Score-Aware Policy-Gradient Methods and Performance Guarantees using Local Lyapunov Conditions

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Reinforcement learning

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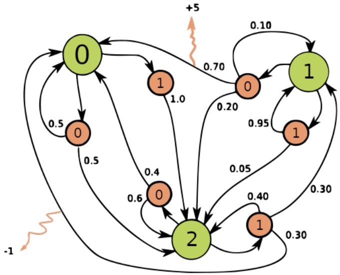

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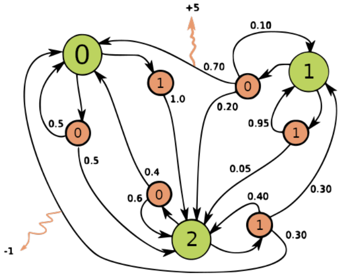

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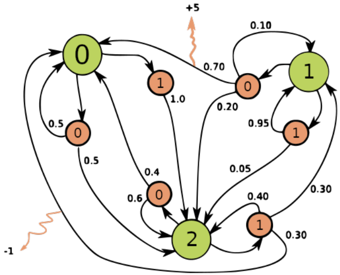

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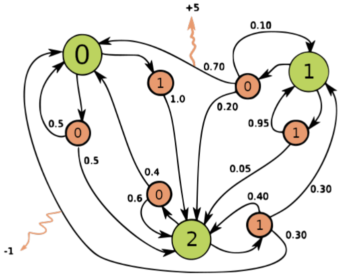

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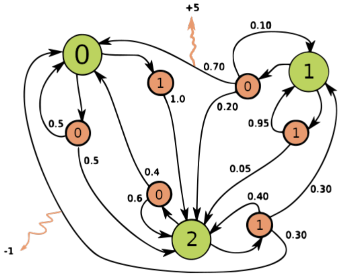

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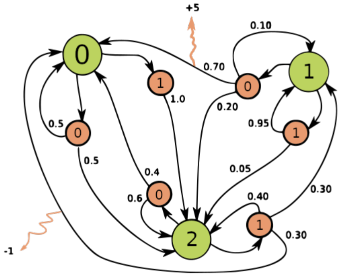

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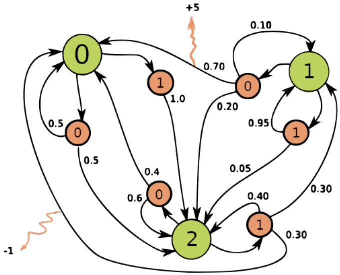
Stationary distribution of $(S_t, t \ge 0)$ under $\pi(a|s,\theta)$



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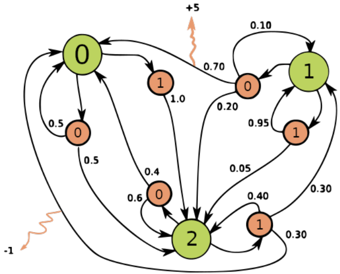
- We estimate the **policy gradient** $\nabla J(\theta)$ and apply **stochastic gradient ascent**



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Policy-gradient algorithms

- Typical **policy-gradient algorithm**:

1: Initialize S_0 and Θ_0
2: **for** $t = 0, 1, 2, \dots$ **do**
3: Sample $A_t \sim \pi(\cdot|S_t, \Theta_t)$
4: Take action A_t and observe R_{t+1}, S_{t+1}
5: Estimate $\nabla J(\Theta_t)$ using the history $S_0, \Theta_0, A_0, R_1, \dots, S_t, \Theta_t, A_t, R_{t+1}, S_{t+1}$
6: Update $\Theta_{t+1} \leftarrow \Theta_t + \alpha \nabla J(\Theta_t)$
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- **Actor-critic** applies the policy-gradient theorem (Sutton and Barto, 2018):

$$\nabla J(\theta) \propto \mathbb{E}\Big[\Big(R - J(\theta) + v(S'|\theta) - v(S|\theta)\Big)\nabla \log \pi(A|S, \theta)\Big],$$

with $(S, A, R, S') \sim \textsc{stat}(\theta)$: $\mathbb{P}[S = s, A = a, R = r, S' = s'] = p(s|\theta)\pi(a|s, \theta)P(r, s'|s, a)$.

