t}_{r/\mathbf{c}_i} = 0 \end{cases} \tag{9}$$

According to Poinsot's theorem, we subdivide the central axes Δ_g given by (7) into two classes: Δ_i are the central axes of the wrench $\mathbf{w}_i$ and Δ_r are the central axes of $\mathbf{w}_r$. The second condition ($\mathbf{t}_{r/\mathbf{c}_i} = \mathbf{0}$) in (9) defines a subclass of central axis Δ_r^* with zero torques and passing through $\mathbf{c}_i$. We denote Δ_g^* the union of Δ_i and Δ_r^* ($\Delta_g^* = \Delta_i \cup \Delta_r^*$). Now, we put forward the following proposition for n-finger equilibrium grasps.

Proposition 1: *A grasp can achieve equilibrium iff the $n-1$ first fingers can generate, at least, one central axis of class Δ_r^* that is opposite to one central axis of class Δ_i which is generated by the last ith finger.*

Proof: *i) Sufficiency*: when the wrenches $\mathbf{w}_r = (\mathbf{f}_r, \mathbf{t}_{r/\mathbf{c}_i})^T$ applied by the $n-1$ fingers can generate one central axis Δ_r^*, the torque around this axis $\mathbf{t}_{r/\mathbf{c}_i}$ is zero. Hence, the second condition in (9) is satisfied. The direction of the central axis Δ_r^* is defined by that of $\mathbf{f}_r$ and the direction of the central axis Δ_i is that of $\mathbf{f}_i$. So, if Δ_r^* and Δ_i have opposite directions, we can write $\mathbf{f}_r = -\delta\mathbf{f}_i$ with $\delta > 0$. Then, the wrenches $\mathbf{w}_r$ can produce forces that assure equilibrium.

ii) *Necessity*: we now consider the case where the wrench $\mathbf{w}_r$ is unable to generate any central axis of class Δ_r^* that passes through point $\mathbf{c}_i$. Thus, the forces $\mathbf{f}_r$ produce a non-zero torque $\mathbf{t}_{r/\mathbf{c}_i}$ w.r.t. $\mathbf{c}_i$. This torque cannot be balanced by the ith finger because the torques $\mathbf{t}_{i/\mathbf{c}_i}$ produced by $\mathbf{f}_i$ w.r.t. $\mathbf{c}_i$ are zero. Therefore, the grasp is not in equilibrium and the condition of Proposition 1 is necessary. □

4.2 Necessary and Sufficient Force-Closure Condition

Force-closure is often used to characterize a grasp intended to immobilize an object. This is equivalent to saying that the grasp wrench $\mathbf{w}_g$ can balance any task wrench $\mathbf{w}_t$ [29]. Using this definition, we state a necessary and sufficient condition on the grasp wrench $\mathbf{w}_g$ to achieve FC.
Proposition 2: *Given an arbitrary point (e.g.. $\mathbf{c}_i$), an n-finger grasp is force-closure iff*:
(i)- all grasp-wrench central axes of class Δ_g^* can positively span $\mathbb{R}^3$ at $\mathbf{c}_i$, and
(ii)- the torque applied by the n fingers, positively span $\mathbb{R}^3$ at $\mathbf{c}_i$.
Proof: *i*) *Sufficiency*: at least, 4 central axes are needed to positively span $\mathbb{R}^3$ at the point $\mathbf{c}_i$. So, we consider 4 forces $\left\{\mathbf{f}_{g1}^*,\ \mathbf{f}_{g2}^*,\ \mathbf{f}_{g3}^*,\ \mathbf{f}_{g4}^*\right\}$ that positively span the entire $\mathbb{R}^3$ at $\mathbf{c}_i$ (Fig. 3). We also consider that the torques $\mathbf{t}_{g/\mathbf{c}_i}$, applied by all contact forces w.r.t. $\mathbf{c}_i$, can positively span $\mathbb{R}^3$ at $\mathbf{c}_i$. Therefore, we have the following wrench w.r.t. $\mathbf{c}_i$:

$$\mathbf{w}_{g/\mathbf{c}_i}^* = \left[\mathbf{f}_g^*\ ,\ \mathbf{t}_{g/\mathbf{c}_i}\right]^T \tag{10}$$

Where $\mathbf{f}_g^*$ denotes grasp forces applied along central axes Δ_g^*, and $\mathbf{t}_{g/\mathbf{c}_i}$ is the torque applied by all contact forces w.r.t. $\mathbf{c}_i$. $\mathbf{f}_g^*$ and $\mathbf{t}_{g/\mathbf{c}_i}$ are independent entities because torques produced by $\mathbf{f}_g^*$ w.r.t. $\mathbf{c}_i$ are zeros. Hence, if conditions (i) and (ii) are satisfied, the wrench $\mathbf{w}_{g/\mathbf{c}_i}^*$ can balance any task wrench $\mathbf{w}_{t/\mathbf{c}_i}$ applied at $\mathbf{c}_i$.

Obviously, if the n fingers can produce independent forces and torques w.r.t. any point $\mathbf{p}$, where the forces and the moments positively span $\mathbb{R}^3$ at $\mathbf{c}_i$, then the grasp is FC. Hence, we will proof that if Proposition 2 is satisfied, the grasp wrench can produce independent forces and torques w.r.t. any point $\mathbf{p}$.

