Stochastic Dynamic Matching

A Mixed Graph-Theory and Linear-Algebra Approach

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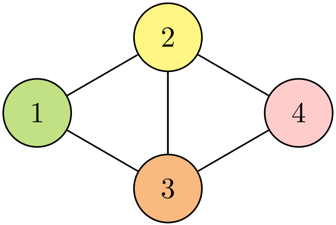
Matching problem

Graph G undirected, connected, without loop

- Nodes $V = \{1, 2, \dots, n\}$ → items
- Edges $E = \{1, 2, \dots, m\}$ → possible matches



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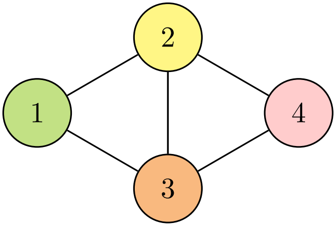

Online stochastic matching problem

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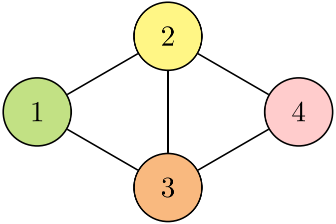

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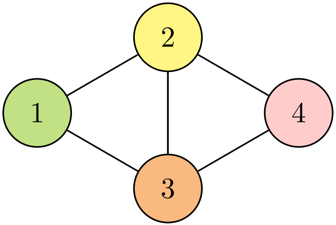

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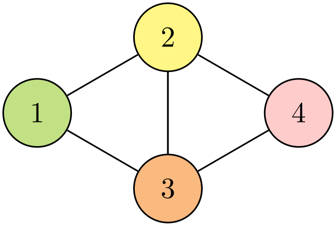

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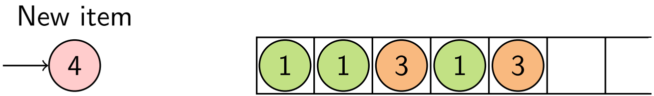

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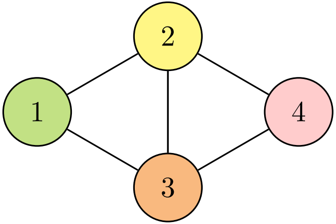

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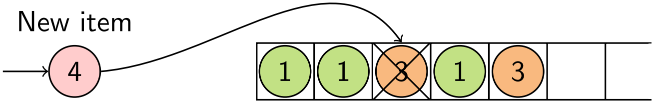

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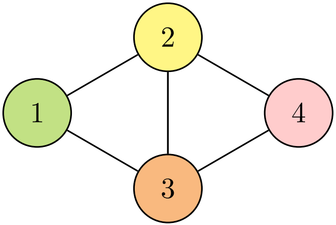

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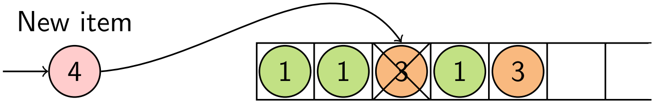
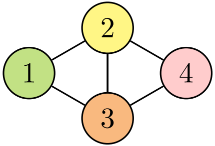


- If the policy is given, we obtain a Markov chain
- We assume stabilizability

Matching rate



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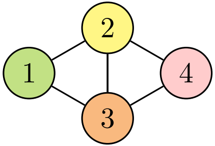

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  mean number of matches per time unit between classes i and j.

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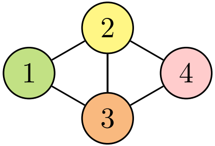

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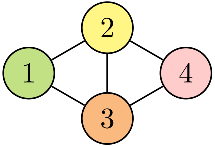

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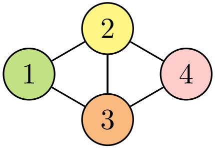

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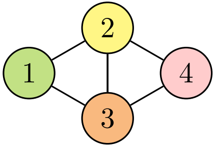

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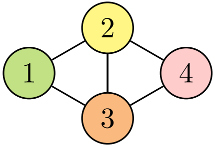
Given a graph $G = (V, E)$ and an arrival-rate vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, what is the set of "achievable" matching-rate vectors $\mu = (\mu_1, \mu_2, \ldots, \mu_m)$?