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- Exploit the product-form to introduce a new **policy-gradient estimator**
- Prove that the corresponding policy-gradient algorithm has nice **convergence properties**
- Outline of the rest of the presentation:
  1. Product-form distributions as exponential families
  2. Score-aware gradient estimator (SAGE)
  3. SAGE-based policy-gradient algorithm
  4. Nonconvex convergence result

① Product-form distributions as exponential families

- **Product-form** distribution

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Example: M/M/1 queue with admission control

- Arrival rate $\lambda > 0$, service rate $\mu > 0$
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- Actions: accept or reject
- Admission reward α per job
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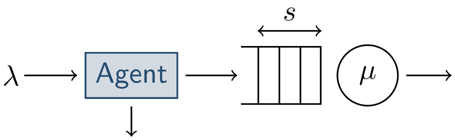

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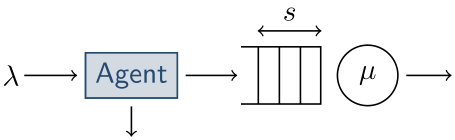

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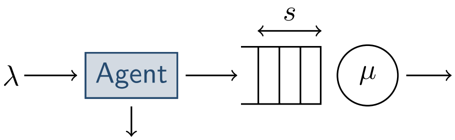

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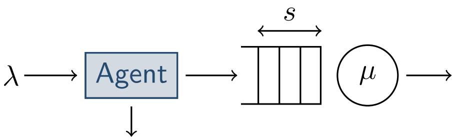

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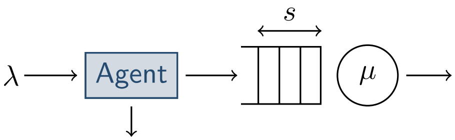

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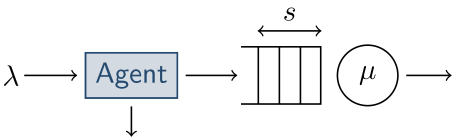

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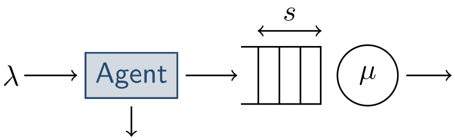

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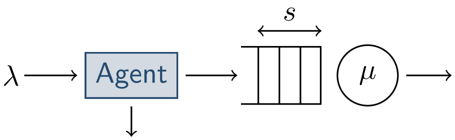

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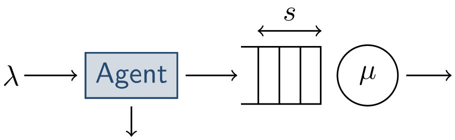

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Theorem

If $p(s|\theta)$ *has a product-form in the sense that* $\log p(s|\theta) = \log \rho(\theta)^\intercal x(s) - \log Z(\theta)$, *then*

$$\nabla \log p(s|\theta) = \mathrm{D} \log \rho(\theta)^\intercal (x(s) - \mathbb{E}[x(S)]),$$
$$\nabla J(\theta) = \mathrm{D} \log \rho(\theta)^\intercal \mathrm{Cov}[R, x(S)] + \mathbb{E}[R \nabla \log \pi(A|S,\theta)],$$

with $(S, A, R, S') \sim \text{STAT}(\theta) : \mathbb{P}[S = s, A = a, R = r, S' = s'] = p(s|\theta)\pi(a|s,\theta)P(r, s'|s,a)$.

