EINDHOVEN
UNIVERSITY OF
TECHNOLOGY

Adaptive Load Balancing under Assignment Constraints

NETWORKS Conference – Session “Scalable Network Algorithms and Performance”

Céline Comte

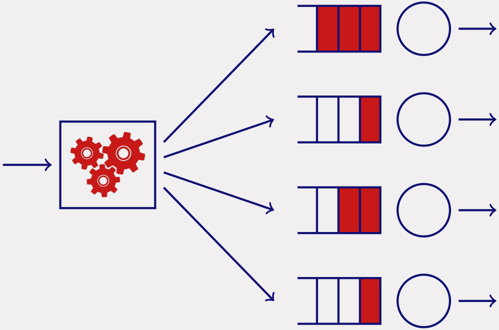
Outline

Outline

Model

Parameters

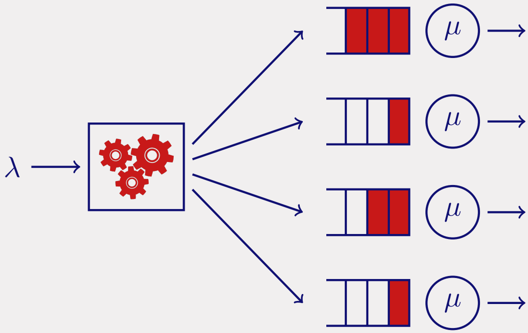
- 1 dispatcher, n servers, jobs



Model

Parameters

- 1 dispatcher, n servers, jobs
- Job arrival rate λ, service rate μ



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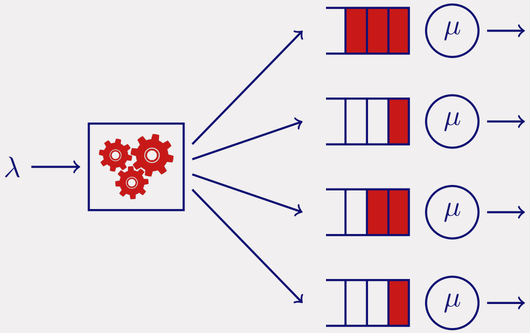
- Stability requires that $\lambda < n\mu$



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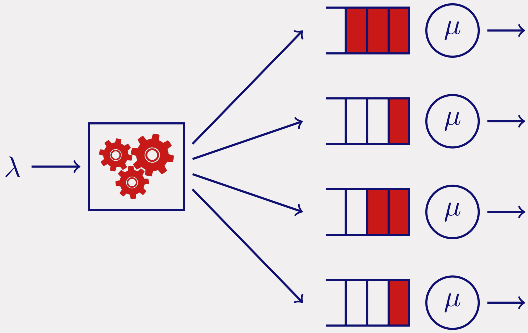
Randomness

- Poisson arrival process



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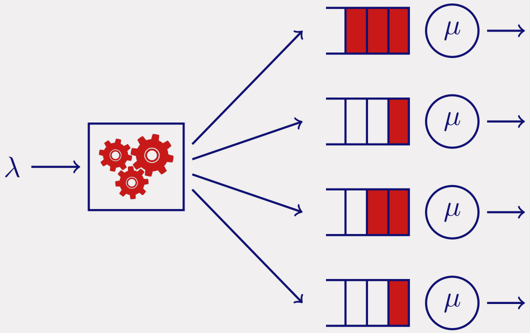
Randomness

- Poisson arrival process
- Exponential service times



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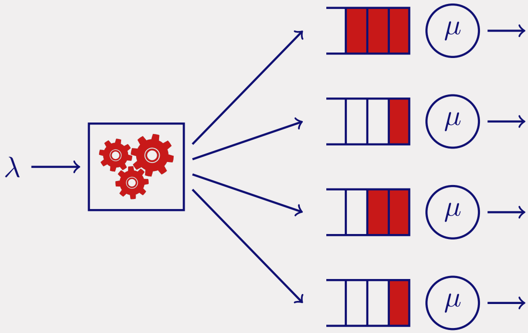
Randomness

- Poisson arrival process
- Exponential service times
- Continuous-time Markov chain



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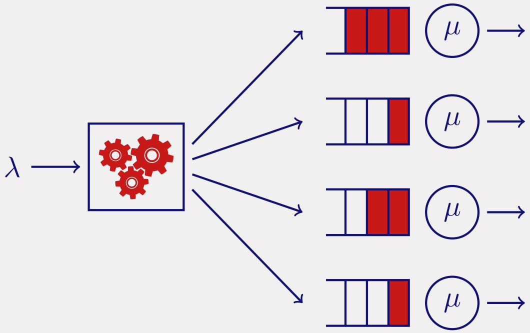
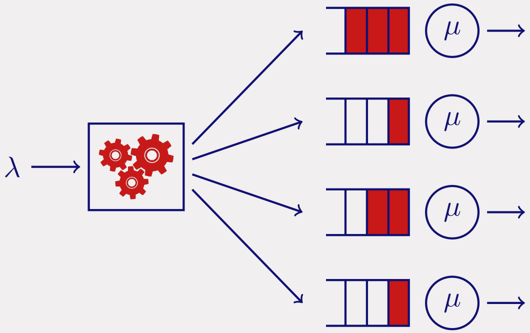
Examples: supermarket, cloud...