Conservation law



The matching rates satisfy the **conservation law**

$$\lambda_i = \sum_{k \in E_i} \mu_k, \quad i \in \{1, 2, \ldots, n\}.$$

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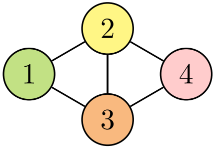

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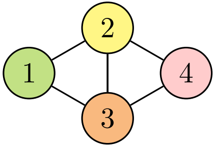

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where A is the **incidence matrix** of the graph G.

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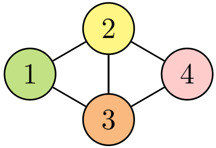

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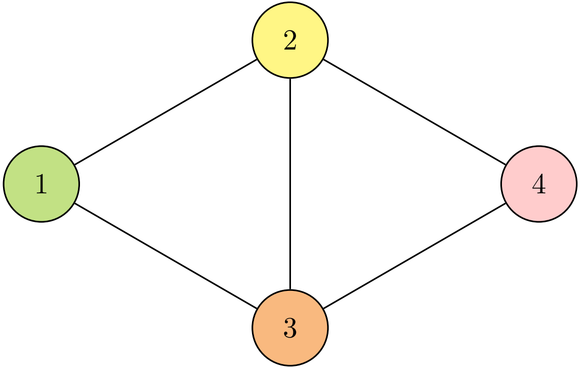
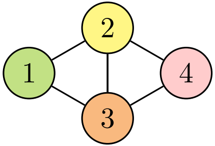
Example: Diamond graph





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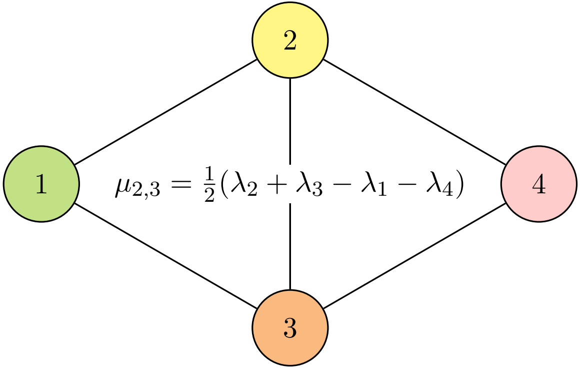

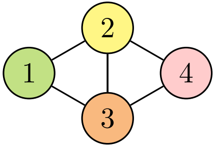

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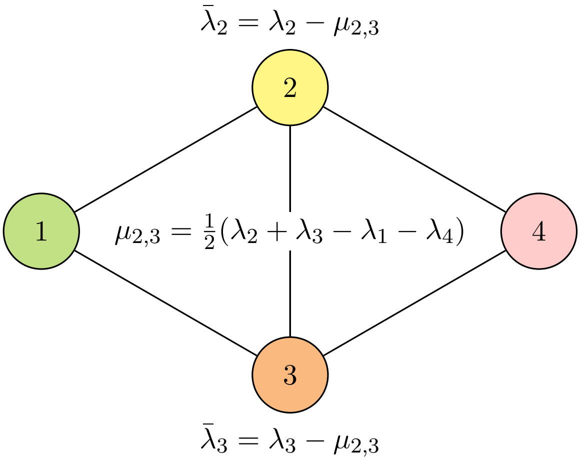