The forces $\mathbf{f}_g^*$ produce a torque $\mathbf{t}_{g/\mathbf{p}}^* = (\mathbf{c}_i - \mathbf{p}) \times \mathbf{f}_g^*$ w.r.t. any $\mathbf{p}$. This torque $\mathbf{t}_{g/\mathbf{p}}^*$ can balance any external torques except those around the axis $\mathbf{c}_i\mathbf{p}$. An external torque around $\mathbf{c}_i\mathbf{p}$ can be balanced by the grasp torque $\mathbf{t}_{g/c_i}$ because it can positively span $\mathbb{R}^3$ at $\mathbf{c}_i$. Then, the

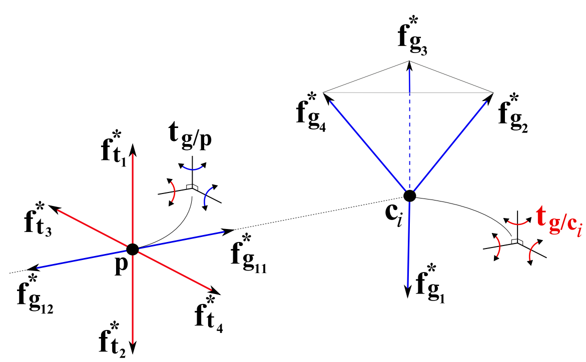


Figure 3: Interpretation of the FC condition, if proposition 2 is satisfied at $\mathbf{c}_i$, the grasp wrench can produce independent forces and torques w.r.t. any point $\mathbf{p}$.

wrench $\mathbf{w}^*_{g/\mathbf{p}}$ can balance any task torques applied at any $\mathbf{p}$. The torques $\mathbf{t}_{g/c_i}$ can be expressed by several forces acting on the point $\mathbf{p}$ (e.g. $\{\mathbf{f}^*_{t1},\ \mathbf{f}^*_{t2},\ \mathbf{f}^*_{t3},\ \mathbf{f}^*_{t4}\}$). These forces can balance any task forces except those along $\mathbf{c}_i\mathbf{p}$. Task forces along $\mathbf{c}_i\mathbf{p}$ can be balanced by $\mathbf{f}^*_g$ because they can produce two central axis along this direction ($\{\mathbf{f}^*_{g11},\ \mathbf{f}^*_{g12}\}$). We conclude that, at any $\mathbf{p}$, $\mathbf{w}^*_{g/\mathbf{p}}$ can balance any arbitrary task wrench $\mathbf{w}_{t/\mathbf{p}}$ and the spatial grasp is FC.
ii) *Necessity*: obviously, if condition (i) is not satisfied, there exist task forces that cannot be balanced by $\mathbf{f}^*_g$. Further, when the torques produced by the n fingers w.r.t. an arbitrary point cannot have both signs (around any direction), the grasp is non FC. Hence, the two conditions of Proposition 2 are necessary. □

With Coulomb friction, if a given grasp is in equilibrium and the central axis Δ^*_r is pointing strictly within the negative ith friction cone, the condition (i) is automatically satisfied. According to (7), the central axes Δ_i of wrenches $\mathbf{w}_{i/\mathbf{c}_i} = (\mathbf{f}_i, \mathbf{0})^T$ are the contact forces in the ith friction cone. Hence, if one central axis of class Δ^*_r that passes through $\mathbf{c}_i$ and pointing strictly within the negative ith friction cone (cone pointing outside the object) exists, the central axes Δ^*_g can positively span $\mathbb{R}^3$ at $\mathbf{c}_i$. Consequently, condition (i) can be formulated by the following system:

$$\begin{cases} \sum_{j=1}^{n} \mathbf{f}_j = -\delta \mathbf{z}_i \\ \sum_{j=1}^{n} ((\mathbf{c}_j - \mathbf{c}_i) \times \mathbf{f}_j) = \mathbf{0} \end{cases} \quad (\delta > 0) \tag{11}$$

4.3 Linear Algorithm for Testing FC grasps

The problem of computing n-finger FC grasps is simplified by using Proposition 2. We start by verifying that all central axes Δ^*_g produced by the grasp wrench can positively span $\mathbb{R}^3$ at $\mathbf{c}_1$. The second step consists of computing the resulting torque applied by the n friction cones w.r.t. the first contact point $\mathbf{c}_1$. If this torque can positively span $\mathbb{R}^3$ then the grasp is FC. We recall that the quadratic friction cone is approximated by an m-sided polyhedral convex cone (Fig. 1-b).

A set of vectors positively span $\mathbb{R}^3$ if any vector in $\mathbb{R}^3$ can be written as a positive combination of the given vectors[17]. Hence, Δ^*_g can positively span $\mathbb{R}^3$ at $\mathbf{c}_1$ iff one central axis of class Δ^*_r that passes through $\mathbf{c}_1$ and pointing inside the negative first cone exists. Hence, we can formulate the first condition in Proposition 2 as the following system of equations:

$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} \mathbf{v}_{ij} = -\delta \mathbf{z}_1 \\ \sum_{i=2}^{n} \sum_{j=1}^{m} ((\mathbf{c}_i - \mathbf{c}_1) \times a_{ij} \mathbf{v}_{ij}) = \mathbf{0} \end{cases} \ ; \ a_{ij} \geqslant 0 \ , \ \delta > 0 \tag{12}$$

For spatial frictional grasps, Δ^*_g can positively span $\mathbb{R}^3$ at $\mathbf{c}_1$ iff their positive combination can produce vectors along the axis $-\mathbf{z}_1$. We normalize the torques by r, the maximum radius from the wrench space origin, often the centre of mass. This ensures that the quality of a grasp will be independent of the object scale[6]. Dividing also by δ yields

$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} \mathbf{v}_{ij} = -\mathbf{z}_1 \\ \sum_{i=2}^{n} \sum_{j=1}^{m} \mathbf{d}_i \times x_{ij} \mathbf{v}_{ij} = \mathbf{0} \end{cases} \ ; \ x_{ij} \geqslant 0 \tag{13}$$

where $\mathbf{d}_i = \frac{1}{r}(\mathbf{c}_i - \mathbf{c}_1)$ and $x_{ij} = \frac{a_{ij}}{\delta}$.