② Score-aware gradient estimator (SAGE)

- The **score** is the gradient of the log-likelihood with respect to the parameter vector:

$$\text{“Likelihood”} = p(s|\theta) \quad \rightarrow \quad \text{“Score”} = \nabla \log p(s|\theta).$$

Theorem

If $p(s|\theta)$ *has a product-form in the sense that* $\log p(s|\theta) = \log \rho(\theta)^\intercal x(s) - \log Z(\theta)$, *then*

$$\begin{aligned} \nabla \log p(s|\theta) &= \mathrm{D} \log \rho(\theta)^\intercal (x(s) - \mathbb{E}[x(S)]), \\ \nabla J(\theta) &= \mathrm{D} \log \rho(\theta)^\intercal \mathrm{Cov}[R, x(S)] + \mathbb{E}[R \nabla \log \pi(A|S,\theta)], \end{aligned}$$

with $(S, A, R, S') \sim \textsc{stat}(\theta) : \mathbb{P}[S = s, A = a, R = r, S' = s'] = p(s|\theta)\pi(a|s,\theta)P(r,s'|s,a)$.

- **Main take-away:** If we can compute $\mathrm{D} \log \rho(\theta)$, we have an estimator for $\nabla J(\theta)$.

③ SAGE-based policy-gradient algorithm

- Typical **policy-gradient algorithm**:

1: Initialize S_0 and Θ_0
2: **for** $t = 0, 1, 2, \dots$ **do**
3: Sample $A_t \sim \pi(\cdot|S_t, \Theta_t)$
4: Take action A_t and observe R_{t+1}, S_{t+1}
5: Estimate $\nabla J(\Theta_t)$ using the history $S_0, \Theta_0, A_0, R_1, \dots, S_t, \Theta_t, A_t, R_{t+1}, S_{t+1}$ How?
6: Update $\Theta_{t+1} \leftarrow \Theta_t + \alpha \nabla J(\Theta_t)$
7: **end for**

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- Instead of applying actor-critic, we estimate $J(\Theta_t)$ with a **SAGE**:

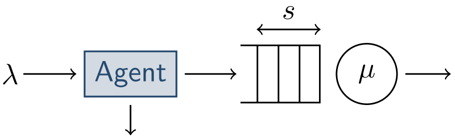
$$\nabla J(\theta) = \mathrm{D} \log \rho(\theta)^\intercal \mathrm{Cov}[R, x(S)] + \mathbb{E}[R \nabla \log \pi(A|S, \theta)],$$

with $(S, A, R, S') \sim \textsc{stat}(\theta)$: $\mathbb{P}[S = s, A = a, R = r, S' = s'] = p(s|\theta)\pi(a|s, \theta)P(r, s'|s, a)$.

Example: M/M/1 queue with admission control

Stable/Possibly-unstable case

- Arrival rate $\lambda = 0.7/1.4$, service rate $\mu = 1$
- Admission reward $\alpha = 5$, holding cost rate $\eta = 1$
- Initial policy: admit with probability $\frac{1}{2}$



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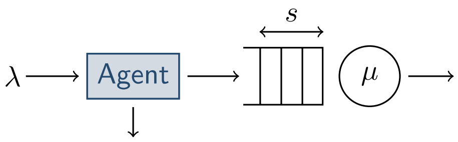

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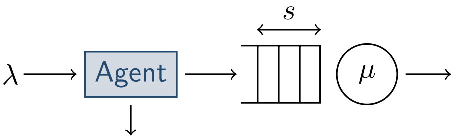
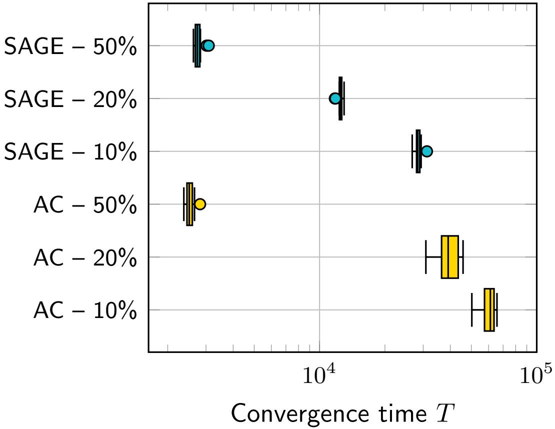


Algorithms

- SAGE-based policy-gradient
- Actor-critic without eligibility traces (Sutton and Barto, Section 13.6)
- Gradient update at every step, with step size $\alpha = 10^{-3}$