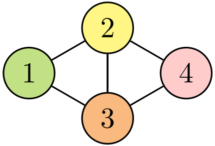

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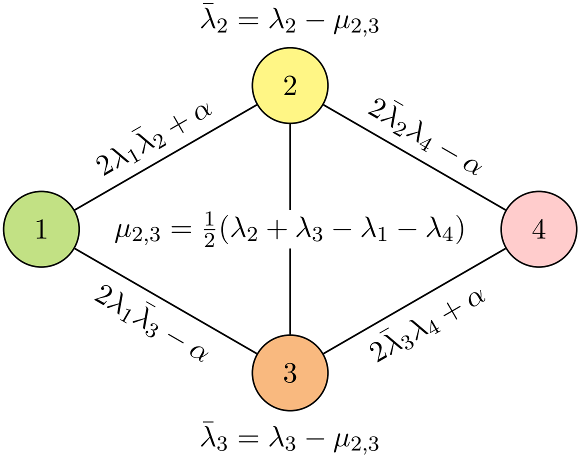

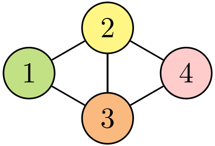

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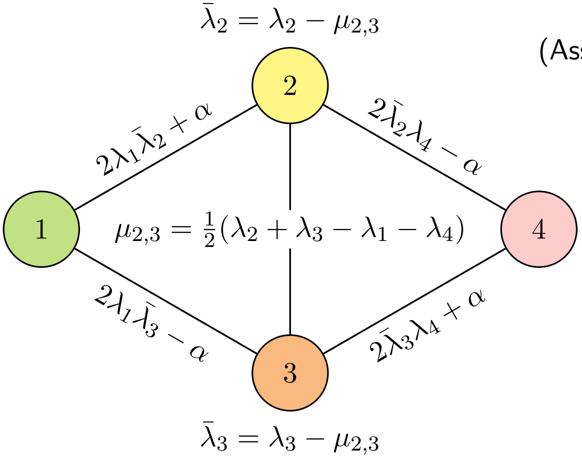
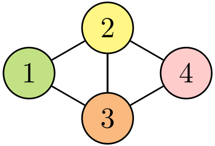


(Assuming $\lambda_1 + \lambda_4 = \frac{1}{2}$)



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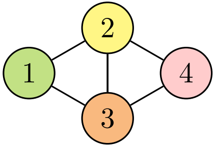
All solutions	
Non-negative solutions	
Achievable solutions	



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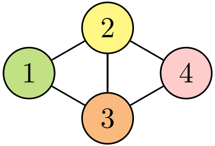

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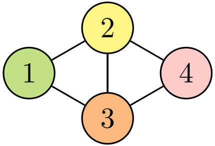

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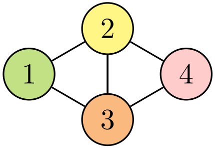

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with $d =$ (number of edges) $-$ (number of nodes)

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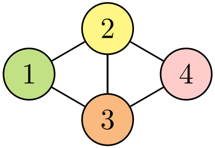

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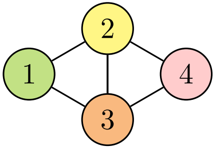

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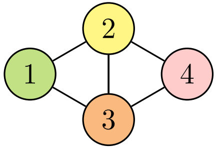
- **Contributions**
  - Characterization of surjective/injective/bijective graphs G.
  - Stabilizability conditions that are easier to verify.
  - Almost complete characterization of the set of achievable matching rate vectors.
  - Filtering (resp. semi-filtering) policies to achieve (resp. approach) vertices of the polytope.

