Dynamic Load Balancing with Tokens

Céline Comte

Young European Queueing Theorists XII
December 3, 2018

Introduction

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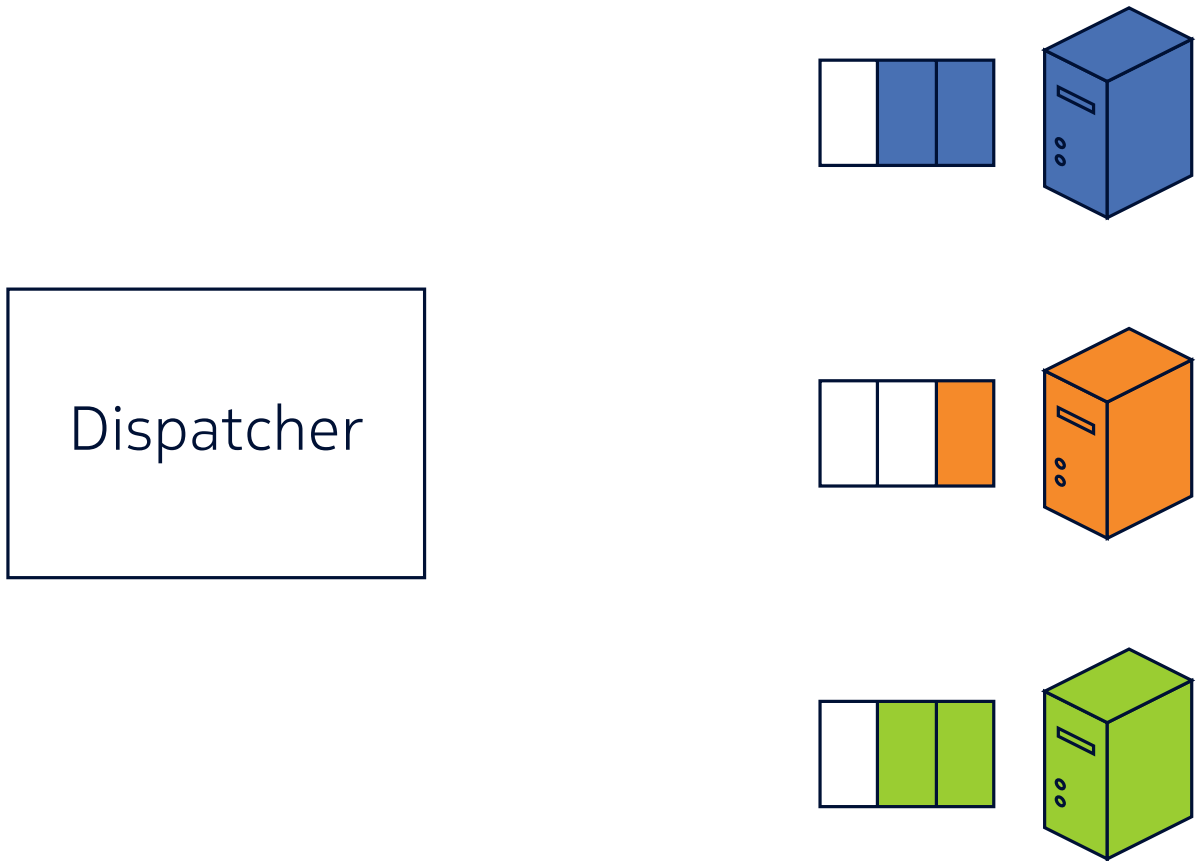
Airport

Abstraction



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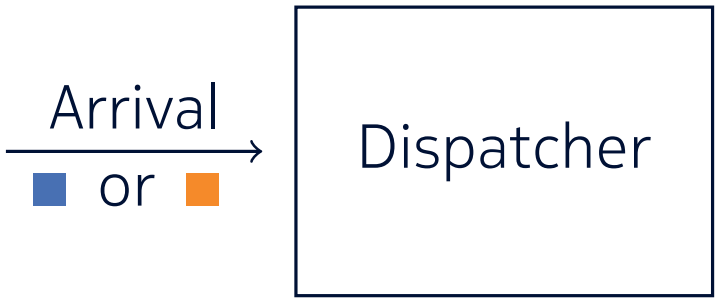

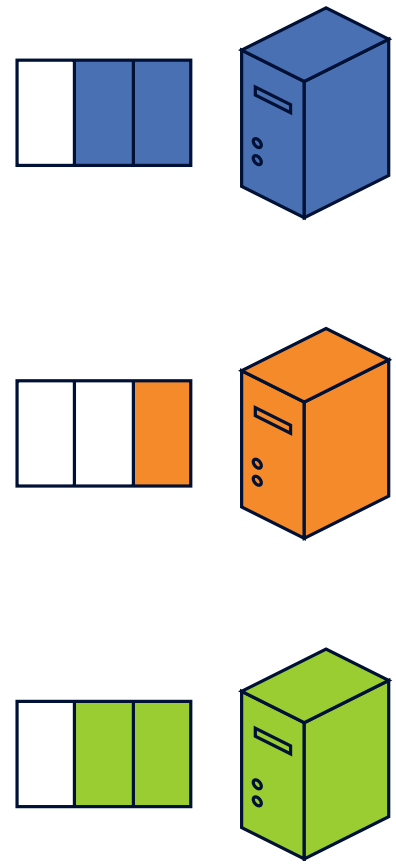

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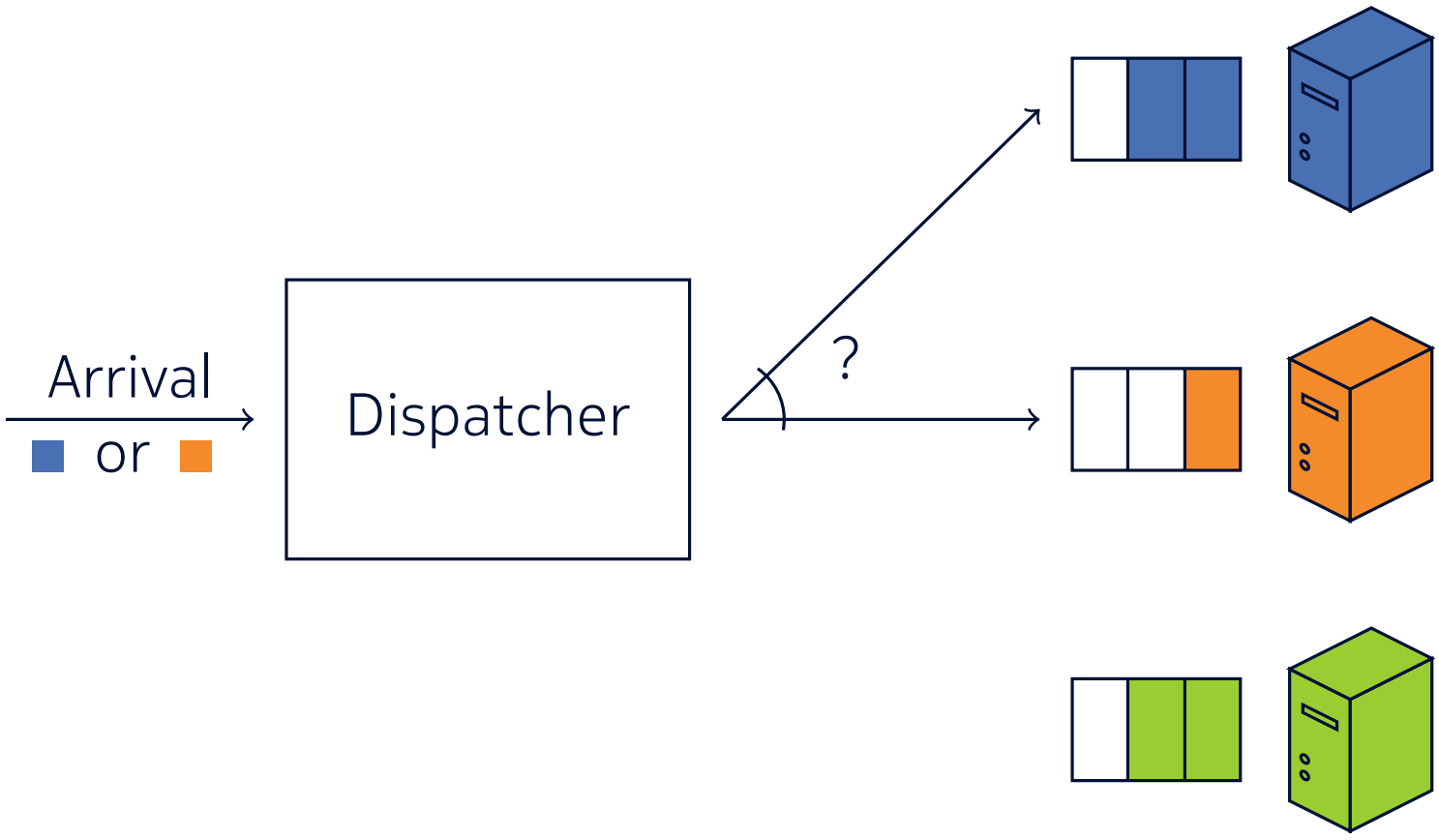

Our contributions

Load balancing algorithm

Robust and adaptative, yet simple

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Robust and adaptative, yet simple

Queueing analysis
Under a Poisson arrival process

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Relate several existing works

- Join-Idle-Queue (Lu et al., 2011)
- Assign to the longest idle server (ALIS) and FCFS-ALIS (Adan and Weiss, 2012) based on order-independent queues (Berezner et al., 1995)
- Insensitive load balancing (Bonald et al., 2004) based on Whittle networks (Whittle, 1985)

Outline

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Queueing analysis

Numerical results

Extensions

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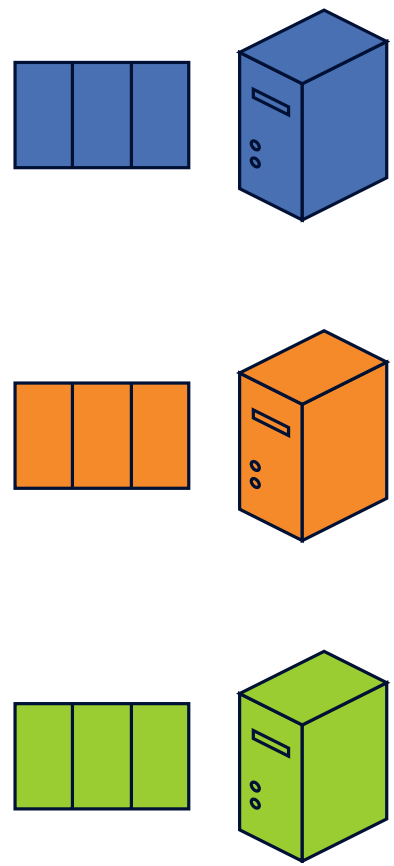