Model

Non-anticipating policies

- First-come-first-served scheduling
- Immediate, irrevocable assignment



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Non-anticipating policies

- First-come-first-served scheduling
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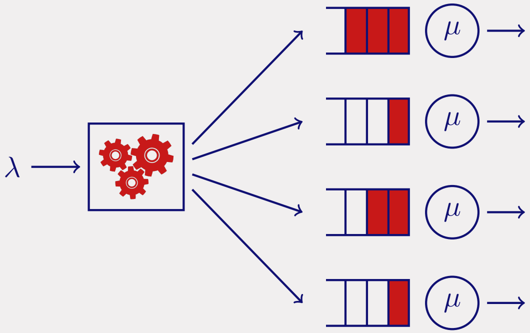
Performance metrics

- Waiting probability
- Mean waiting time
- Communication cost (per job)



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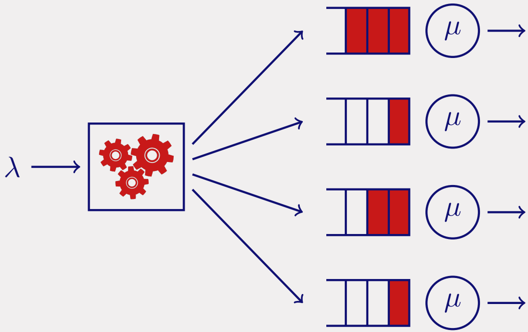
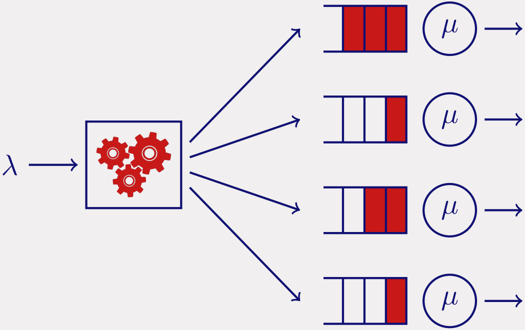
Variants: heterogeneity, scale...



Research questions

Algorithm design

- Efficient use of resources
- Scalability
- Predictable performance



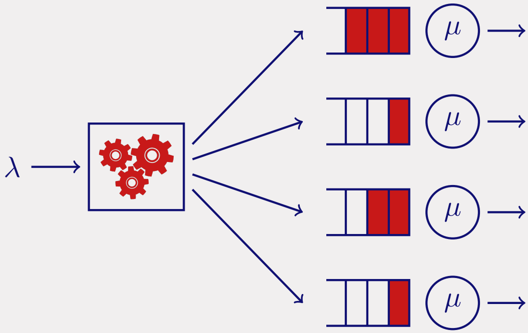
Research questions

Algorithm design

- Efficient use of resources
- Scalability
- Predictable performance

Performance evaluation

- Dimension infrastructure
- Compare algorithms
- Guarantee performance

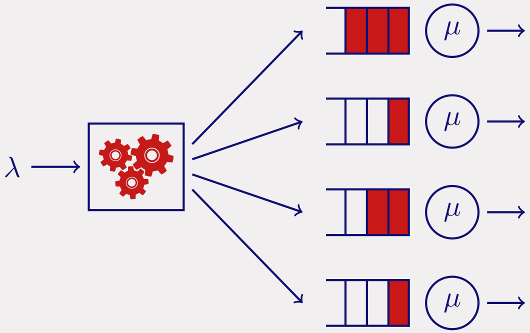


Outline

Join-idle-queue (JIQ)

Definition

- Each server sends a token to the dispatcher when it becomes idle



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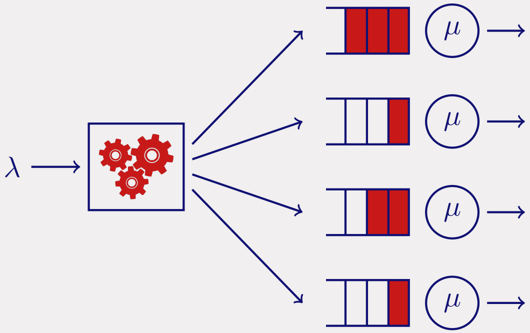
- Each server sends a token to the dispatcher when it becomes idle
- Assign incoming job to the longest idle server, if any



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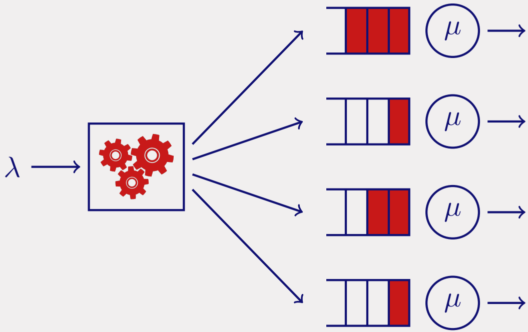