Affine space of all solutions of $\lambda = A\mu$



We define

- μ° = a particular solution, built using the pseudo-inverse of A
- d = (number of edges) − (number of nodes)
- $\{b_1, b_2, \ldots, b_d\}$ = basis of kernel(A), built using a spanning tree of G (Doob, 1973)

Proposition

The solution set Π *of the equation* $\lambda = A\mu$ *is the* **d-dimensional affine space**

$$\Pi = \left\{ \mu^\circ + \alpha_1 b_1 + \alpha_2 b_2 + \ldots + \alpha_d b_d : (\alpha_1, \alpha_2, \ldots, \alpha_d) \in \mathbb{R}^d \right\}.$$

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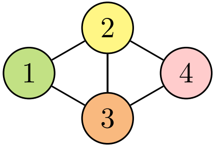

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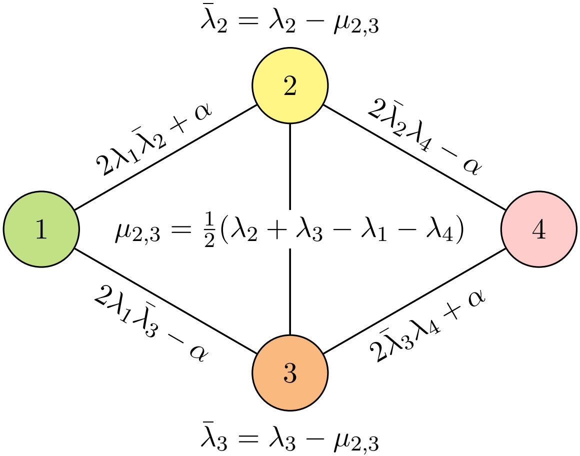
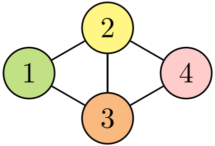
Two coordinate systems:

- **Edge coordinates:** $\mu = (\mu_1, \mu_2, \ldots, \mu_m) \in \mathbb{R}^m$
- **Kernel coordinates:** $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_d) \in \mathbb{R}^d$

Example: Diamond graph





$$
\begin{cases}
\lambda_1 = \mu_{1,2} + \mu_{1,3} \\
\lambda_2 = \mu_{1,2} + \mu_{2,3} + \mu_{2,4} \\
\lambda_3 = \mu_{1,3} + \mu_{2,3} + \mu_{3,4} \\
\lambda_4 = \mu_{2,4} + \mu_{3,4}
\end{cases}
$$

$$
\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix} \mu_{1,2} \\ \mu_{1,3} \\ \mu_{2,3} \\ \mu_{2,4} \\ \mu_{3,4} \end{bmatrix}
$$

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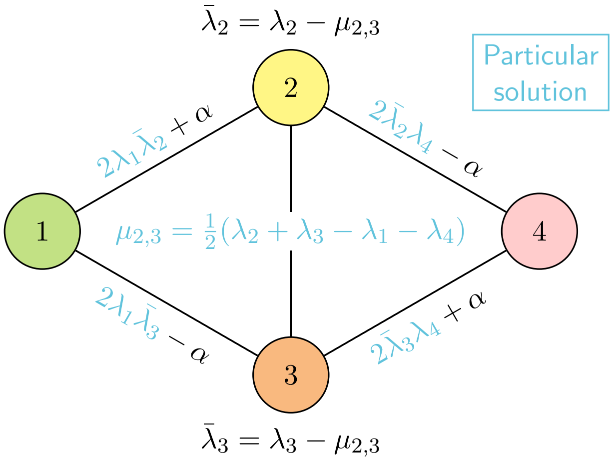

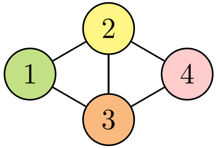

$$\begin{cases} \lambda_1 = \mu_{1,2} + \mu_{1,3} \\ \lambda_2 = \mu_{1,2} + \mu_{2,3} + \mu_{2,4} \\ \lambda_3 = \mu_{1,3} + \mu_{2,3} + \mu_{3,4} \\ \lambda_4 = \mu_{2,4} + \mu_{3,4} \end{cases}$$