Example: M/M/1 queue with admission control

Stable case – Convergence time
(Time T such that $J(\Theta_t) > J^* - \epsilon$ for each $t \in \{T, T+1, \dots, 10^6\}$)



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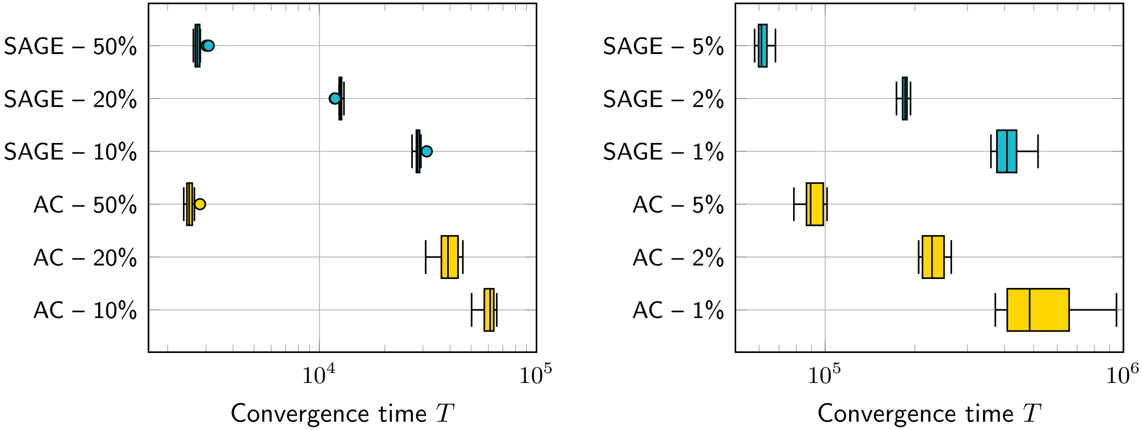

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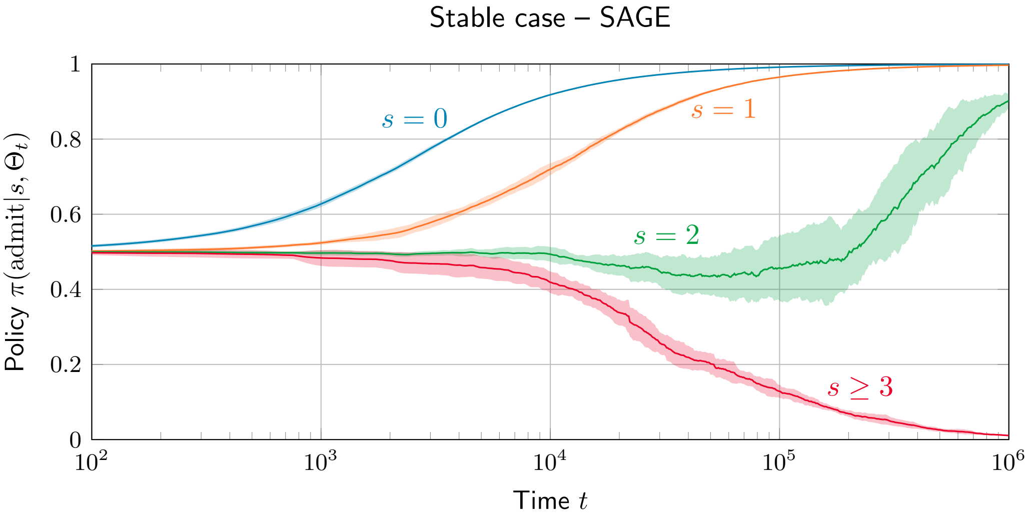