In the grasp planning process, we have to quantify the FC in order to optimize the generated grasps. So, we reformulate (13) by the following linear program:

$$\min_{\mathbf{x}=(x_{11},x_{12},\cdots,x_{2m},\cdots,x_{nm})^T} \left\{ \boldsymbol{f}^T \mathbf{x} \ : \ A\mathbf{x} = -\mathbf{z}_1, \ \mathbf{x} \geqslant \mathbf{0} \right\} \tag{14}$$

the matrix A of dimension $(6 \times mn)$ is given by

$$A = \begin{pmatrix} \mathbf{v}_{11} & \cdots & \mathbf{v}_{1m} & \mathbf{v}_{21} & \cdots & \cdots & \mathbf{v}_{nm} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{t}_{21} & \cdots & \cdots & \mathbf{t}_{nm} \end{pmatrix} \tag{15}$$

where $\mathbf{t}_{ij} = \mathbf{d}_i \times \mathbf{v}_{ij}$.

If a given grasp is FC and the first finger must participate to generate resultant force along the negative direction of $\mathbf{z}_1$ axis (e.g.. Fig. 7-b) then the remaining $n-1$ fingers cannot produce resultant force along $-\mathbf{z}_1$ and the grasp will be weak. In order to quantify this grasp, the coefficients of x_{1j} in the vector $\boldsymbol{f}$ must be greater than the remaining coefficients (of x_{ij} with $i \neq 1$). In this work, the coefficients of x_{1j} are arbitrary set to n, the other coefficients are set to 1.

The torque applied by the n friction cones w.r.t. $\mathbf{c}_1$ can positively span $\mathbb{R}^3$ if the contact point $\mathbf{c}_1$ lies strictly inside the convex hull $H(\mathbf{t}_{ij})$ of $\mathbf{t}_{ij}$. This can be verified using matrix computations[27] or by computing the sphere in three-dimensional moment space. Hence, the following necessary condition must be satisfied:

$$\mathbf{c}_1 \in H(\mathbf{t}_{ij}) \tag{16}$$

To solve the proposed linear program, we have used the Interior Point Method (IPM) which is very effective in discovering linear program unfeasibility[30]. This characteristic is very suitable for a grasp planner in order to rapidly test the FC property. If the linear program is consistent then the optimal solution is $\boldsymbol{f}^T\mathbf{x}^* = \frac{\boldsymbol{f}^T\mathbf{a}^*_{ij}}{\delta^*}$. The proposed FC quality gives the minimal contact forces $\mathbf{a}^*_{ij}$ that contribute to obtain δ^*, the maximum of the force along $-\mathbf{z}_1$.

Algorithm 1 : Q_{LP} = `FC_Test`$(n,\ \mathbf{c}_i,\ \mathbf{z}_i,\ \mu)$; $(i = 1 \ldots n)$

Require: $n \geqslant 3$
Ensure: Q_{LP};

```
if n < 3 then
    Q_LP ← +∞ {the grasp is not FC}
    return Q_LP
else
    if (16) is not satisfied then
        Q_LP ← +∞ {the grasp is not FC}
        return Q_LP
    else
        if (14) is inconsistent then
            Q_LP ← +∞ {the IPM returns the infeasibility of the LP}
            return Q_LP
        else
            Q_LP ← f^T x* {the grasp is FC and the IPM returns the optimal solution}
            return Q_LP
        end if
    end if
end if
```

In Algorithm 1, we describe the proposed n-finger FC test (where $n \geqslant 3$ contact points). Thus, if the condition (16) is not satisfied or if the LP given by (14) is inconsistent then, the given grasp is not FC. Otherwise, the grasp achieves FC and its quality Q_{LP} is equal to the optimal solution of (14).

5 Optimal FC Grasps Synthesis

Miller *et al.*[25] have proposed a grasp planner that begins by generating a set of starting grasp locations based on a simplified object model (such as spheres, cylinders, cones and boxes) and, moving the preshaped hand along predetermined directions toward the object in order to generate automatic grasps. For each grasp, the grasp planner computes the six-dimensional convex hull of the primitive contact wrenches. If the origin is not contained within this convex hull then, the given grasp does not have FC. Otherwise, the grasp achieves FC and, its quality is the radius r_{ball} of the largest ball inscribed inside the convex hull.

Inspired by the idea of this work[25], we synthesize stable grasps by automatically generating the hand starting posture. Our main idea rests on a stochastic technique to generate these postures, for which the proposed FC qualitative test is included. This new approach makes it possible to obtain feasible grasps without solving the kinematics of the mechanical hand and without simplifications of the object model. The main differences between the proposed approach and the one advanced in[25] are: First, the proposed approach does not required simplifications in the model of the grasped object, which reduce the required computational time. Second, the starting postures of the hand are not fixed in advance but, we use a stochastic technique to globally explore the object's surface. This technique offers more chance to avoid local optima traps. Third, the test and the evaluation of the FC grasps are based on the new FC test algorithm described in Section 4. The proposed FC test algorithm is computationally more efficient and the optimal solution of the proposed LP defines a quantitative measure of the FC grasp. Finally, we recall that the proposed approach can easily adapt to various hand kinematic structures.

In this Section, we begin by expressing the different parameters defining the hand's starting posture w.r.t. the object's coordinate frame. Secondly, we present the formulation of the optimization problem and finally, we summarize the main steps of the proposed grasp planner.

5.1 Grasp Generation

Kinematically speaking, a multifingered hand has a *tree topology*[34]. It has the structure of a palm generally attached to a six degrees of freedom (DOF) manipulator, and a set of fingers. A finger consists of a number (two to four) of revolute coupled links playing the role of phalanges. In many instances[34], the joints of a finger are not independently actuated.