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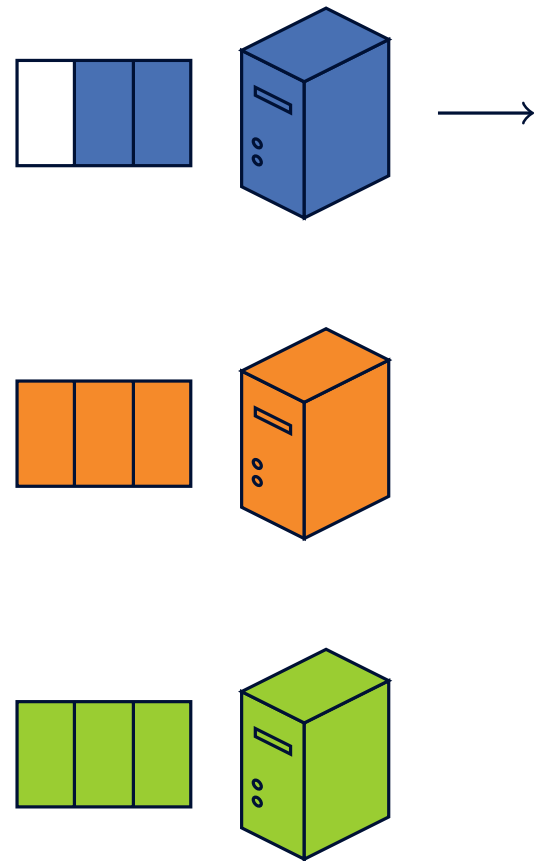

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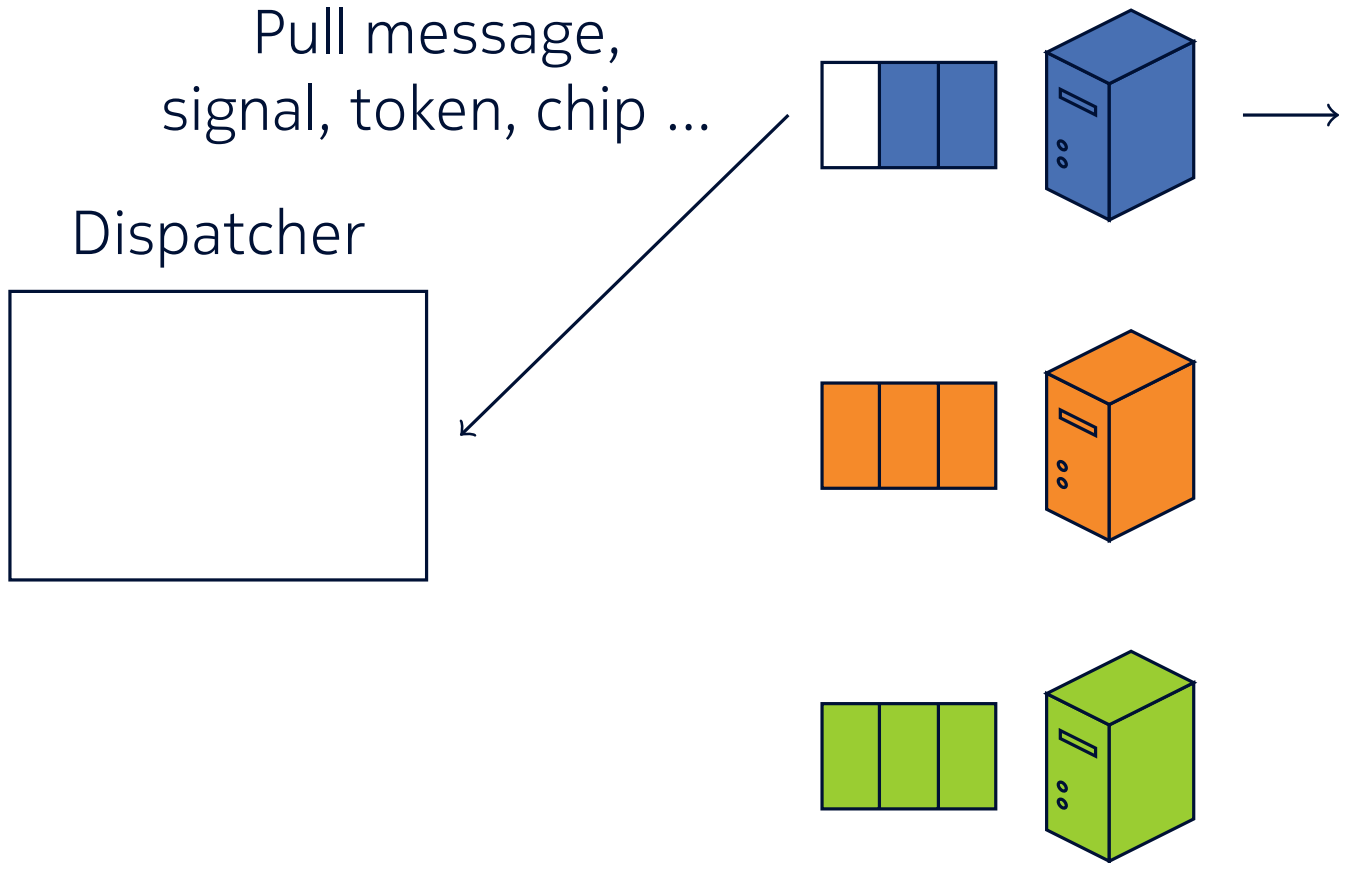

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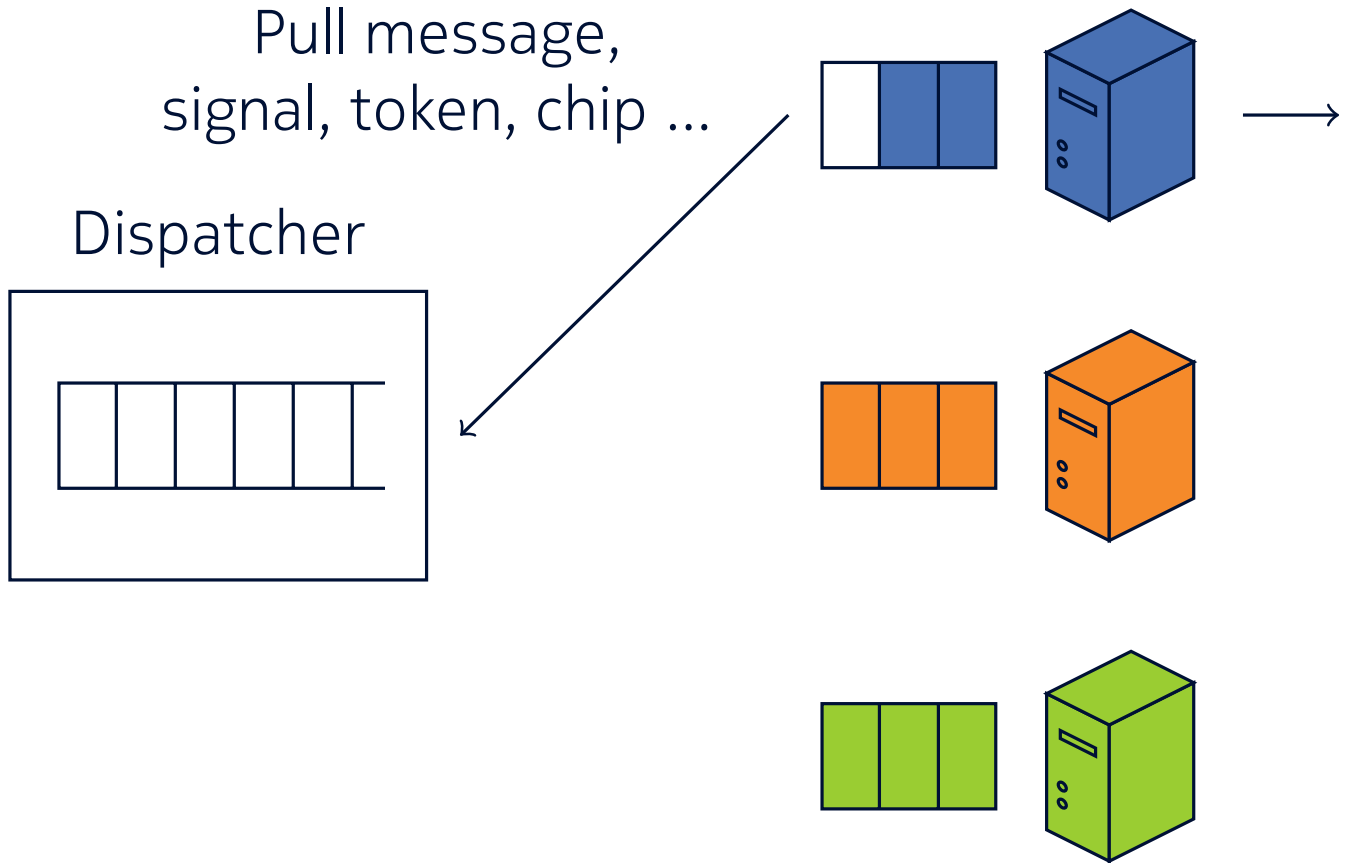

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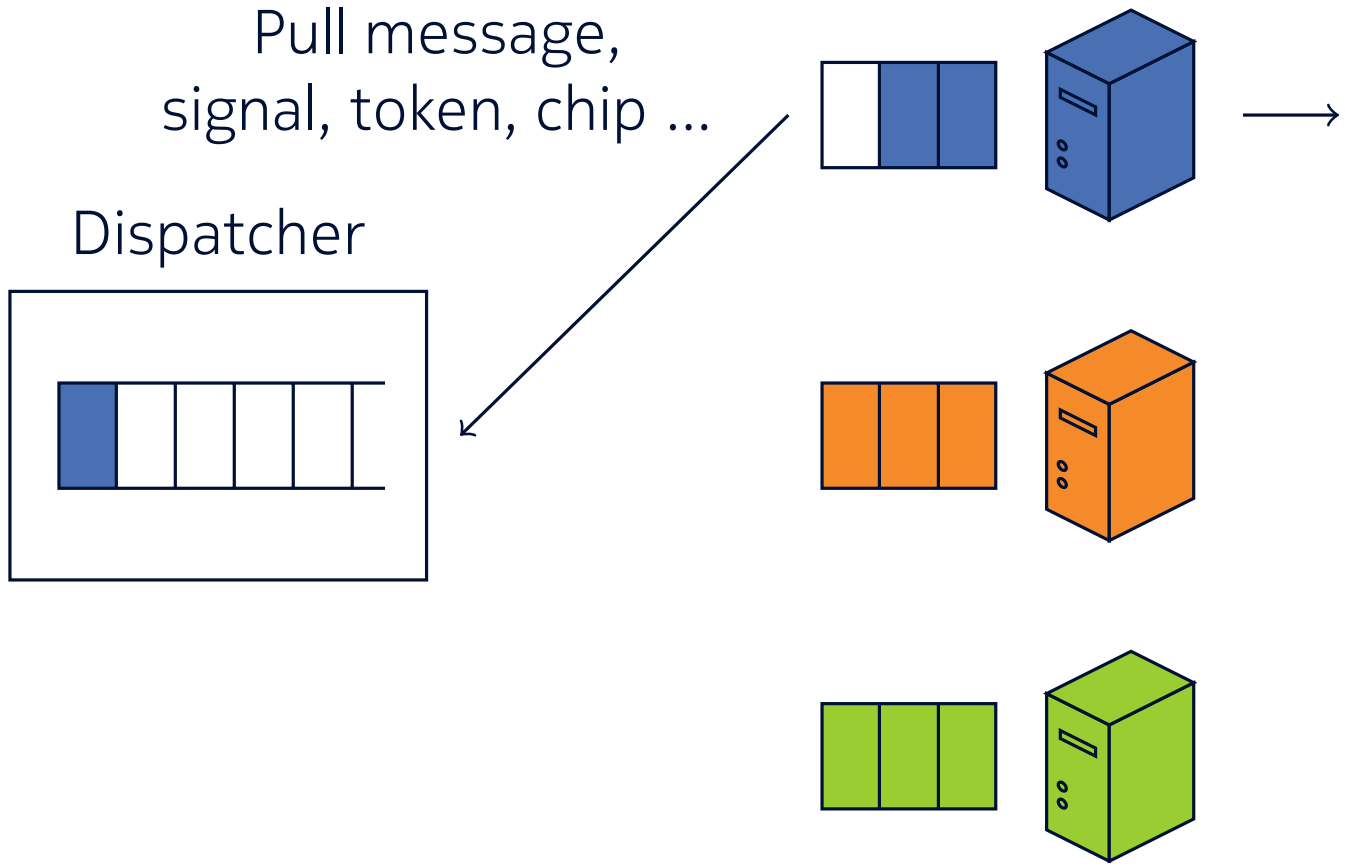