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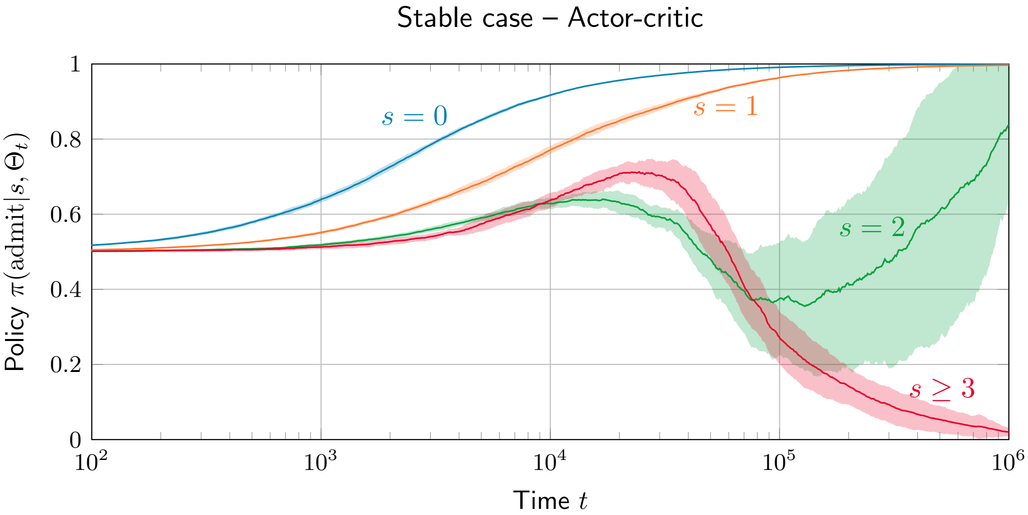

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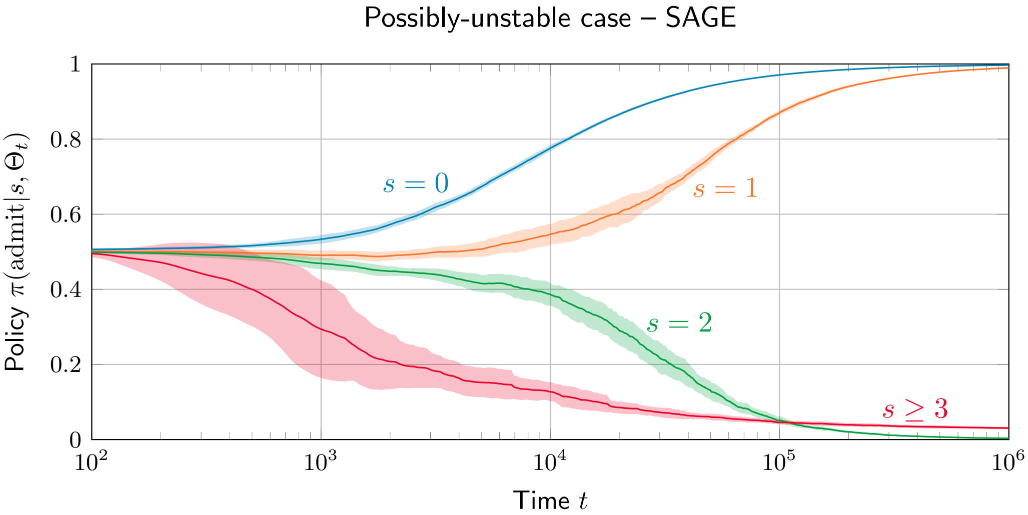