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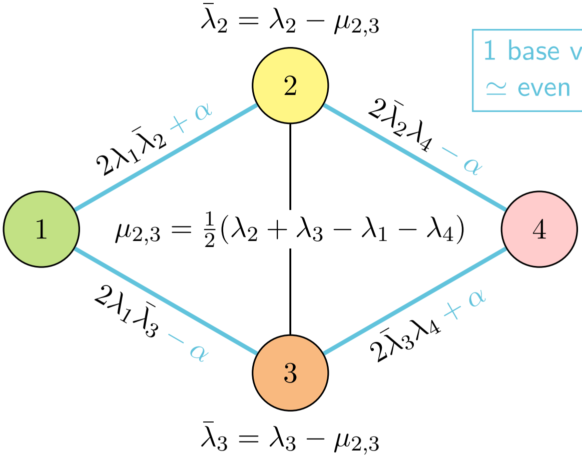
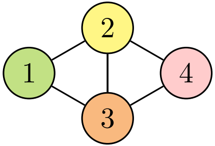


1 base vector
$\simeq$ even cycle



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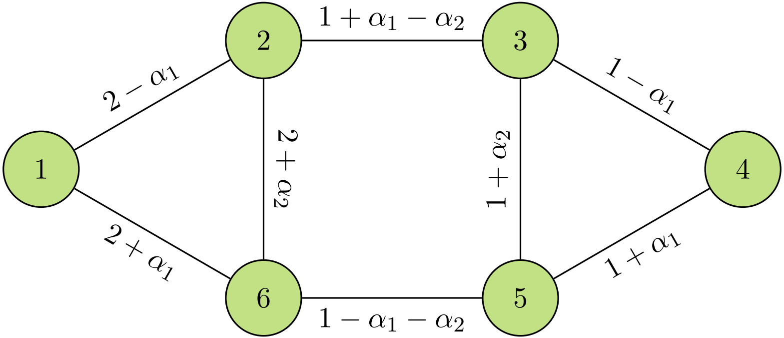

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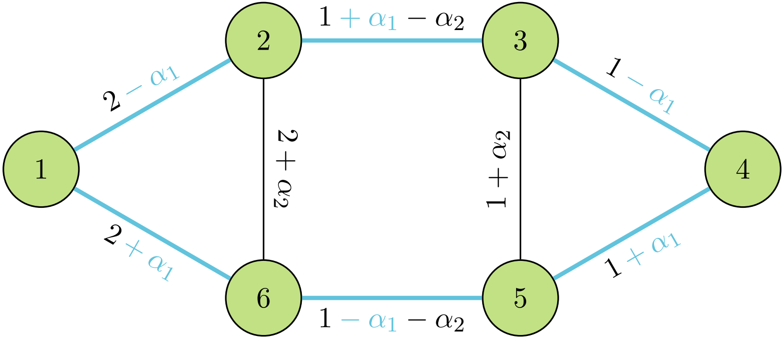

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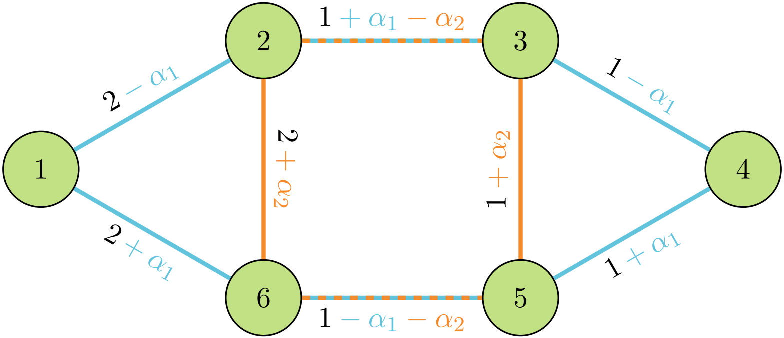

Convex polytope of non-negative solutions of $\lambda = A\mu$

Corollary

The set $\Pi_{\geq 0}$ *of non-negative solutions of* $\lambda = A\mu$ *is the* **d-dimensional convex polytope**

$$\Pi_{\geq 0} = \Pi \cap \mathbb{R}^d_{\geq 0} \simeq \left\{ \alpha \in \mathbb{R}^d : \mu^\circ + \alpha_1 b_1 + \ldots + \alpha_d b_d \geq 0 \right\}.$$