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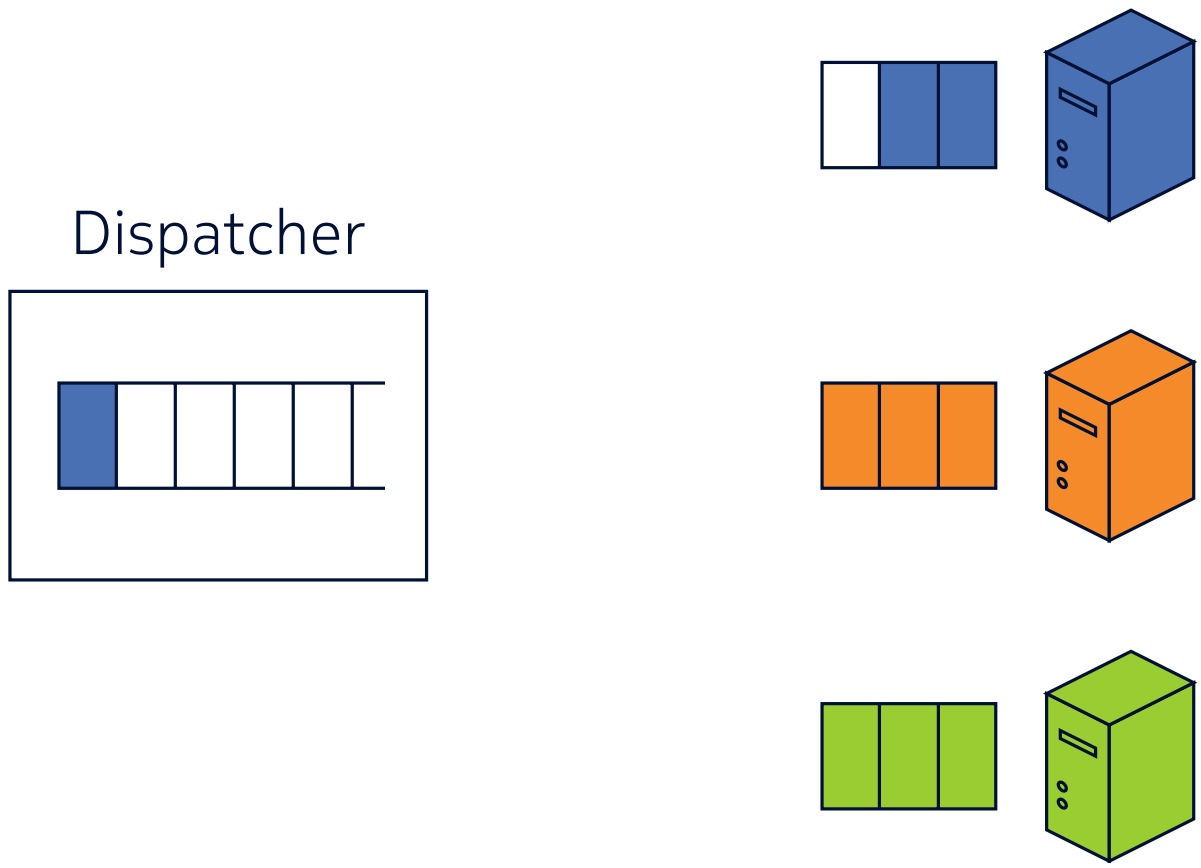

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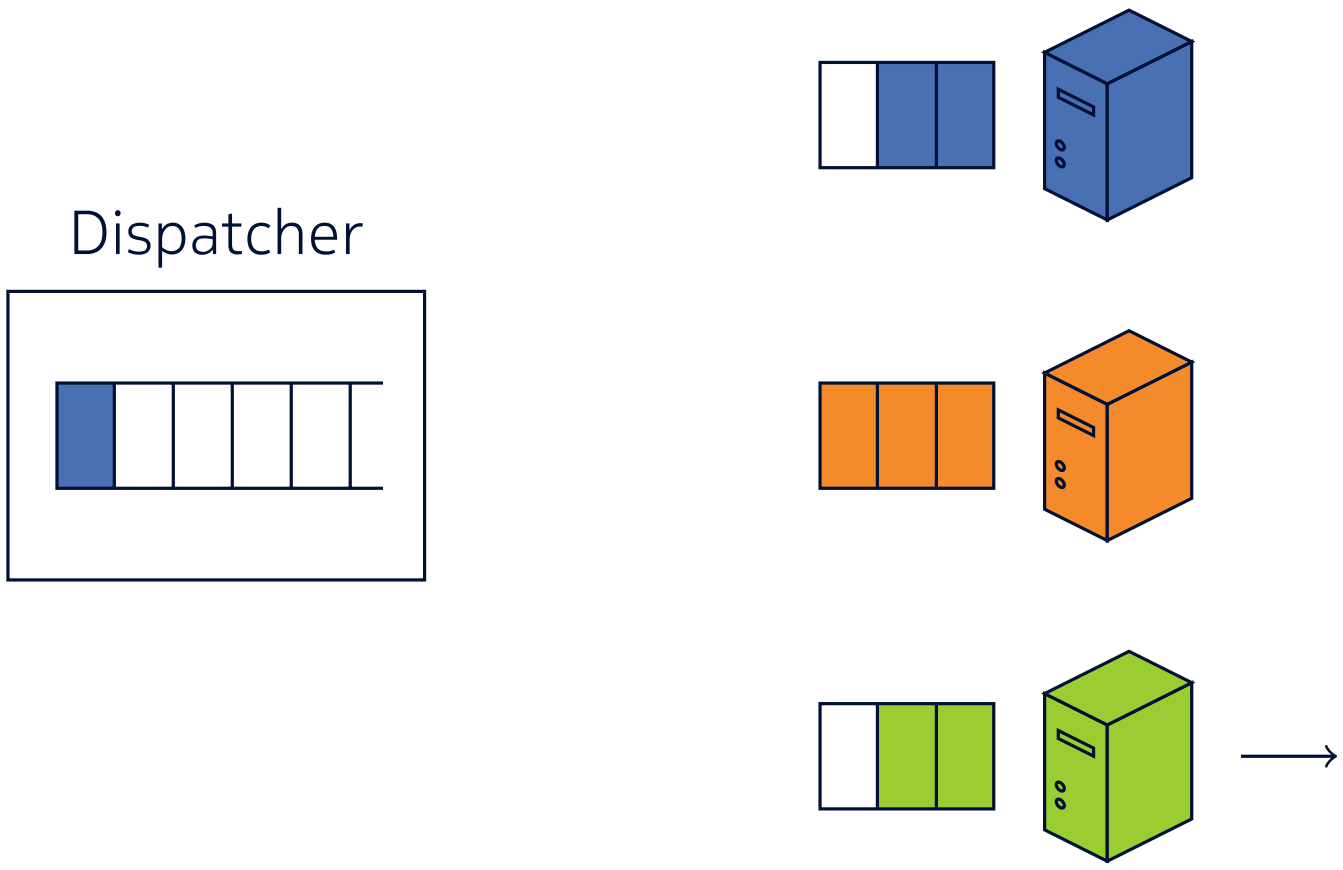

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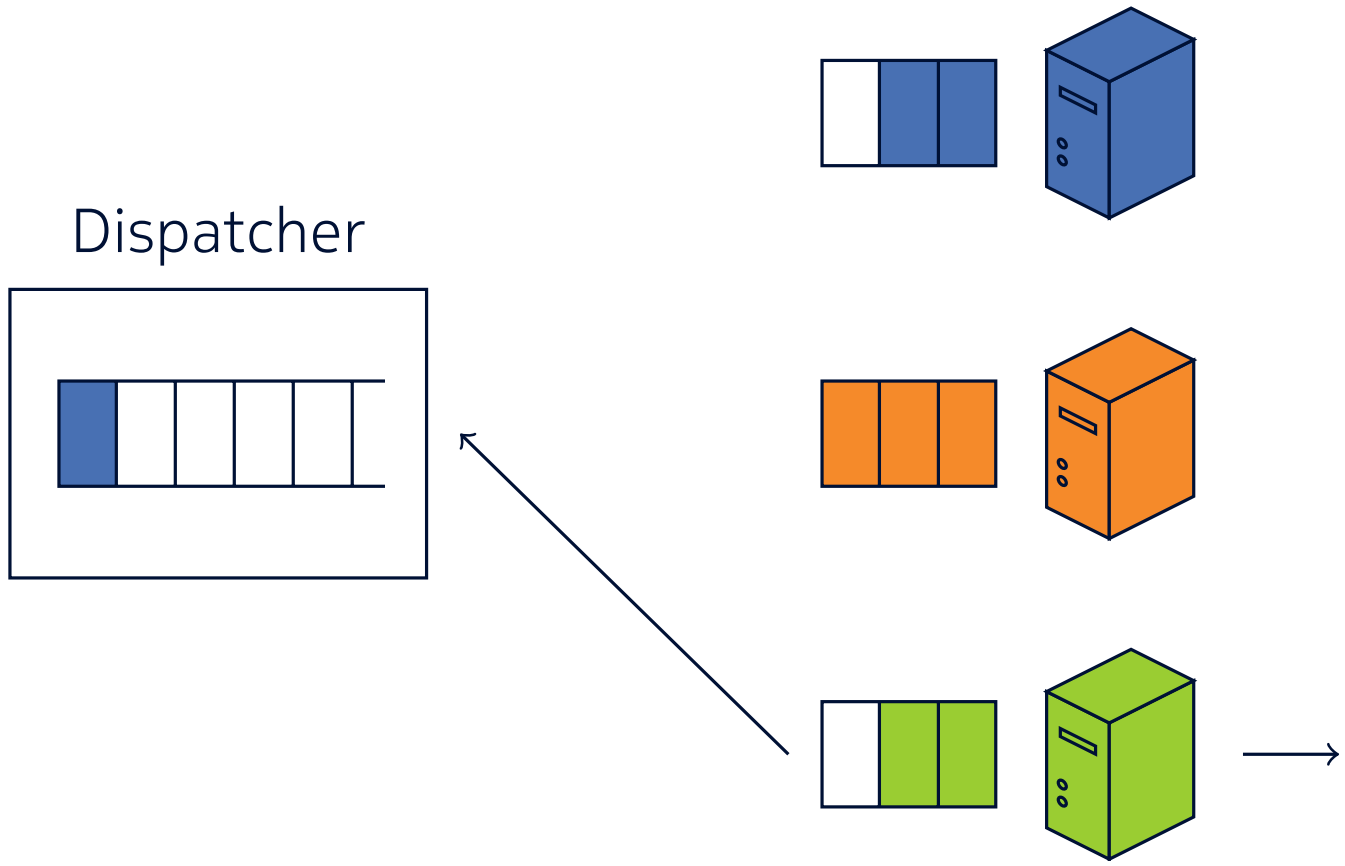