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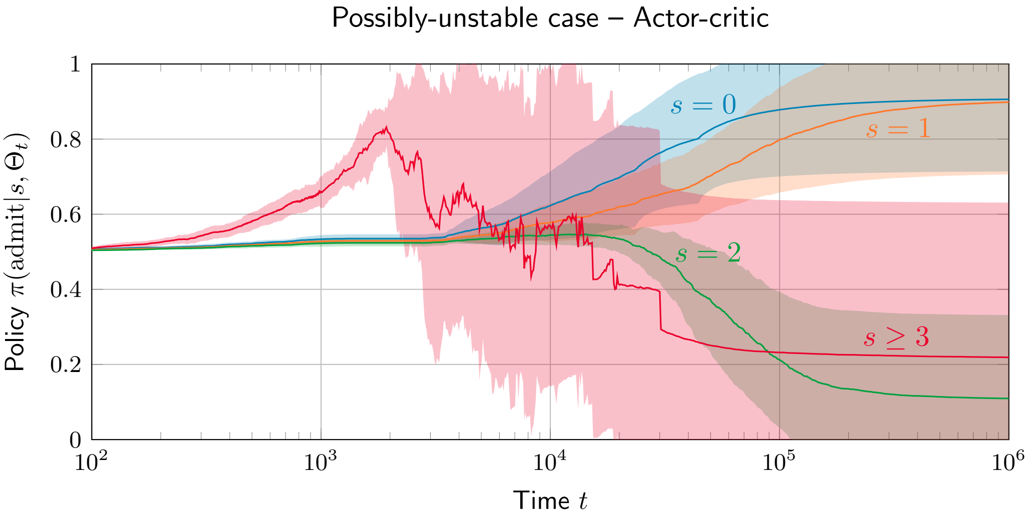

④ Local convergence result

(Sketch of) Theorem

Under additional assumptions, a batch variant of the algorithm that starts in a basin of attraction of a global maximizer will converge to a global maximizer with large probability.

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What are these "additional assumptions"?

- The step sizes are decreasing and the batch sizes are increasing.
- There exists a neighborhood of the global maximizer where:
  - The Markov chain of state-action pairs is geometrically ergodic.
  - The objective function behaves approximately in a convex manner in directions that are perpendicular to the set of global maximizers.
  - The function $\mathrm{D} \log \rho$ is bounded and the functions x, r, and $r \nabla \log \pi$ grow slowly enough.

Conclusion

- **Main contributions**
  1. Product-form distributions as exponential families
  2. Score-aware gradient estimator (SAGE)
  3. SAGE-based policy-gradient algorithm
  4. Nonconvex convergence result

Product-form stationary distribution

$$\log p(s|\theta) = \log \rho(\theta)^\intercal x(s) - \log Z(\theta)$$

$$\downarrow$$

$$\nabla \log p(s|\theta) = \mathrm{D} \log \rho(\theta)^\intercal (x(s) - \mathbb{E}[x(S)])$$

Score-aware gradient estimator (SAGE)

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  - (Ongoing) Run extensive numerical results on larger and more challenging examples.

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Score-aware gradient estimator (SAGE)

- **Future research directions**
  - (Ongoing) Run extensive numerical results on larger and more challenging examples.
  - Find better estimators for covariance and expectation, such as robust estimators.
  - Apply to MDPs where the stationary distribution is known only *up to a multiplicative constant*.

June–December 2024 in Toulouse

- 5 international workshops
- Atelier en Évaluation des Performances
- Invited researchers: Vivek Borkar, Itai Gurvich, Sean Meyn, and Adam Wierman

Call for abstract for the RL workshop until February 29!

More information on the webpage

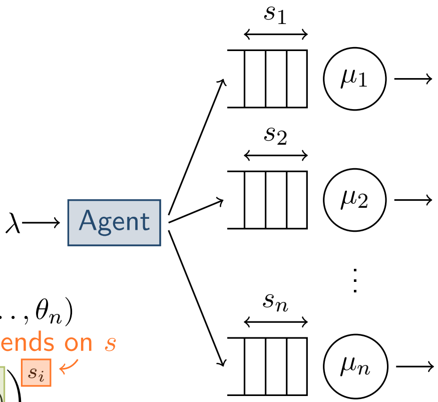
`https://indico.math.cnrs.fr/category/683/`

Example: Static load balancing

- Jobs arrive as a Poisson process with rate λ
- M/M/1 queues with service rates μ_1, μ_2, ..., μ_n
- Maximum c jobs in the system
- State vector $s = (s_1, s_2, \ldots, s_n)$ of queue sizes
- Actions are to assign to some server i
- Admission reward 1 per job
- Policy $\pi(\text{server } i | \cdot, \theta) = \dfrac{e^{\theta_i}}{\sum_{j=1}^{n} e^{\theta_j}}$ with $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$
- Stationary distribution $p(s|\theta) \propto \prod_{i=1}^{n} \left(\dfrac{\lambda}{\mu} \pi(\text{server } i | \cdot, \theta) \right)^{s_i}$