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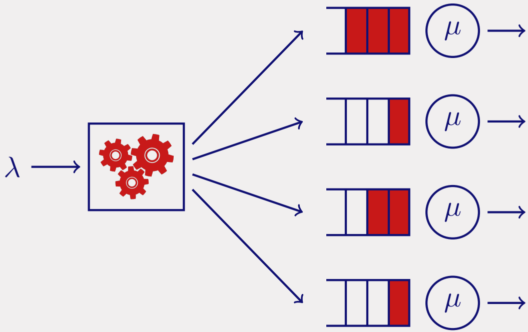
Performance

- Vanishing waiting probability



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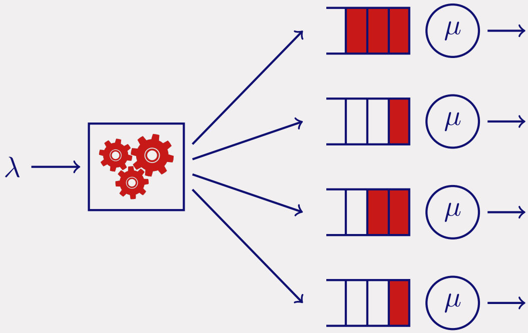
Performance

- Vanishing waiting probability
- Communication $\leq$ 1 per job



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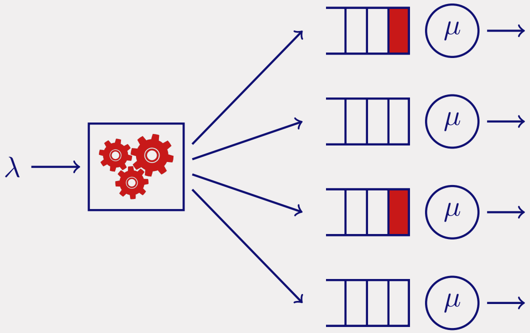

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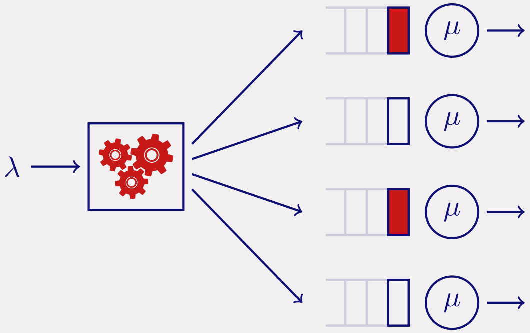

Performance

- Vanishing waiting probability
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Assign-to-the-longest-idle-server (ALIS)

Definition

- Each server sends a token to the dispatcher when it becomes idle
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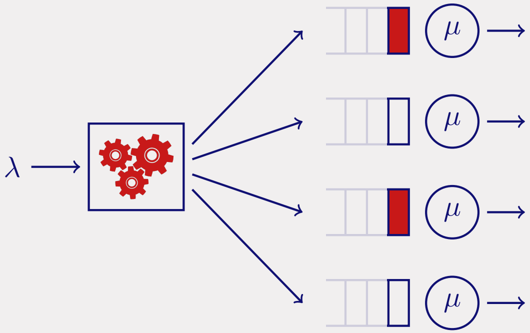

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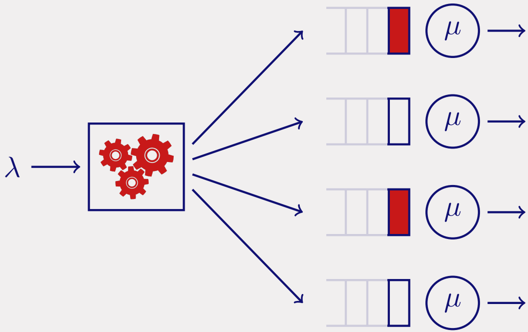


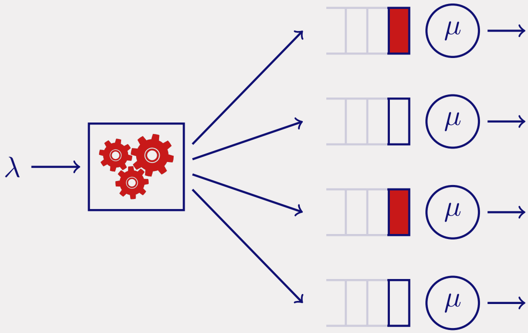
Performance

- Vanishing blocking probability
- Communication $=$ 1 per accepted job

Assign-to-the-longest-idle-server (ALIS)

Mathematical tractability

- Product-form stationary distribution (Erlang loss model)
- Performance is *insensitive* to the job size distribution



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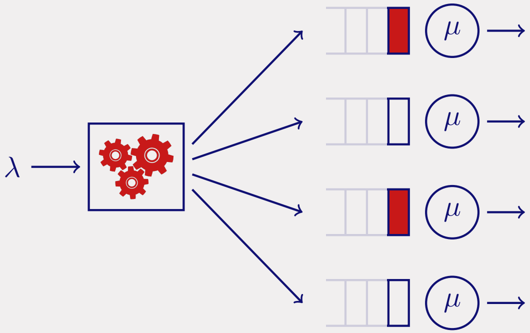
Generalizations