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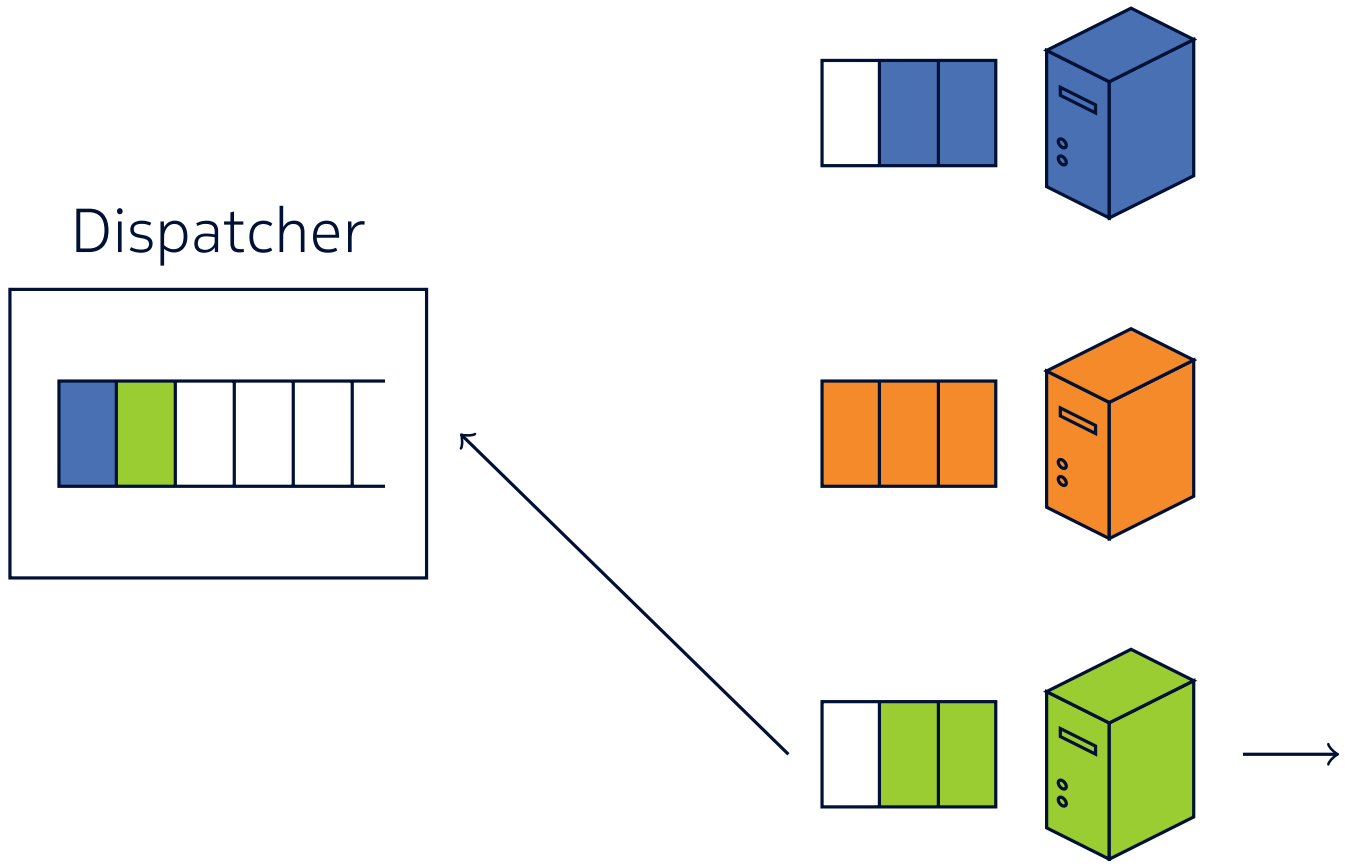

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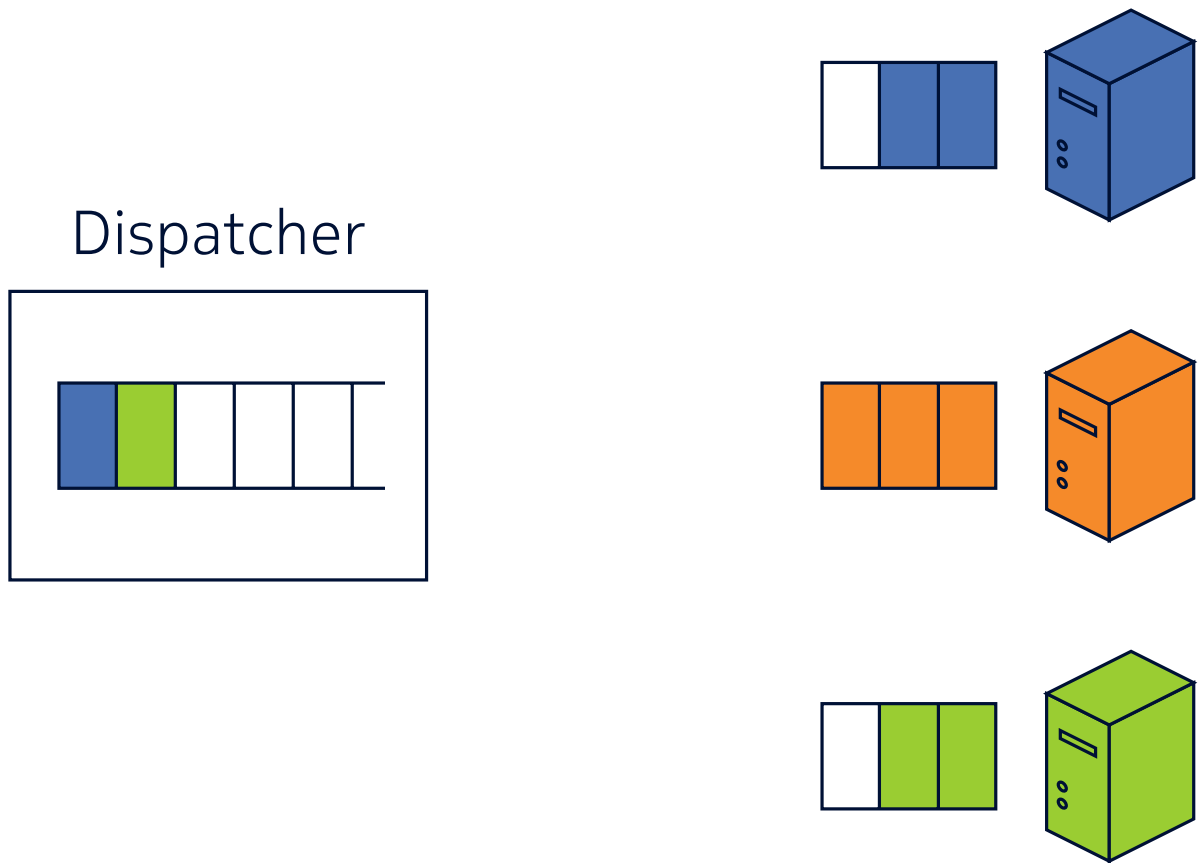

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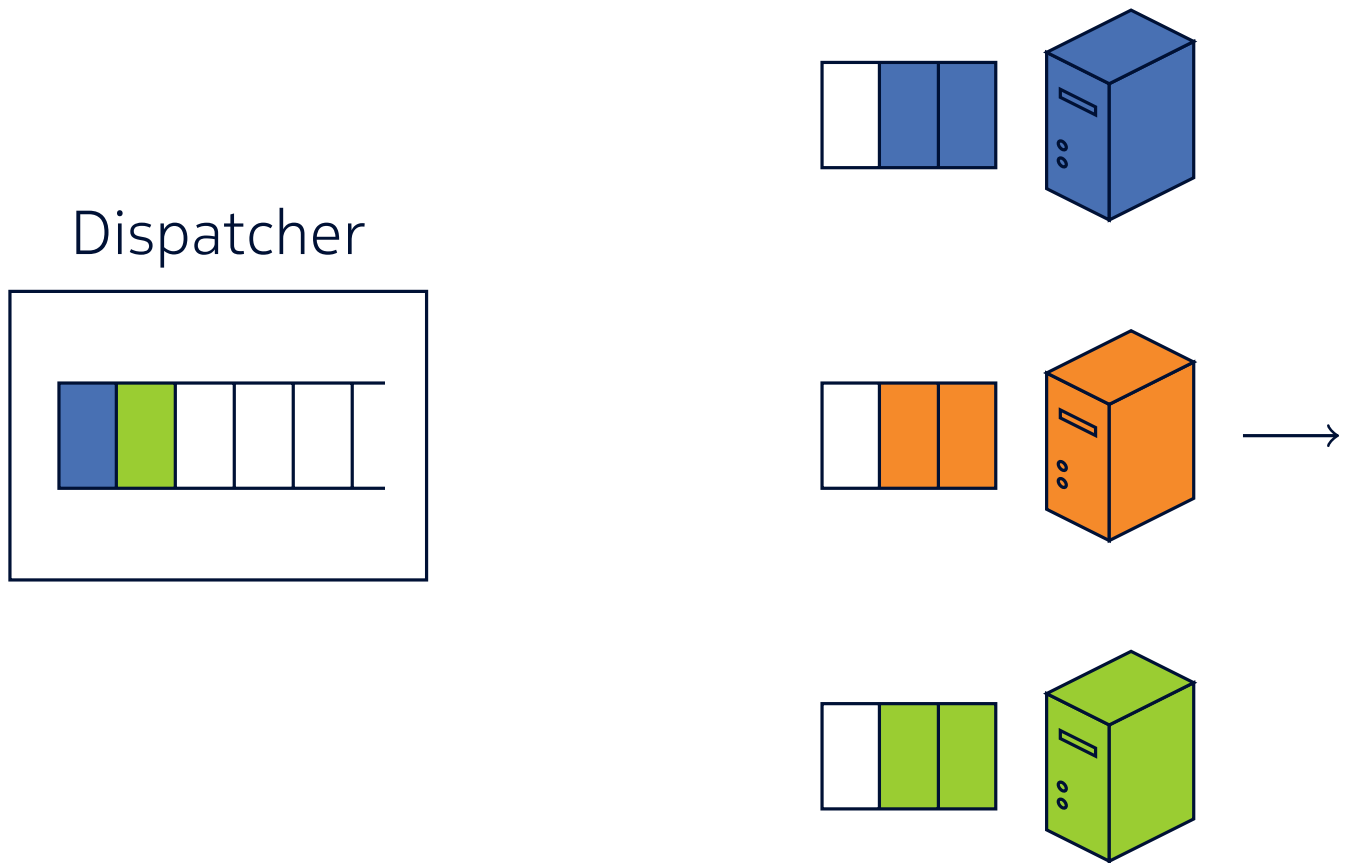