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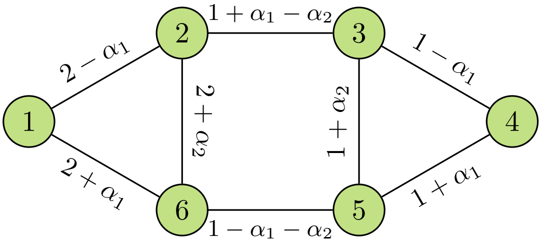

(a) General solution of $\lambda = A\mu$

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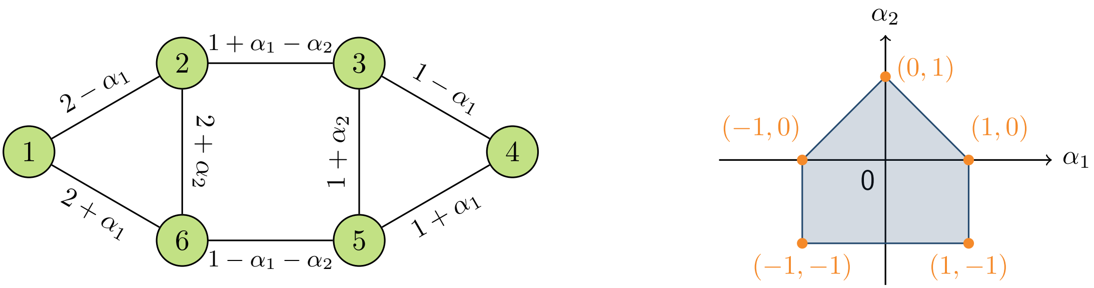

(a) General solution of $\lambda = A\mu$ (b) Convex polytope of non-negative solutions

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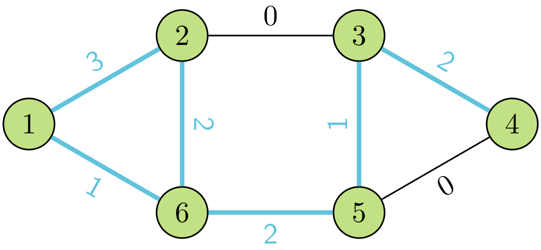

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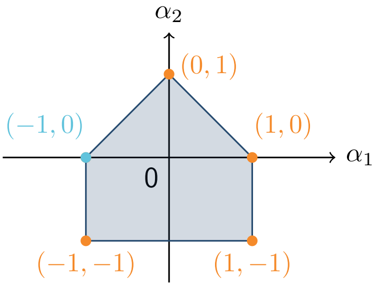

(b) Convex polytope of non-negative solutions

Achievable solutions of $\lambda = A\mu$

Theorem

Let μ be a vertex of $\Pi_{\geq 0}$ and $p =$ number of (strictly) positive coordinates of μ.

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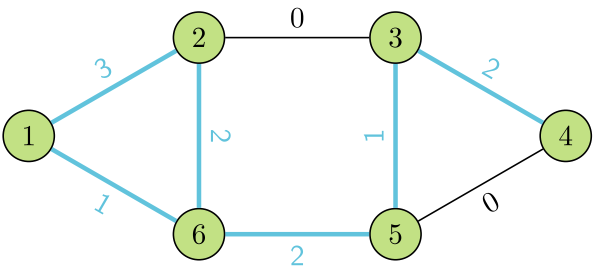

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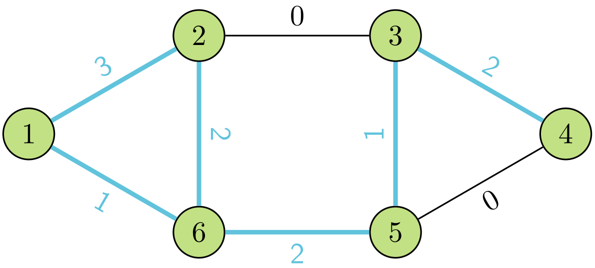
- *If $p <$ number of nodes in G, then μ cannot be achieved by a stable policy.*