- Heterogeneous servers
- Assignment constraints
- Longer queues



Related work

Join-idle-queue (JIQ)

- M. van der Boor et al. “Scalable load balancing in networked systems: A survey of recent advances”. (June 14, 2018). arXiv: 1806.05444
- Y. Lu et al. “Join-Idle-Queue: A novel load balancing algorithm for dynamically scalable web services”. *Performance Evaluation*. Special Issue: Performance 2011 68.11 (Nov. 1, 2011), pp. 1056–1071
- M. van der Boor et al. “Load balancing in large-scale systems with multiple dispatchers”. IEEE INFOCOM 2017 - IEEE Conference on Computer Communications. May 2017, pp. 1–9

Related work

Assign-to-the-longest-idle-server (ALIS) and insensitive algorithms

- T. Bonald et al. "Insensitive Load Balancing". SIGMETRICS '04/Performance '04. New York, NY, USA: ACM, 2004, pp. 367–377
- I. Adan and G. Weiss. "A Loss System with Skill-based Servers under Assign to Longest Idle Server Policy". *Probability in the Engineering and Informational Sciences* 26.3 (July 2012), pp. 307–321
- M. Jonckheere and B. J. Prabhu. "Asymptotics of insensitive load balancing and blocking phases". *Queueing Systems* 88.3 (Apr. 1, 2018), pp. 243–278
- C. Comte. "Dynamic load balancing with tokens". *Computer Communications* 144 (Aug. 15, 2019), pp. 76–88

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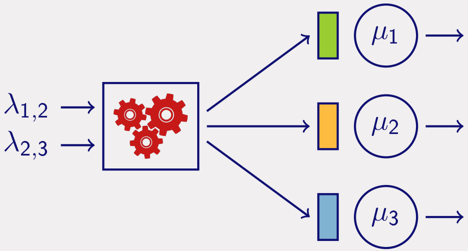

Model

- Arrival rates $\lambda_{1,2}$ and $\lambda_{2,3}$
- Service rates μ_1, μ_2, and μ_3

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Load-balancing algorithm

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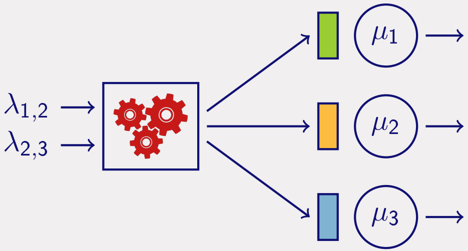

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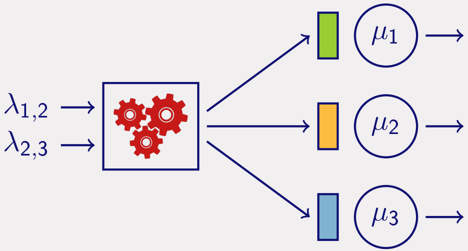
Contributions

- Product-form stationary distribution



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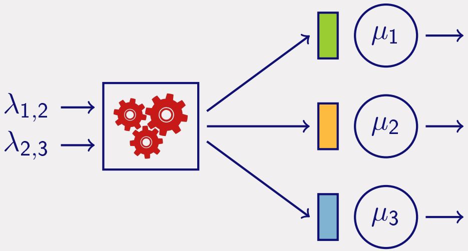

Contributions

- Product-form stationary distribution
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Assign-to-the-longest-available-token

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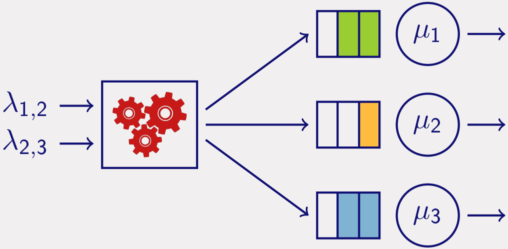

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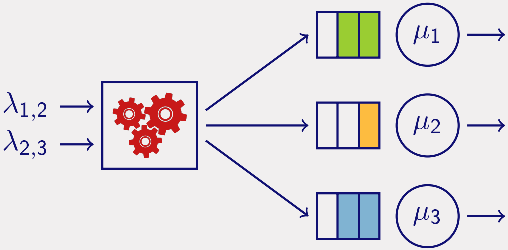

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Assign-to-the-longest-available-token

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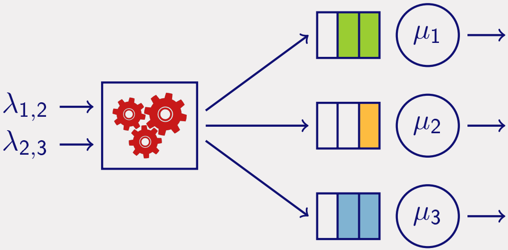
$$\pi(c_1, \ldots, c_n) \propto \prod_{p=1}^{n} \frac{\mu_{c_p}}{\lambda(\{c_1, \ldots, c_p\})}$$