In this Section, we have chosen (without loss of generality) the Barrett hand to state our approach. This hand, produced by Barrett Technology, is based on a design developed at the University of Pennsylvania[35]. It is a three-fingered mechanical hand with each finger having two revolute joints (Fig. 4). One finger is stationary and the two others can spread synchronously about the palm by an angle $\theta_3 \in [0, \pi]$ rad. The Barrett hand has four actuated joints. Each of the three fingers has one actuated proximal link θ_{i1} $(i = 1 \cdots 3)$, and a coupled distal link which rotates at a fixed rate θ_{i2} with the proximal link. A clutch mechanism allows the distal link to continue its rotation if the proximal link's motion is obstructed. Therefore, the Barrett hand has four internal DOF: one for the spread angle of the fingers θ_3, and three for the angles of the proximal links θ_{i1}.

5.1.1 Starting Posture

In Fig.4, two right-hand rectangular coordinate systems are given. The first coordinate system $(\mathbf{o}, \mathbf{x}_o, \mathbf{y}_o, \mathbf{z}_o)$ is attached to the grasped object, the origin $\mathbf{o}$ coincides with the object center of mass (the point $\mathbf{o}$ is also considered as the origin of the fixed reference frame). The second coordinate system $(\mathbf{o}_h, \mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$, is attached to the palm of the Barrett hand with the coordinate axis $\mathbf{z}_h$ perpendicular to the palm's surface. We define the position of point $\mathbf{o}_h$ w.r.t the object's coordinate frame as follows:

$$\mathbf{o}_h = \rho_h \left[\ \sin\theta_2\cos\theta_1, \quad \sin\theta_2\sin\theta_1, \quad \cos\theta_2\ \right]^T \tag{17}$$

where ρ_h is selected sufficiently high to assure collision avoidance between the Barrett hand and the grasped object.

The rotations of the hand's reference frame w.r.t the object-attached frame, is defined by the angle θ_4 and the the spherical coordinates of the starting point $\mathbf{o}_s$. So, we get:

$$\mathbf{o}_s = \rho_s \left[\ \sin q_2\cos q_1, \quad \sin q_2\sin q_1, \quad \cos q_2\ \right]^T \tag{18}$$

where point $\mathbf{o}_s$ belongs to the grasped object. So, the positive parameter ρ_s cannot exceed the maximal radius r of the object from the origin $\mathbf{o}$.

Thus, using the coordinates of points $\mathbf{o}_h$ and $\mathbf{o}_s$ given in (17) and (18) respectively, we define the components of the vector $\mathbf{z}_h$ as follows:

$$\mathbf{z}_h = \frac{\mathbf{o}_s - \mathbf{o}_h}{\|\mathbf{o}_s - \mathbf{o}_h\|} \tag{19}$$

Then, the two coordinate axes $\mathbf{x}_h$ and $\mathbf{y}_h$ are determined using a basic rotation matrix for a rotation about $\mathbf{z}_h$ axis with θ_4 angle.

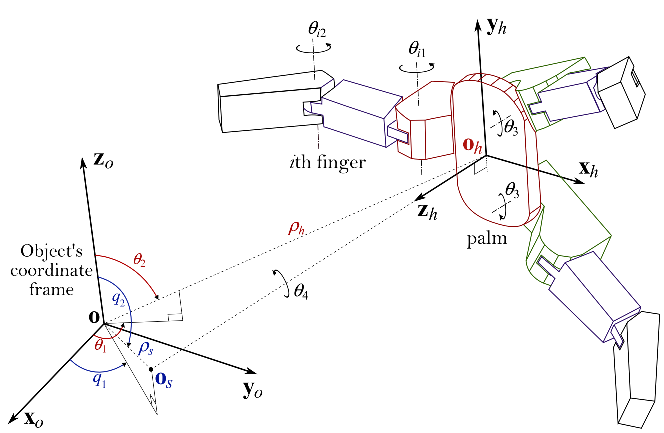


Figure 4: The different parameters which define the starting posture of the Barrett hand w.r.t the object coordinate frame.

We can notice that, with the variation of the two parameters θ_1 and θ_2, the origin $\mathbf{o}_h$ of the palm can sweep a sphere's surface of fixed radius ρ_h all around the object. Adding the variation of starting point $\mathbf{o}_s$ locus and the rotation about $\mathbf{z}_h$ axis, we provide the mechanical hand with the ability to explore the object's geometry to look for grasps along all possible directions.

For the internal DOF of the Barrett hand, the spread angle θ_3 is also considered as a decision variable. With $\theta_3 \in [0, \pi]$ rad, the starting posture of the Barrett hand can have different preshapes. The three angles of the proximal links θ_{i1} are initially set to zero.

Finally, we denote $\mathbf{x}_{sp}$ the vector of the geometric parameters that describe the starting posture of the mechanical hand, w.r.t. the object-attached frame. For the Barrett hand, we define the components of this vector as follows:

$$\mathbf{x}_{sp} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & q_1 & q_2 & \rho_s \end{bmatrix}^T \tag{20}$$

5.1.2 Grasp Testing and Evaluation

To perform a grasp, the Barrett hand is initially positioned at the starting posture. The hand is then translated along the coordinate axis $\mathbf{z}_h$ until it is prevented from moving further by a contact. Third, the fingers are closed around the object until contacts or joint limits (θ_{i1}) prevent from further motion. If at least three contacts between the hand and the object exist, the obtained grasp is evaluated using the proposed FC test (Algorithm 1). Otherwise, the grasp is not FC. Fourth, the angles of the proximal links (θ_{i1}) are initialized, the hand is slightly backed away from the object along the axis $\mathbf{z}_h$ with a small distance d_{back} and the fingers are closed again. Finally, This backing off iteration continues until the fingers reach the object and the corresponding grasp can be evaluated. The back distance d_{back} can be selected according to the hand scale.