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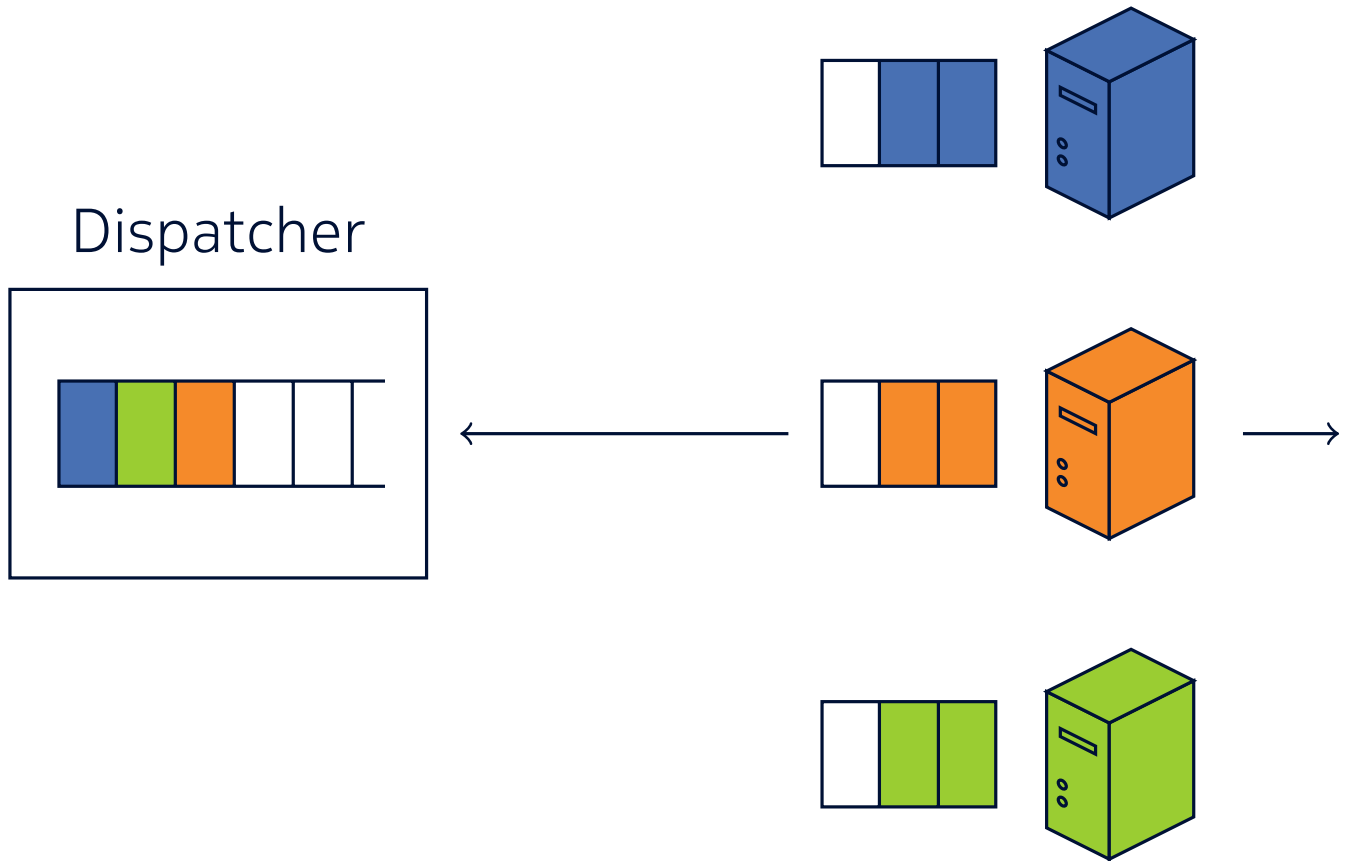

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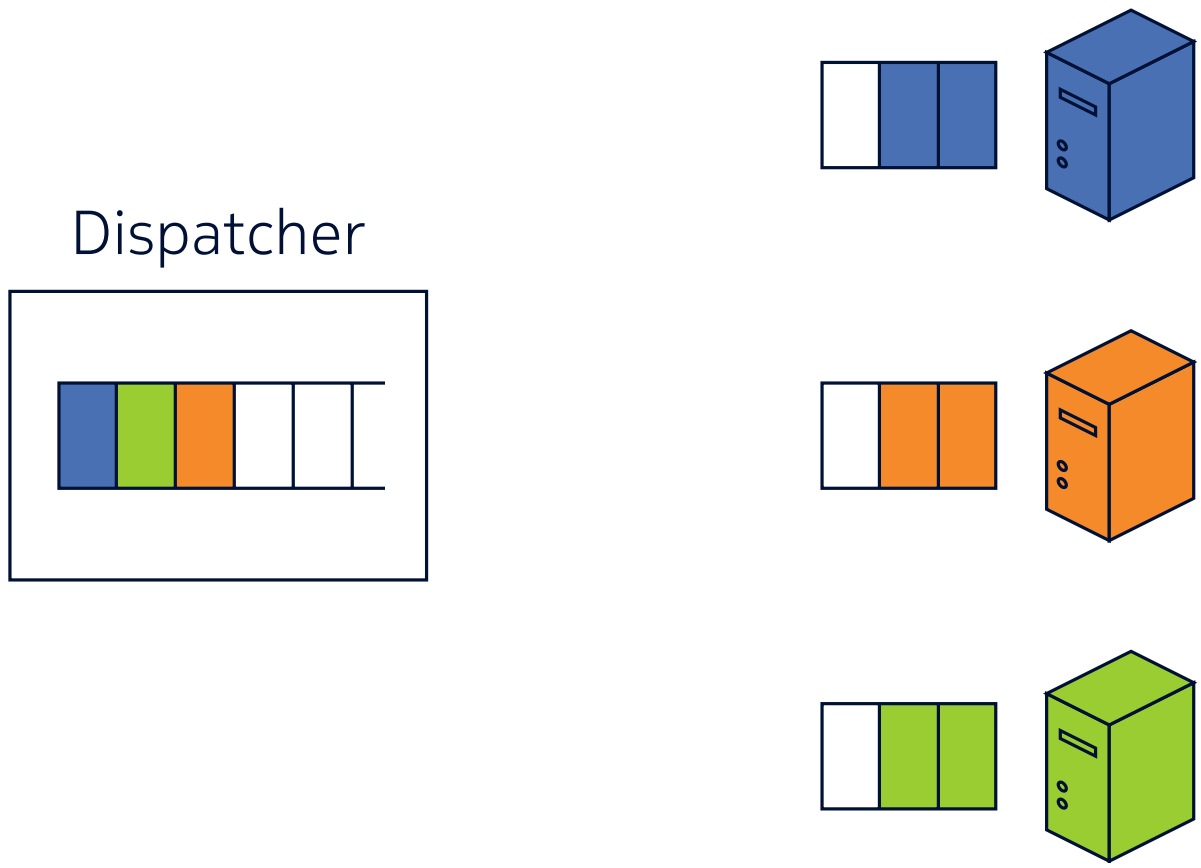

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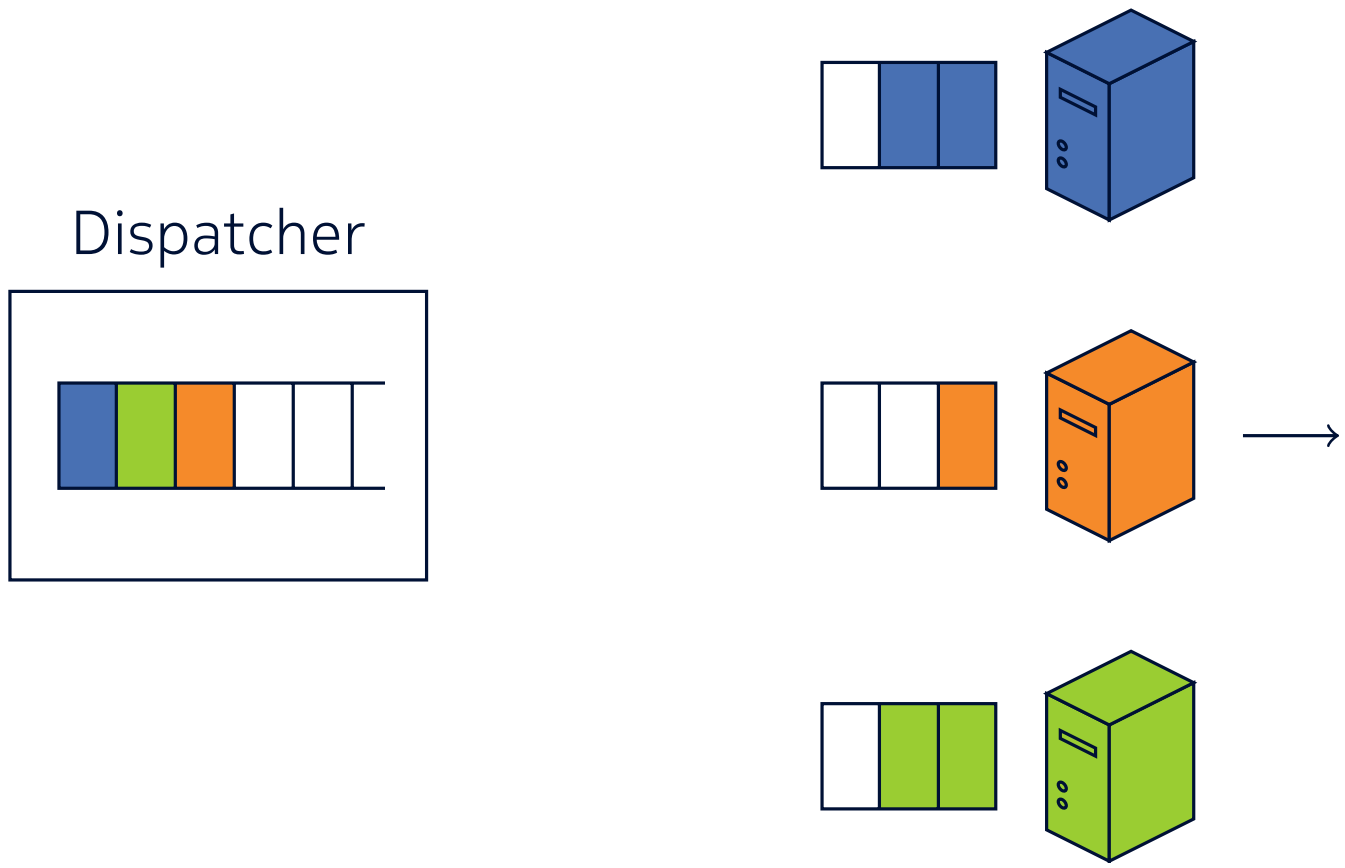