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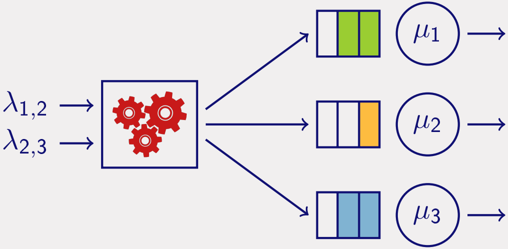

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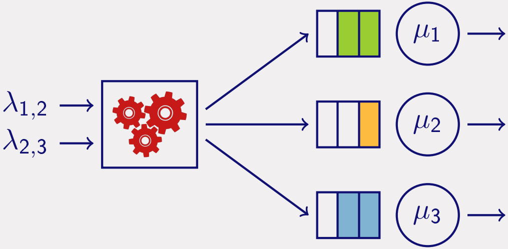
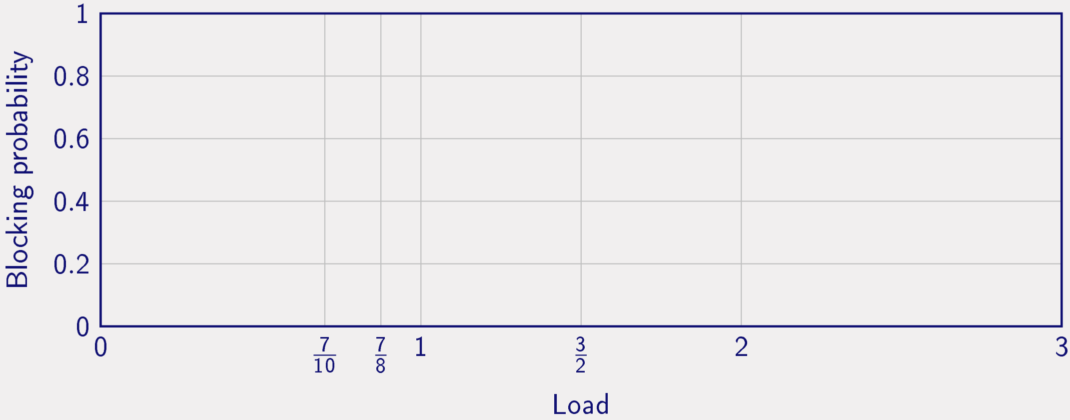
- Performance is insensitive to the job size distribution if the scheduling policy is processor-sharing
- Connection with existing queueing models and algorithms: order-independent queues, Whittle networks, ...



Numerical results

$\mu_1 = \mu_2 = \ldots = \mu_{10} = 1 \qquad \ell_1 = \ell_2 = \ldots = \ell_{10} = 6 \qquad \lambda_{1,2,\ldots,7} = 4\lambda_{4,5,\ldots,10}$



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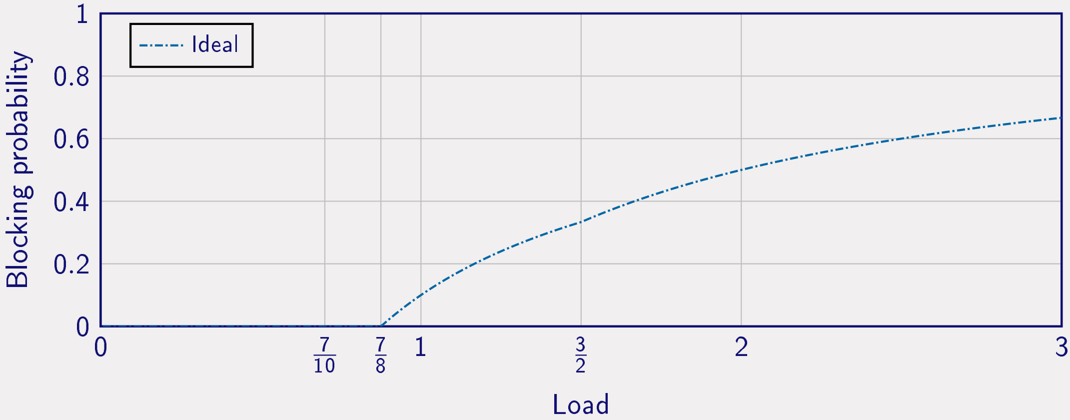