5.2 Grasp Optimization

It is clear that using an arbitrary starting posture (Fig. 4), the object can be grasped by the Barrett hand from any direction. Thus, the hand can better explore the object in order to find feasible grasps. However, the main problematic is how to find the best starting posture of the Barrett hand that may lead to an optimal grasp. Therefore, we reformulate the grasp planning as an optimization problem.

5.2.1 Problem statement

The optimization problem can be stated as follows: let $\mathbf{x}_{sp}$ denote the vector of the geometric parameters which define the initial posture of the Barrett hand w.r.t. the object's coordinate system and, try to find the optimal solution $\mathbf{x}_{sp}^*$. This solution corresponds to the optimal FC grasp according to a given performance criterion while all constraints are respected.

5.2.2 Constraints

Generating feasible grasps with the Barrett hand is restricted by numerous constraints that must be satisfied. These constraints concern boundary conditions and non-collision conditions between the mechanical hand and the task environment.

- **Boundary conditions**: They allow the mechanical hand to look for feasible grasps in the interior of a subspace which includes the grasped object. Additional constraints that may be considered represent the physical limitations of the mechanical hand. Hence, the vector of the geometric parameters must satisfy the following constraints:

$$\mathbf{x}_{sp} \leqslant \left[\begin{array}{ccccccc} 2\pi & \pi & \pi & 2\pi & 2\pi & \pi & r \end{array}\right]^T \tag{21}$$

where the components of $\mathbf{x}_{sp}$ are non-negative real numbers. Besides these constraints, the extrema of all joint trajectories must be within the geometric limits of each joint. The actuated-joint limits are given by:

$$0 \leqslant \theta_{i1} \leqslant 0.8\pi \;\; ; \;\; (i = 1 \cdots 3) \tag{22}$$

where the linear equations that give unactuated joints θ_{i2} as a function of the actuated ones θ_{i1} have the form:

$$\theta_{i2} = \frac{\theta_{i1}}{3} + \frac{\pi}{4} \tag{23}$$

• **Collision conditions**: In pick and place tasks, it is required that the hand configuration at both the initial and final locations must be specified before planning motion trajectory. In this paper, we are focused on determining the relative configuration of the hand w.r.t. the object's frame, while this latter is being grasped. Thus, when the hand is grasping an object, if collisions between the hand's bodies and the environment are discovered, then the current grasp is rejected and the planning procedure evolves toward the search for other possible solutions.

5.2.3 Performance Index

Many criteria can be chosen as performance index for the proposed optimization problem. Different grasp parameters such as location of contact points, hand configuration or limitation on finger forces, can be involved in the quality measure. A global quality measure obtained through the combination of several criteria can be also considered. In this work, the optimal solution Q_{LP} described in Section 4 is used as a cost function in the optimization problem. The proposed FC quality gives the minimal contact forces that contribute to force magnitude maximization along the negative direction of $\mathbf{z}_1$ axis (the normal at the first contact point

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Algorithm 2 : $[F_{obj}, \mathcal{G}^*_{iter}] =$ `Cost_Function`$(\mathbf{x}_{sp})$

Ensure: $[F_{obj}, \mathcal{G}^*_{iter}]$

1: Set_Initial_Configuration($\mathbf{x}_{sp}$) {setting the initial posture of the Barrett hand}
2: Translate_Forward($\mathbf{z}_h$) {translation along $\mathbf{z}_h$ until contact}
3: Close_Fingers() {close the fingers around the object}
4: **while** $n > 2$ **do**
5: F_{obj}=FC_Test() {test and evaluate the grasp}
6: Update_Quality() {save the best grasp $[F_{obj}, \mathcal{G}^*_{iter}]$}
7: Open_Fingers() {the actuated joints (θ_{i1} and θ_3) are initialized}
8: Translate_Backward(z_h,d_{back}) {translation along $-\mathbf{z}_h$ with a distance d_{back}}
9: Close_Fingers()
10: **end while**
11: **return** $[F_{obj}, \mathcal{G}^*_{iter}]$

$\mathbf{c}_1$). From a physical point of view, it means the minimal contact forces that contribute to the obtention of stable grasps which are far from FC loss.

The proposed grasp planner returns the configuration of the mechanical hand when it is grasping an object. The stochastic optimization is performed to find a good global solution using an annealing algorithm. For each initial configuration $\mathbf{x}_{sp}$, many grasps are evaluated using backward iterations (§5.1.2). The algorithm 2 describes this backward mechanism and, returns the best grasp obtained for a given initial configuration $\mathbf{x}_{sp}$. This grasp is returned as a vector $\mathcal{G}^*_{iter}$. For the Barrett hand, the components of $\mathcal{G}^*_{iter}$ describe the posture of the hand's reference frame $(\mathbf{o}_h, \mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ w.r.t the object-attached frame and the value of the actuated joints (θ_{i1} and θ_3) while the hand is grasping the object.

5.2.4 Stochastic Optimization Technique

The proposed method is based on a stochastic optimization scheme. The simulated annealing technique will control how each trial grasp is being generated and will decide, via its built-in Metropolis decision algorithm, if a candidate should be accepted or rejected. It will also check for constraints and will keep track, of course, of the best candidate. At each iteration of the SA algorithm, a new grasp is randomly generated. The distance of the new grasp from the current one is based on a probability distribution. The algorithm accepts all new grasps that decrease the objective function F_{obj}, but also, with a certain probability, grasps that increase it. Therefore, the algorithm avoids being trapped in local minima, and it is able to explore the object surface globally for more solutions. While the SA technique is unlikely to find the optimal solution, it can often find a very good one. Hence, calculated grasps are expected to be suboptimal.