Depends on s

Depends on θ



Example: Static load balancing

Simulation setup

- Number $n = 4$ of servers
- Arrival rate $\lambda = 0.7$
- Service rates $\mu_1 = 0.4$, $\mu_2 = 0.3$, $\mu_3 = 0.2$, and $\mu_4 = 0.1$
- Total capacity $c = 10$
- 10^6 simulation steps

Algorithms

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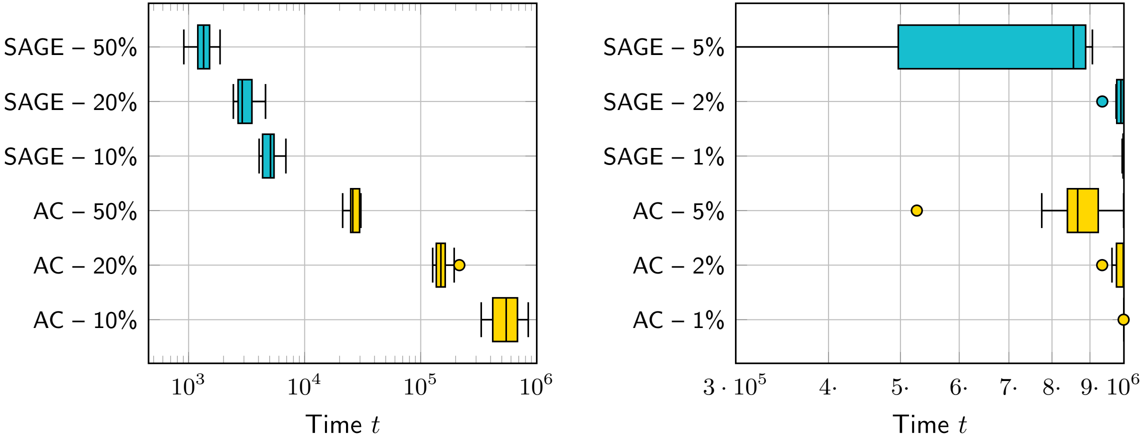
Convergence times



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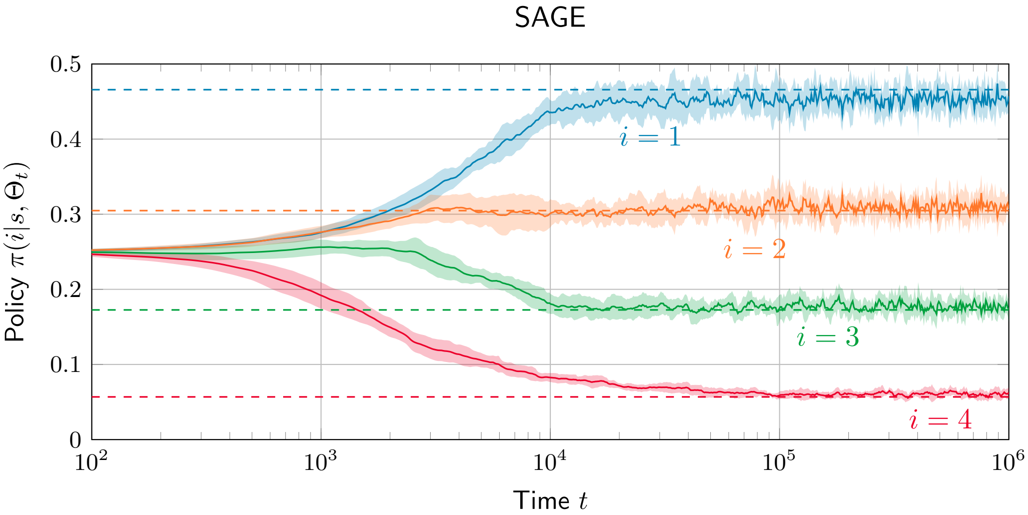

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