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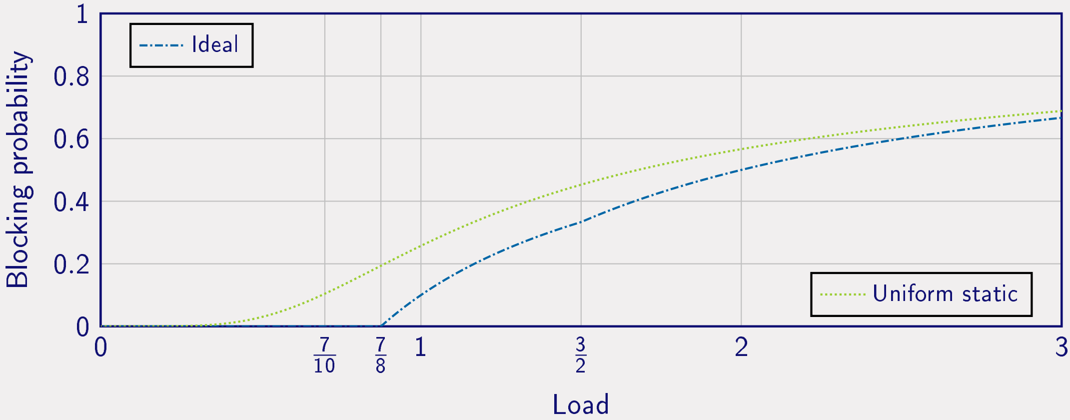

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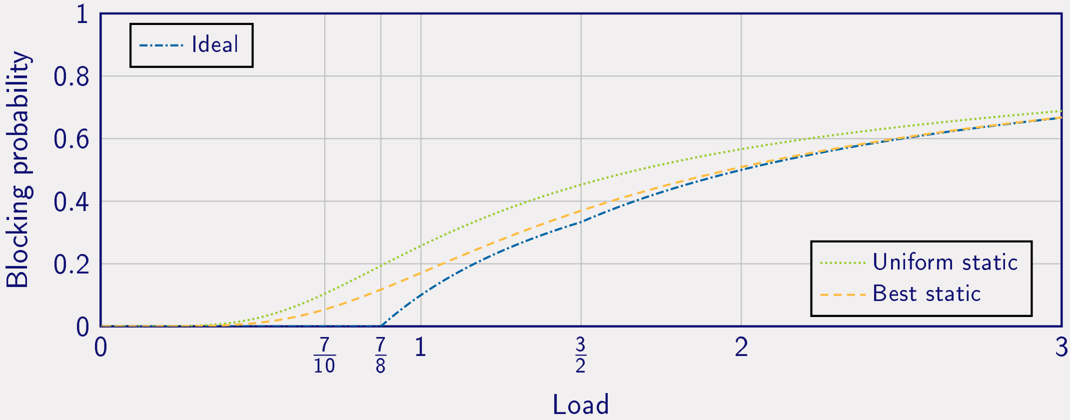

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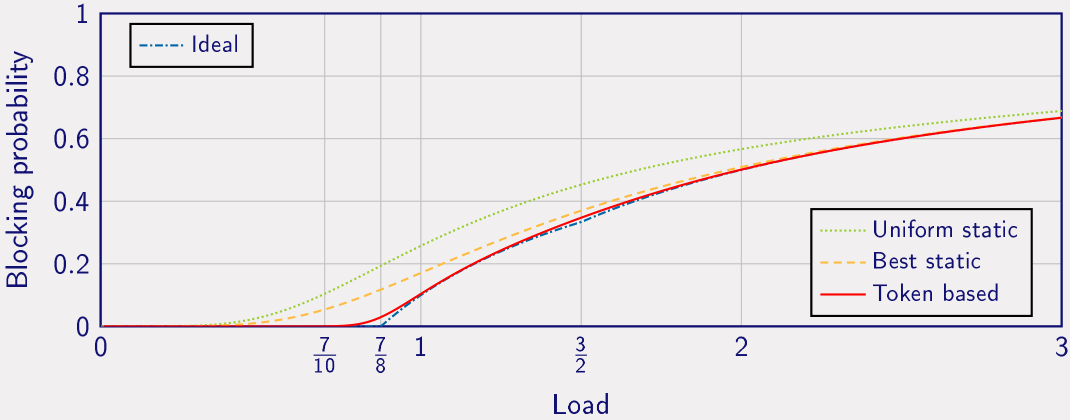

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Based on a joint work with Mark van der Boor that will be presented at IWQoS 2021.

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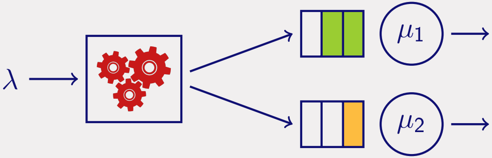
- Arrival rate λ
- Service rates μ_1 and μ_2
- Queue lengths ℓ_1 and ℓ_2 with $\ell_1 + \ell_2 = L$



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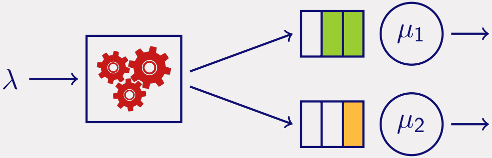

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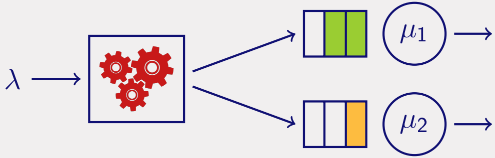

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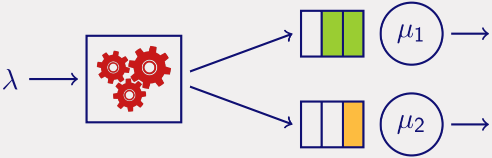
Question: Which queue length minimizes the blocking probability?