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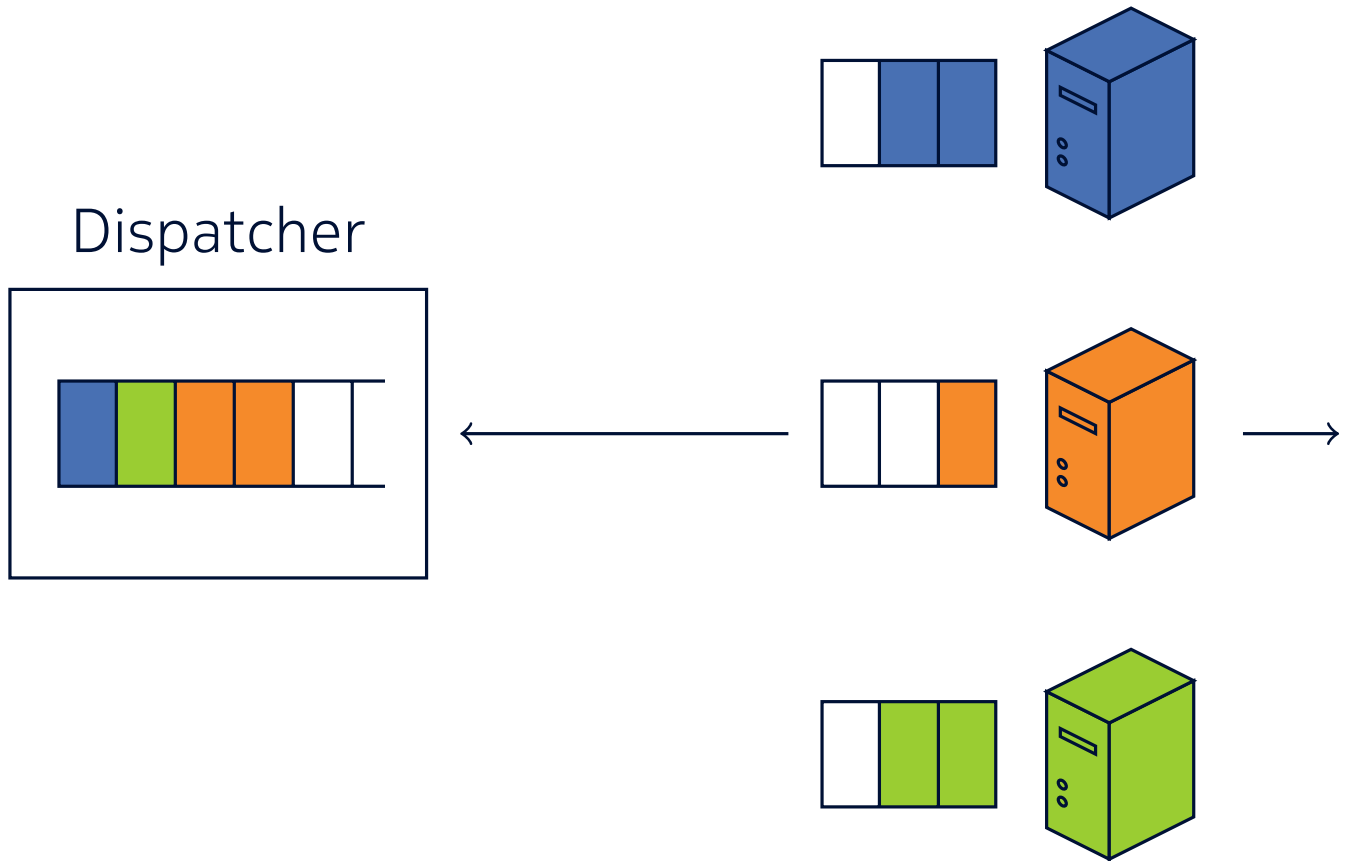

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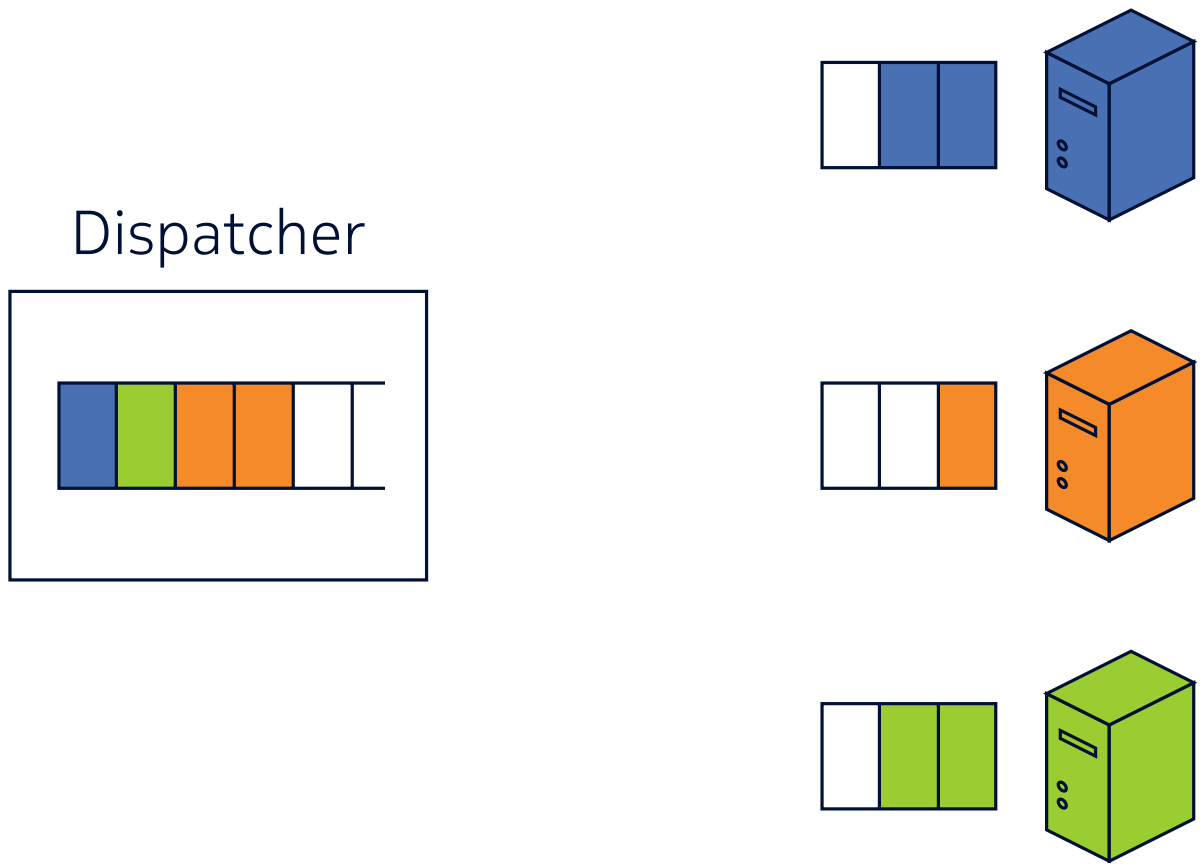

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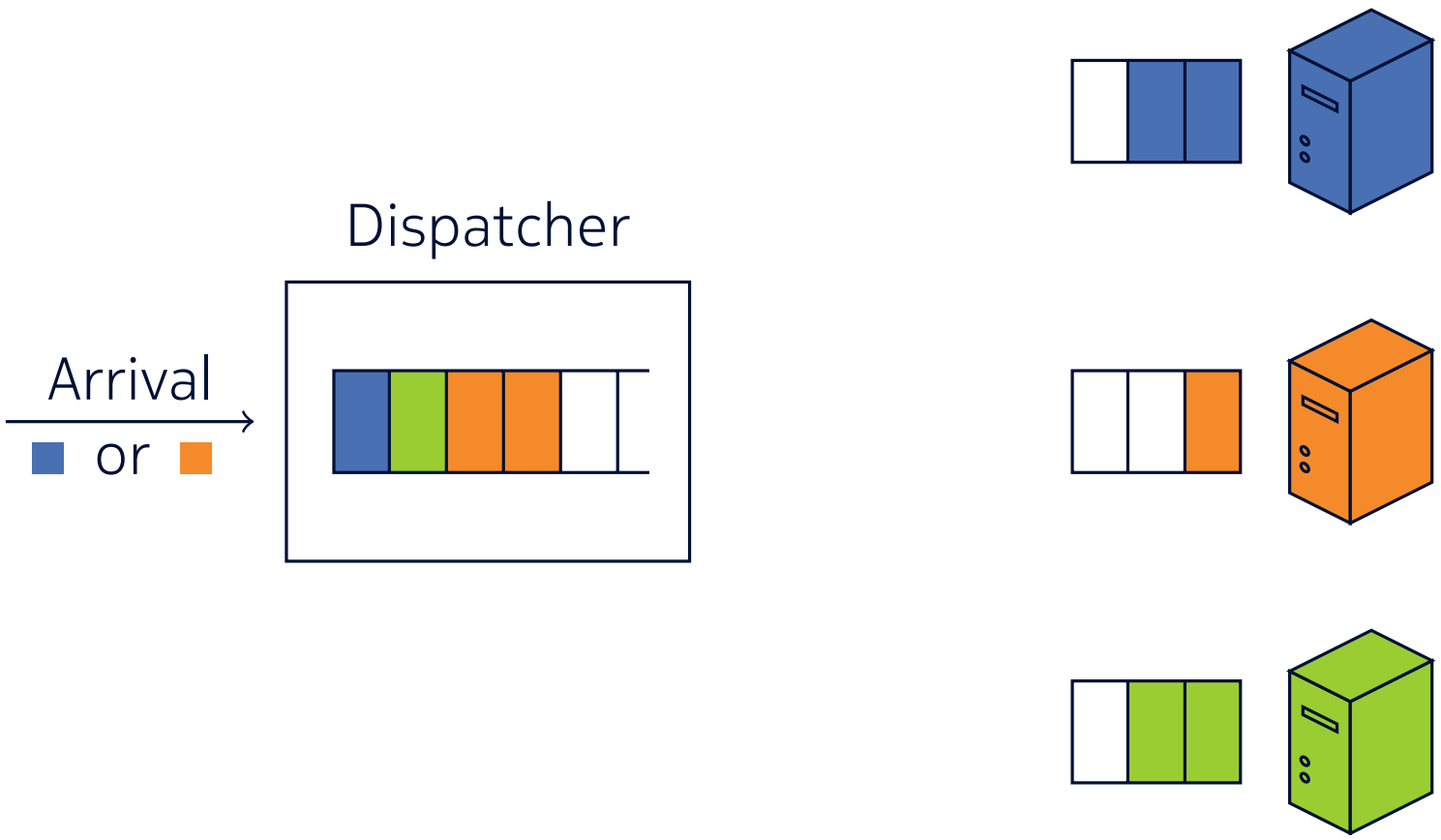