6 Implementation and Results

We now give numerical results obtained on a Pentium-M laptop (processor 1.7 GHz, 1.5 Go of RAM, OS. Linux). This Section contains two parts: Firstly, we compare the proposed FC test to the algorithm advanced in[28] and, we give a discussion about the proposed quality measure Q_{LP}. The second part deals with the application of the proposed grasp planner for computing FC grasps on polyhedral objects. The approach has been tested on several complex objects and its efficiency in testing and computing FC grasps is confirmed.

6.1 FC-Test Performances

The most known FC qualitative test is proposed by Liu[28]. It is equivalent to the ray-shooting algorithm which starts from linearizing friction cones by a polyhedral convex cone with m sides, then computing mn primitive contact wrenches. Finally, it solves a linear programming problem (with 6 decision variables and nm inequalities) to conclude FC test. Our proposed LP has mn decision variables and 6 equalities. It also starts by computing the grasp primitive contact wrenches. If the LP (14) is inconsistent then the grasp is non FC. Otherwise, we check (16) to conclude the test. We have adopted the IPM because of its effectiveness in discovering linear program unfeasibility[30] while the Simplex method is used to solve the ray-shooting algorithm[28] which is suitable for medium-scale problems. The optimization toolbox of MATLAB was used for the implementation.

We also show that our FC quality measure $Q_{LP} = \boldsymbol{f}^T \mathbf{x}^*$ can be considered as better index from a practical point of view. Further, we give a comparison between Q_{LP} and the radius r_{ball} of the largest ball inscribed inside the convex hull of the primitive contact wrenches, often cited in the literature[15,16].

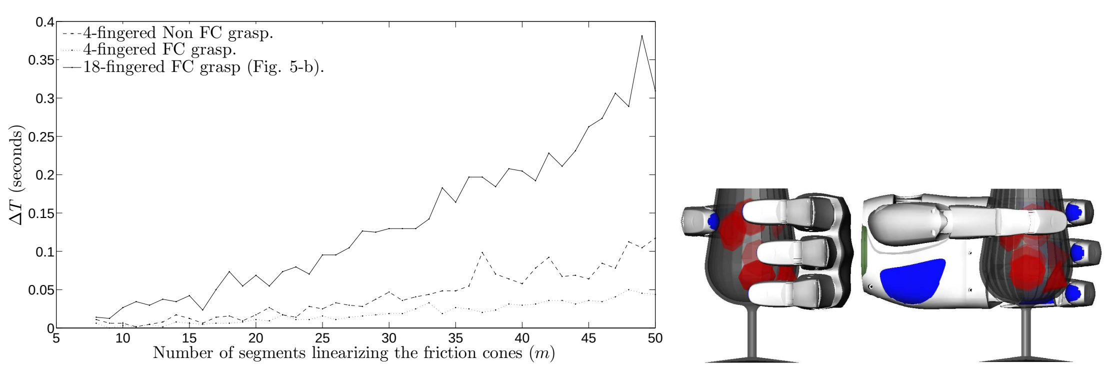
6.1.1 Computation Cost

The first example is from[28], the contact points and the normal vectors were given above (§ 3.2.1).

For a friction coefficient $\mu = 0.3$, the grasp is not FC. Figure 5-a depicts the runtime of the two algorithms versus the number m of facets used for the linearization of the friction cones. In Fig. 5-a (dashed curve), we show that the ray-shooting algorithm requires more time to check non FC grasps[28,31]. The qualitative check is performed efficiently when we use our algorithm. Since the IPM returns the unfeasibility of the LP (14), the runtime is reduced by about 61% in average when $m \in [8, 50]$ sides.

The grasp achieves FC for $\mu = 0.5$. Fig. 5-a (doted curve) plots the runtime of the two algorithms versus m. The proposed qualitative check is performed efficiently and the runtime is reduced by about 43% in average. It should be noted that when the grasp achieves FC, the runtime in our case is the sum of the required time for checking the qualitative test and the time necessary to compute the optimal solution of (14). So, if we use the proposed algorithm only as a qualitative test, the corresponding runtime will be much lower than the values shown on Fig. 5-a (doted curve).

It should be noted that $m \in [8, 20]$ sides leads to acceptable practical results. Figure 5-a shows that for m varying in this interval, the ray-shooting algorithm requires more runtime. The increase of the number of contacts n is also a parameter which must be considered. In Fig. 5-b, we show the Schunk hand grasping a glass with 18 contacts ($\mu = 1$). Figure 5-a (solid curve) plots the runtime of the two algorithms versus m. We can notice that the runtime is



(a) (b)

Figure 5: (a) The difference between the time costs ΔT versus the number of segments linearizing the friction cones m. (b) Operation of grasping a glass with the Schunk hand ($n = 18$, $\mu = 1$).

Table 1: Optimal solution versus the reduction point $\mathbf{c}_i$.

Point	$\mathbf{c}_1$	$\mathbf{c}_2$	$\mathbf{c}_3$	$\mathbf{c}_4$	Average	r_{ball}
Grasp 1	7.758	12.443	8.562	3.109	7.968	0.063
Grasp 2	1.0385	1.0382	1.098	1.100	1.068	0.308

reduced by about 56% in average when $m \in [8, 20]$ sides. Therefore, the ray-shooting algorithm requires higher computational time if n increases.

6.1.2 Quality Measure

The proposed metric $Q_{LP} = \boldsymbol{f}^T \mathbf{x}^*$ is independent of the chosen point $\mathbf{c}_i$. Table 1 summarizes the quality Q_{LP} versus the reduction point $\mathbf{c}_i$. The grasp 1 was described in § 3.2.1 and, the grasp 2 is shown on Fig. 5-b. Indeed, we notice that the second grasp is better than the first one whatever the chosen point $\mathbf{c}_i$. In the last column of table 1, we compute the radius r_{ball} of the largest ball inscribed inside the convex hull, which is considered as the most popular criterion[15,16]. We use the qhull library[32] to compute the six-dimensional convex hull of the primitive contact wrenches. A quality of $r_{ball} \geqslant 0.1$ corresponds to acceptable grasps[26]. The quality criterion r_{ball} indicates that the second grasp is better than the first one, which is also confirmed by Q_{LP} without need to geometric computations on convex hulls. For the grasp 2 (Fig. 5-b), the quality Q_{LP} is computed w.r.t four points but the results are similar for the remainder points.