Optimal queue lengths

Theorem 1: There is $\lambda_* > 0$ such that, for $\lambda \leq \lambda_*$, the blocking probability is minimized when $\frac{\ell_1}{L} \simeq \frac{\mu_1}{\mu_1+\mu_2}$ and $\frac{\ell_2}{L} \simeq \frac{\mu_2}{\mu_1+\mu_2}$.



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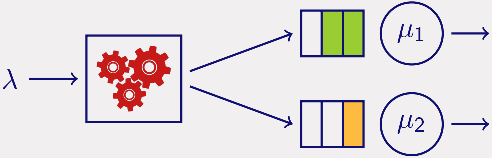
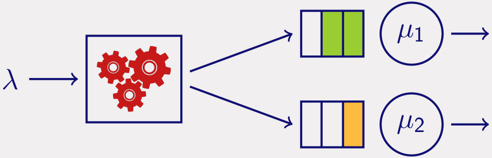
Theorem 2: There is $\lambda^* > 0$ such that, for $\lambda \geq \lambda^*$, the blocking probability is minimized when $\frac{\ell_1}{L} \simeq \frac{1}{2}$ and $\frac{\ell_2}{L} \simeq \frac{1}{2}$.



Optimal queue lengths

Theorem 1: There is $\lambda_* > 0$ such that, for $\lambda \leq \lambda_*$, the blocking probability is minimized when $\frac{\ell_1}{L} \simeq \frac{\mu_1}{\mu_1+\mu_2}$ and $\frac{\ell_2}{L} \simeq \frac{\mu_2}{\mu_1+\mu_2}$.

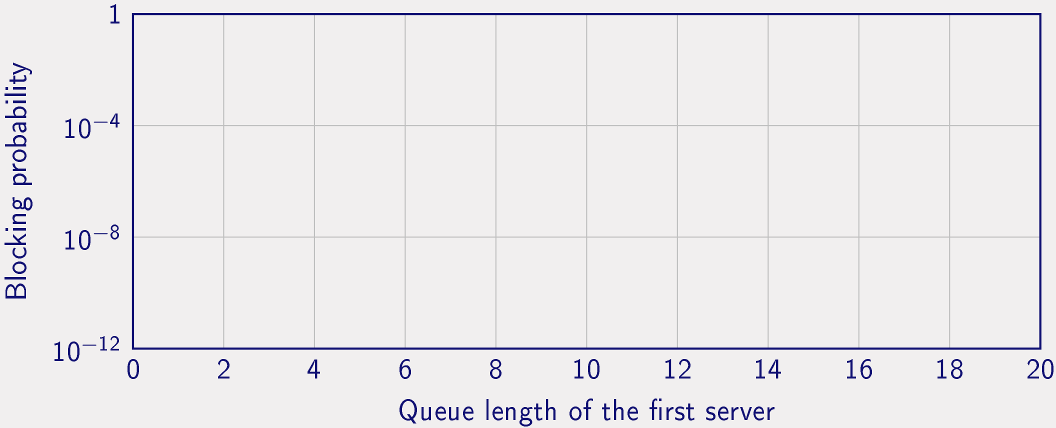
Theorem 2: There is $\lambda^* > 0$ such that, for $\lambda \geq \lambda^*$, the blocking probability is minimized when $\frac{\ell_1}{L} \simeq \frac{1}{2}$ and $\frac{\ell_2}{L} \simeq \frac{1}{2}$.



Theorem 3: The optimal queue length of the fastest server, in terms of the blocking probability, is decreasing with the arrival rate λ.

Numerical results

$L = 20$ $\mu_1 = 0.9$ $\mu_2 = 0.1$



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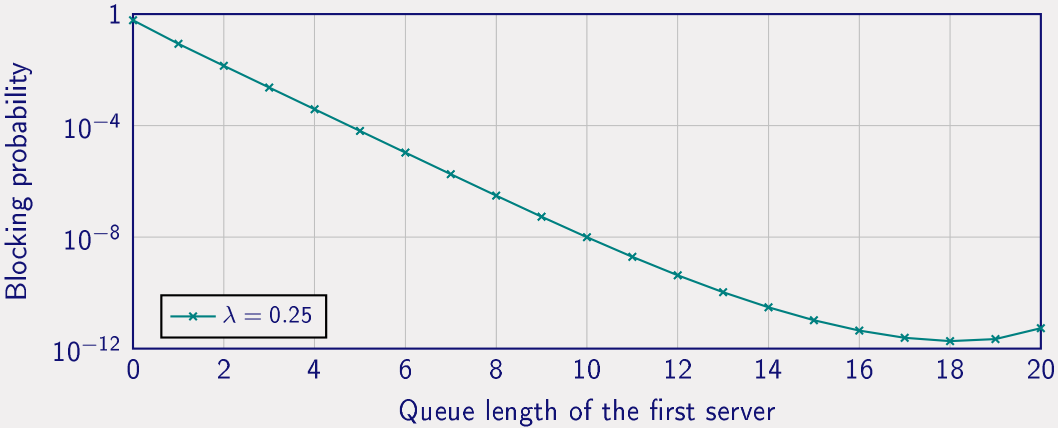