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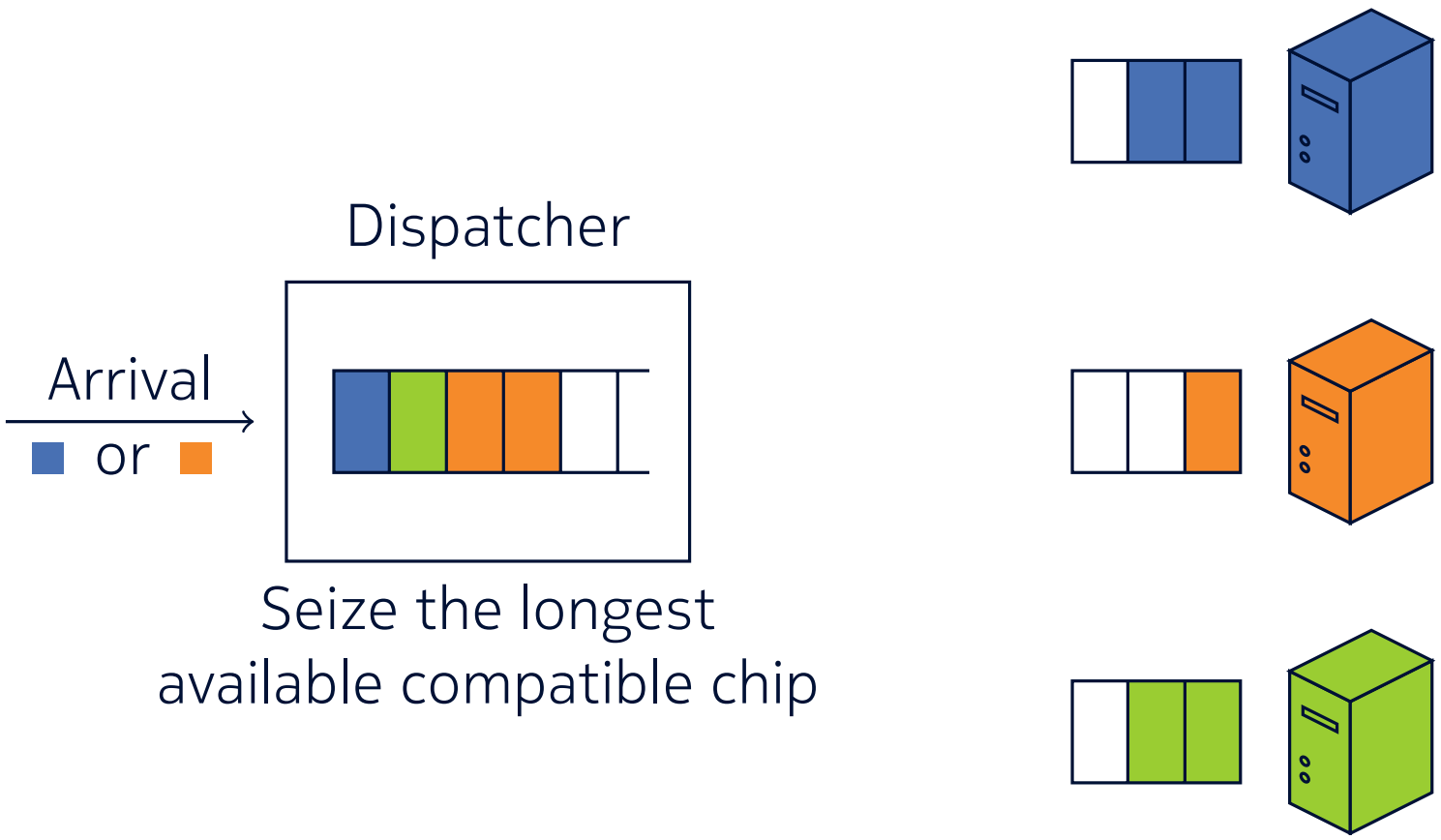

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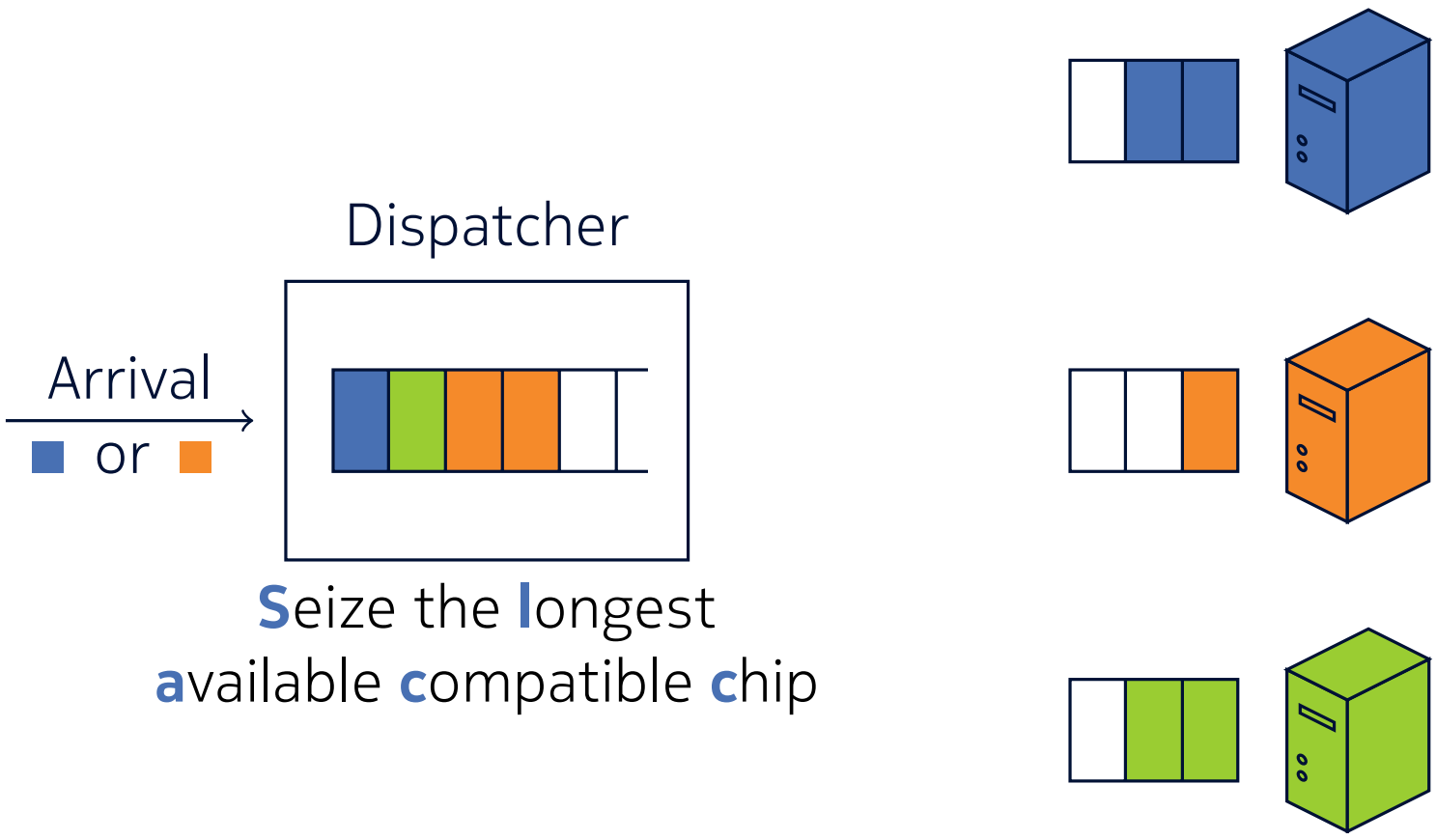

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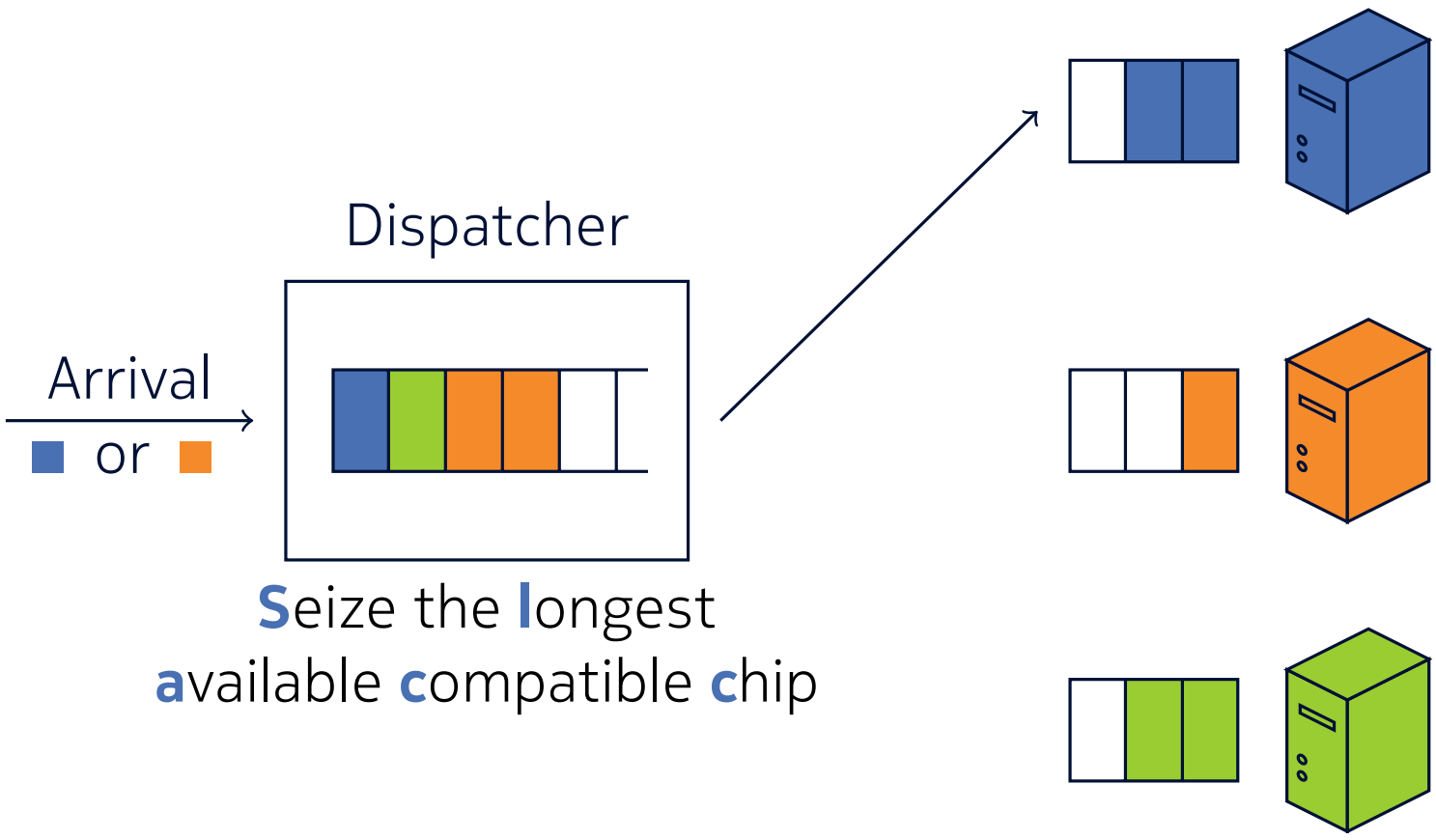