In the sequel, we recapitulate the main advantages of the proposed LP test:

• The qualitative FC test advanced in[28] is based on the optimal solution of the ray-shooting algorithm. Using our linear formulation, the FC qualitative test is based only on the examination of the LP feasibility and the checking of (17). Hence, the proposed LP has less computational cost compared with the ray-shooting algorithm.

• If (14) is consistent and (16) satisfied then the grasp is FC. Furthermore, the optimal solution of (14) gives a quality measure of the FC grasp. Through several numerical simulations, we have noticed that the FC quality criterion $Q_{LP} < 2$ corresponds to acceptable grasps.

• The optimal solution of the LP advanced in[28] is used to test if the origin of wrench space lies strictly inside the convex hull of the primitive contact wrenches. Hence, it cannot give enough information about the quality of the FC grasp. The proposed FC test can also be considered as a quantitative FC test.

6.2 Grasp Planning Results

To test the proposed approach, we planned grasps on various objects. We have used the 3-fingered Barrett hand, but our generic method can easily be adapted to any mechanical hand. Our simulations were performed under the public simulator GraspIt!. The proposed FC test is implemented in GraspIt! using the Gnu Linear Programming Kit (GLPK) package. For the grasps optimization, we used Gnu Scientific Library (GSL) routines for the implementation of the SA technique. Parameters of this technique have been set as indicated in Table 2. Starting from a random initial configuration of the Barrett hand, 70 iterations are performed

Table 2: Parameters for SA run

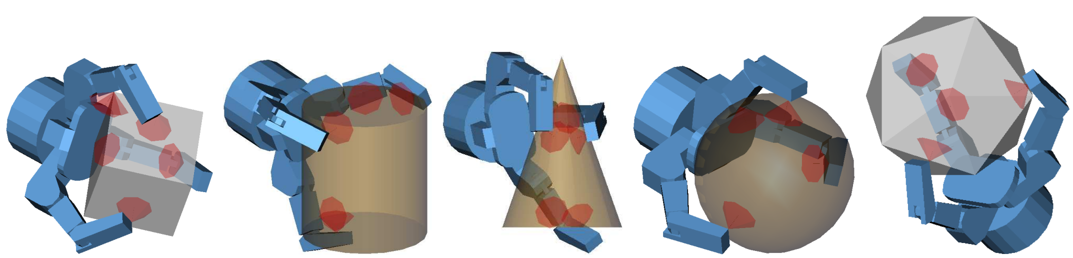
Maximum number of iterations	70
Maximum trying points before stepping	15
Maximum number of iterations for each temperature	5
Damping factor for temperature	1.01
Boltzmann constant	1.0

for each simulation. It is found that this number was sufficient to reach good solutions. In the implementations, the back distance, defined in §5.1.2, is set to $d_{back} = 5$ mm. The friction coefficient is set to 1.0 in all simulations.

We begin by presenting the results of some planned grasps on basic 3D objects. The optimal grasps configuration are shown in Fig. 6. For each grasped object, the evolution of the best quality versus the number of iterations is plotted in Fig. 7. We notice that, for different objects, various acceptable FC grasps are found in few iterations (typically less then 40 iterations).

Our grasp planner is also tested to generate optimal grasps on real objects models. In Fig. 8, we present the obtained grasps after 70 iterations. As shown in Fig. 9, we can see that 20 iterations are sufficient to generate good grasps on different object's geometries. For the example shown in Fig. 8-e, we plot the quality position in each iteration for grasping the airplane model. For the first three iterations, the generated grasps are non FC, but a suboptimal grasp is obtained at the sixth iteration. In Fig. 11, the grasped objects were placed on environments with other obstacles. For these examples, we notice that the number of feasible grasps is reduced which reduce also the planning times (table 3).

Table 3 summarizes the performances of the proposed approach. For each generated suboptimal grasp, we present the best quality Q_{LP}, the corresponding r_{ball} quality measure and the total runtime required for performing 70 iterations. The number of tested grasps and the FC ones are depicted in the last two columns respectively.



(a) cube (b) cylinder (c) cone (d) sphere (e) polyhedron

Figure 6: Suboptimal grasps produced by the proposed planner for basic 3D objects.

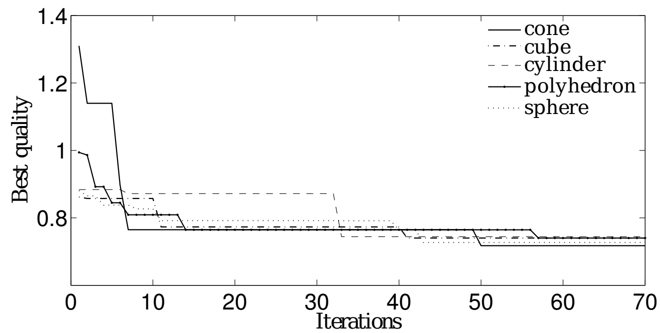
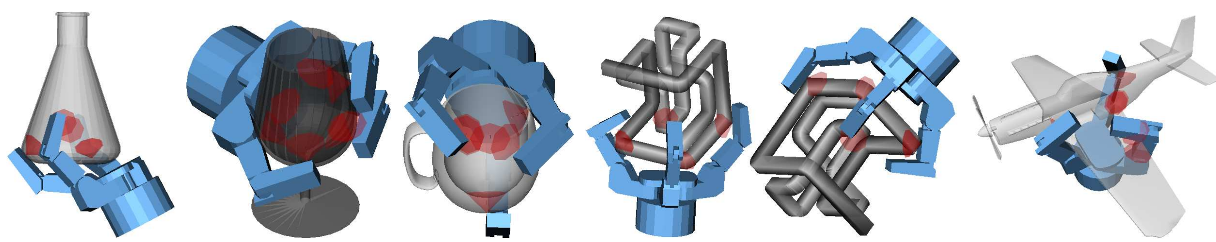


Figure 7: Best quality versus number of iterations for grasps of Fig. 6.