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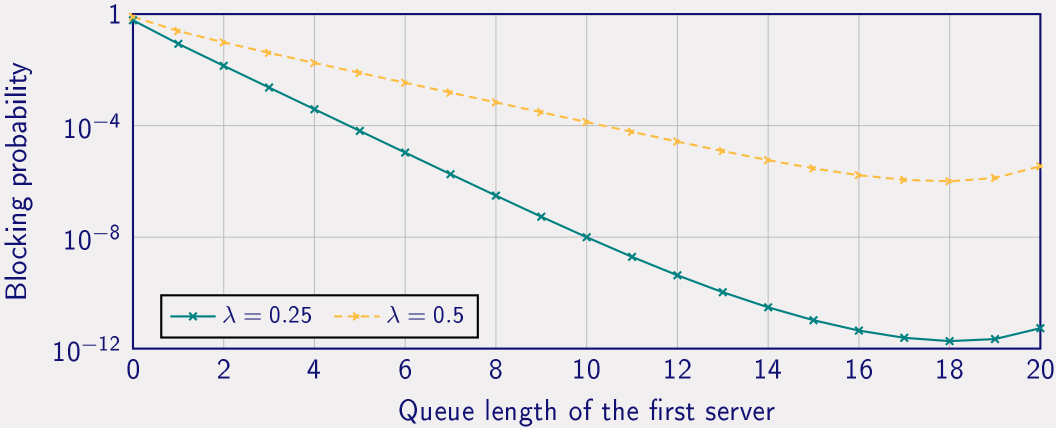

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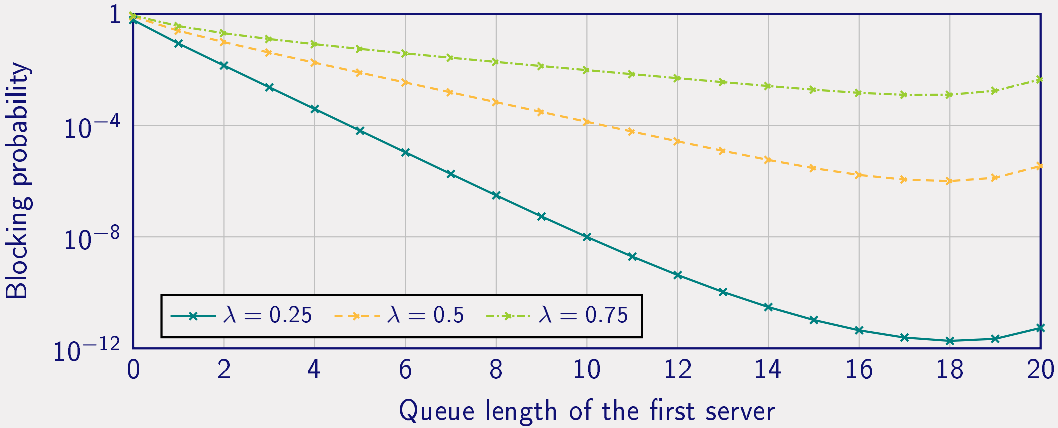

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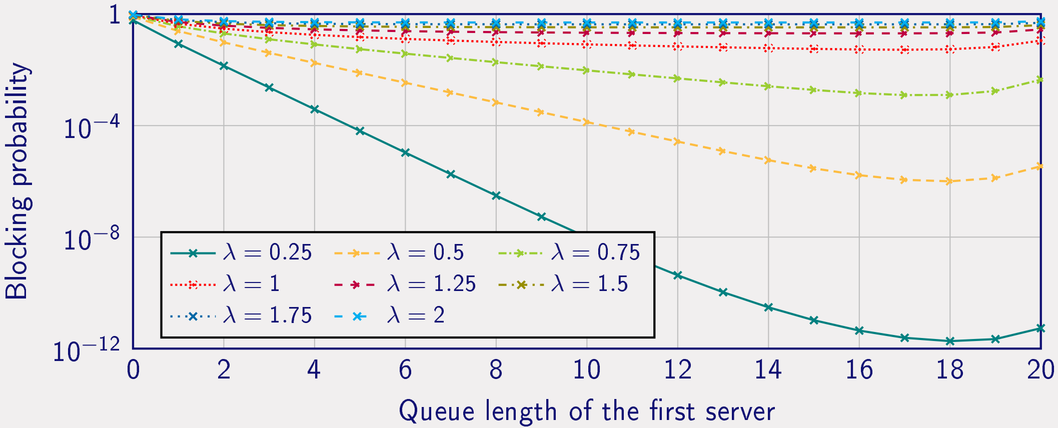

Conclusion

Take-away messages

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- Generalize blocking (insensitive) variants.
- Formalize the relation between JIQ and ALIS.