Non-achievable solutions of $\lambda = A\mu$

Conjecture

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Non-achievable solutions of $\lambda = A\mu$

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Let μ be a vertex of $\Pi_{\geq 0}$. For each $k \in \mathbb{N}$, let Φ_k denote the following "semi-filtering" policy:

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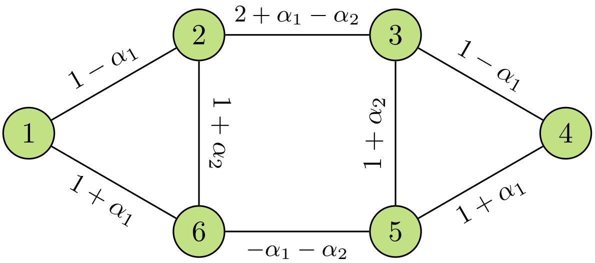

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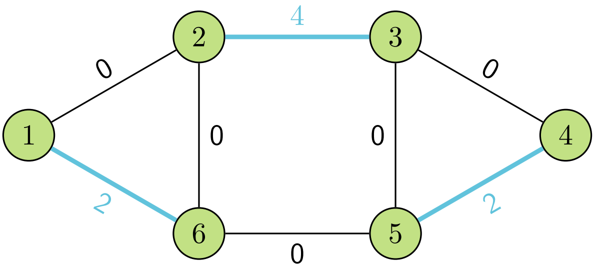
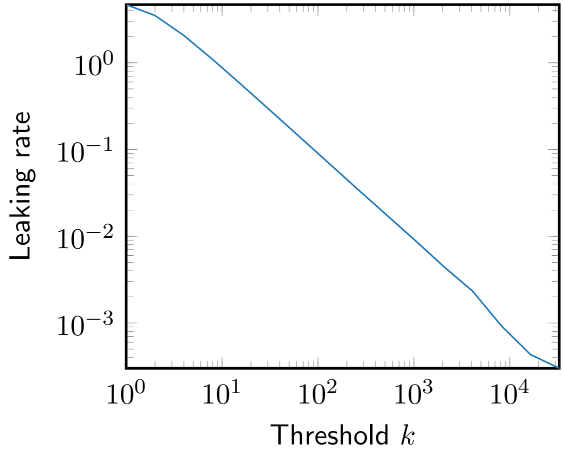
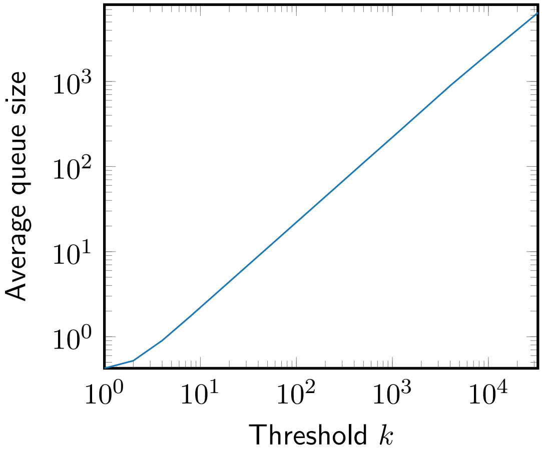


Numerical results: Performance of semi-filtering policies





Conclusion

Contributions

- Characterization of surjective/injective/bijective graphs.
- Stabilizability conditions that are easier to verify.
- Almost complete characterization of the set of achievable matching rate vectors.
- Filtering (resp. semi-filtering) policies to achieve (resp. approach) vertices of the polytope.

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Future works

- More realistic model: hypergraph? reneging?
- What it the arrival rates and/or the graph structure are unknown?