(a) flask (b) glass (c) coffee mug (d) pipe (two views) (e) airplane model

Figure 8: Suboptimal grasps produced by the proposed planner on real objects' models.

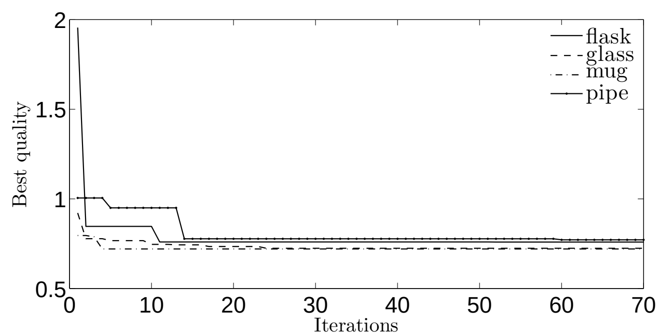


Figure 9: Best quality versus number of iterations for grasps of Fig. 8.

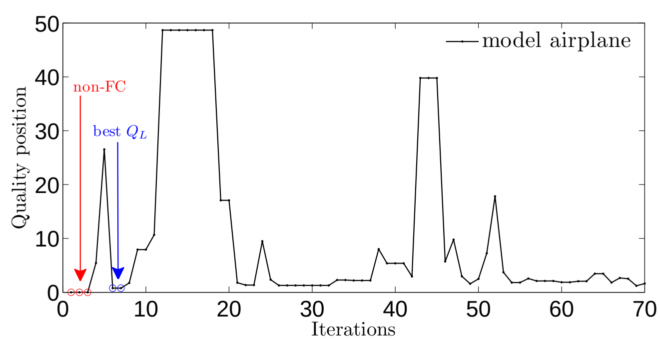
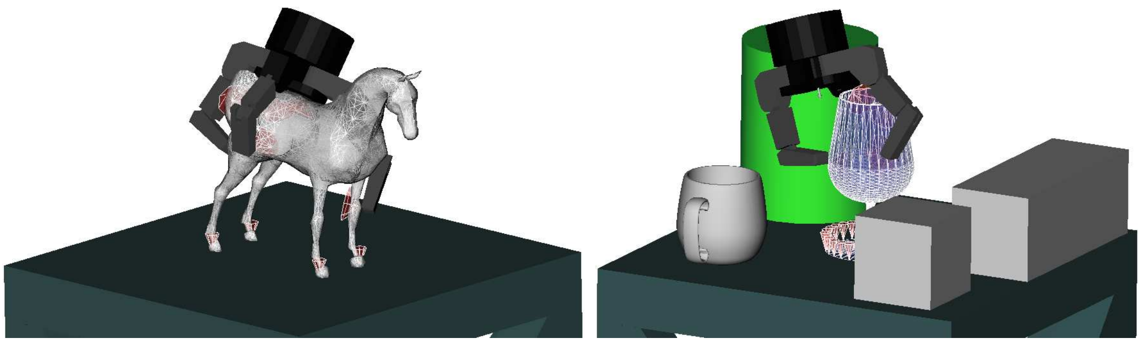


Figure 10: Quality versus number of iterations for the grasp of Fig. 8-e.



(a) horse model on table (b) glass surrounded by obstacles

Figure 11: Best planned grasps of the model horse and the glass in constrained environments.

Table 3: Performance of the proposed grasp planner

Objects	Q_{LP}	r_{ball}	Time (s)	Tested grasps	FC grasps
cube	0.740	0.490	111	4505	2582
cylinder	0.744	0.457	132	4519	2446
cone	0.717	0.169	92	2558	1204
sphere	0.727	0.398	226	5374	3533
polyhedron	0.740	0.286	132	4547	2718
flask	0.759	0.177	152	3294	1843
glass	0.724	0.226	136	3439	2085
mug	0.721	0.427	209	4622	2956
pipe	0.772	0.189	70	1203	409
plane	0.777	0.083	68	936	167
horse on table	0.714	0.171	78	816	686
glass with obstacles	0.711	0.324	91	425	351

7 Conclusion

This paper was focused on grasp analysis and grasp synthesis. We were primarily concerned with the proposition of a new FC test, for which a rigorous proof was provided for the 3D case. Based on the central-axis theory and using friction cone linearization, we have reformulated the proposed FC condition as a new linear programming problem solved using an IPM. Through numerical results, we have confirmed the efficiency of the proposed algorithm in testing the FC property.

Moreover, we have proposed an approach for finding appropriate stable grasps for a robotic hand on arbitrary 3D objects. The presented method is based on a stochastic optimization scheme. We use the SA technique for synthesizing suboptimal grasps. Our algorithm can compute good grasps for multifingered hands on arbitrary objects within a reasonable runtime. The SA technique often prevents being trapped in local minima, and it is able to explore the object surface globally for more solutions. In addition, the proposed approach makes it possible to obtain feasible grasps without solving the kinematics of the mechanical hand and without further simplifications of the object model. Consequently, the computational cost of the grasp planning process is significantly reduced.

As generated grasps must preform different tasks in the environment, our future works will be concentrated on the development of oriented task qualities and on objects' manipulation.

8 Acknowledgment

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