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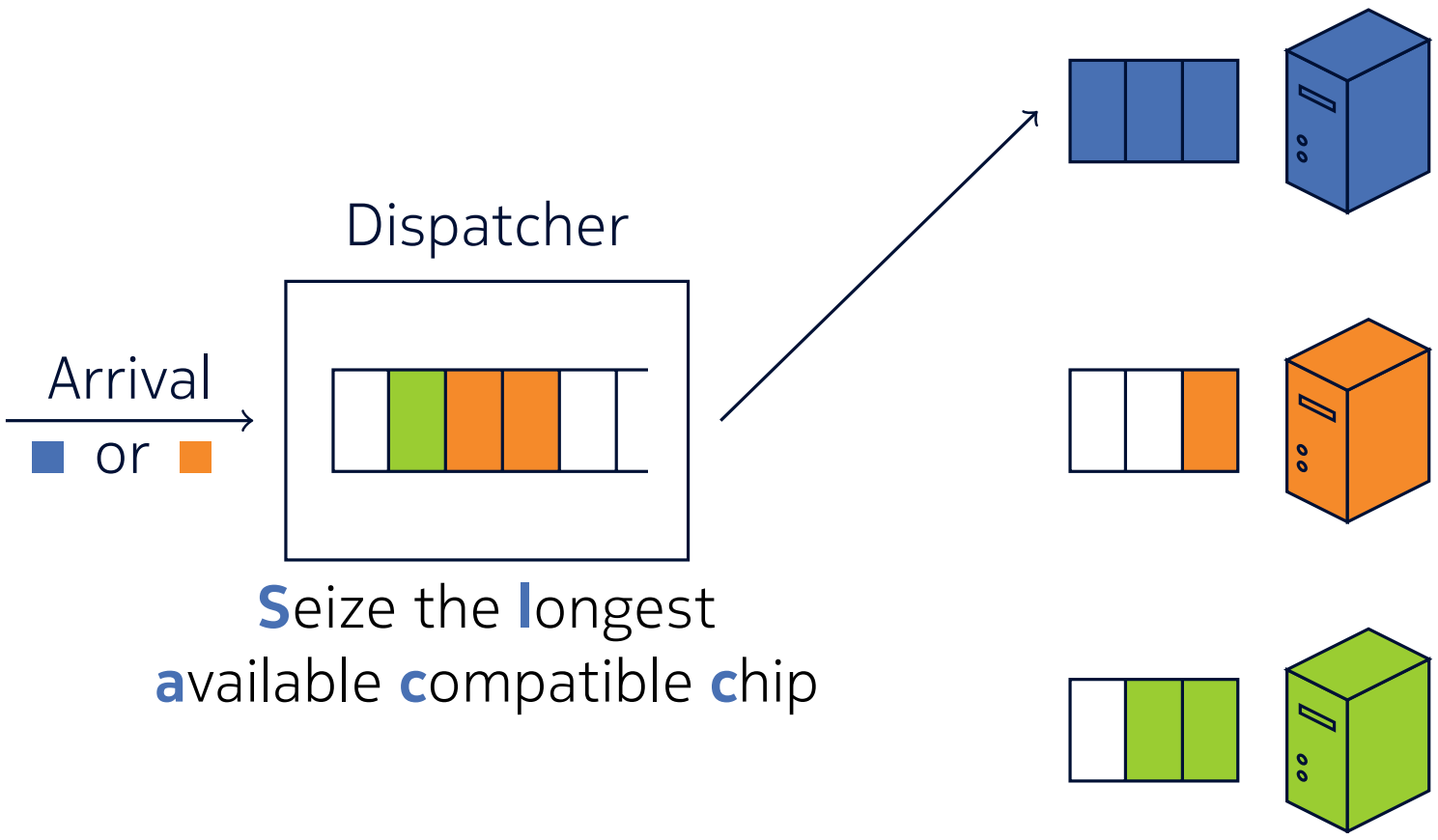

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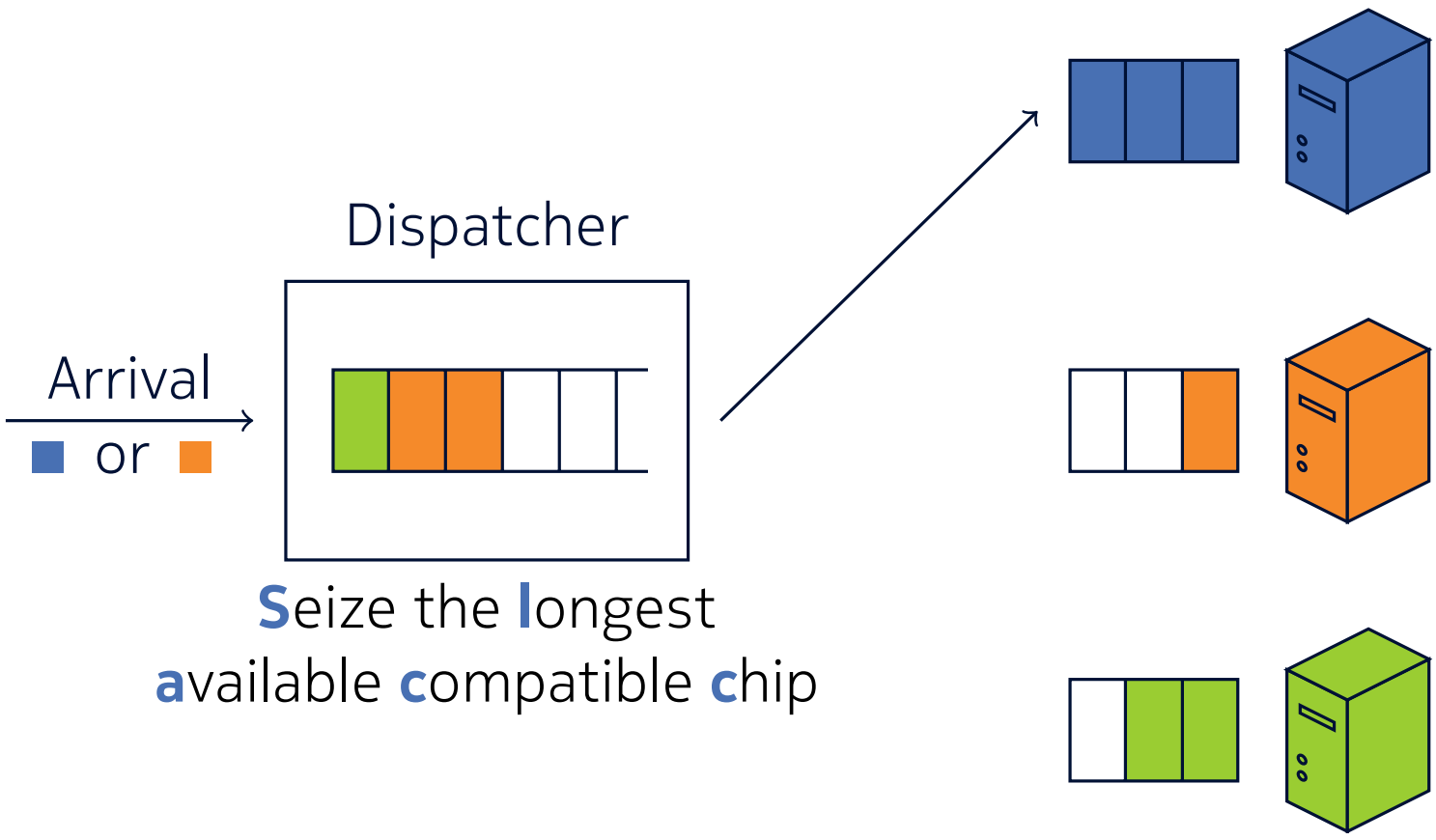

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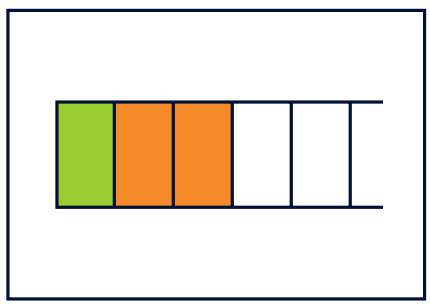
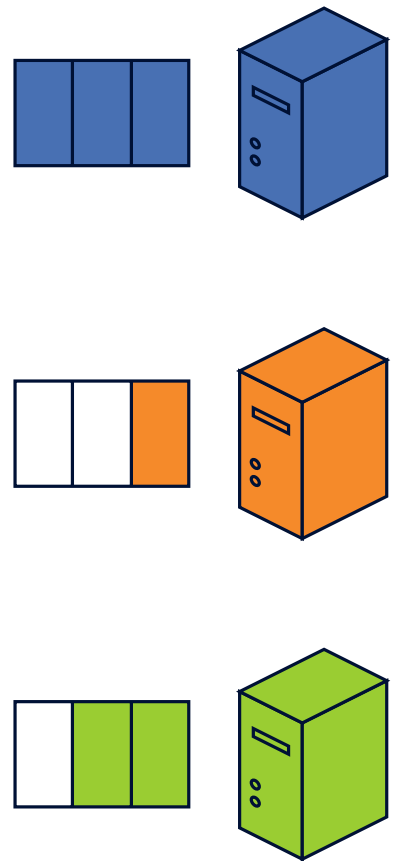


Seize the **l**ongest
available **c**ompatible **c**hip



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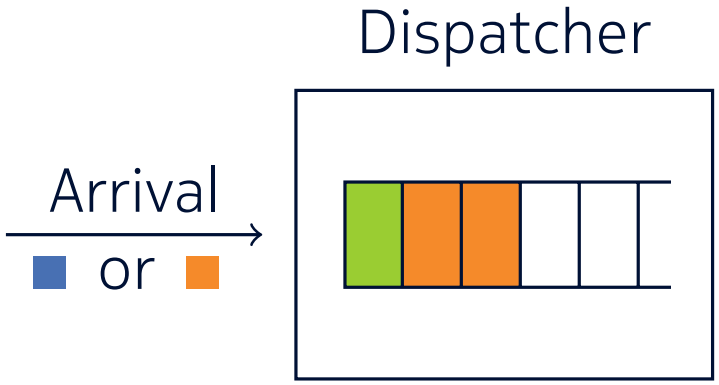

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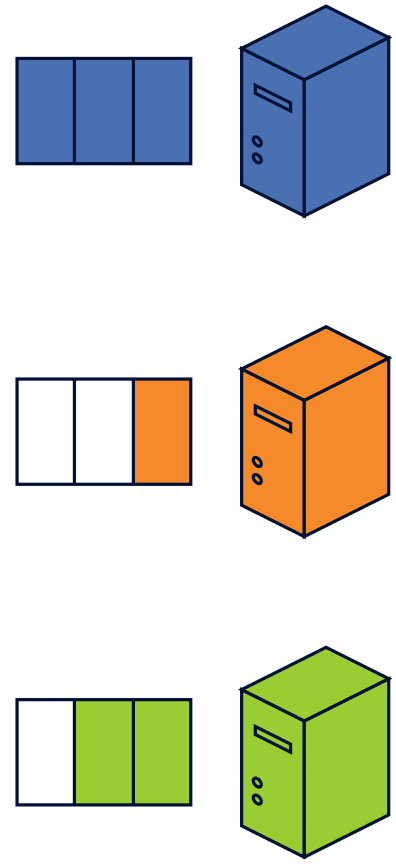

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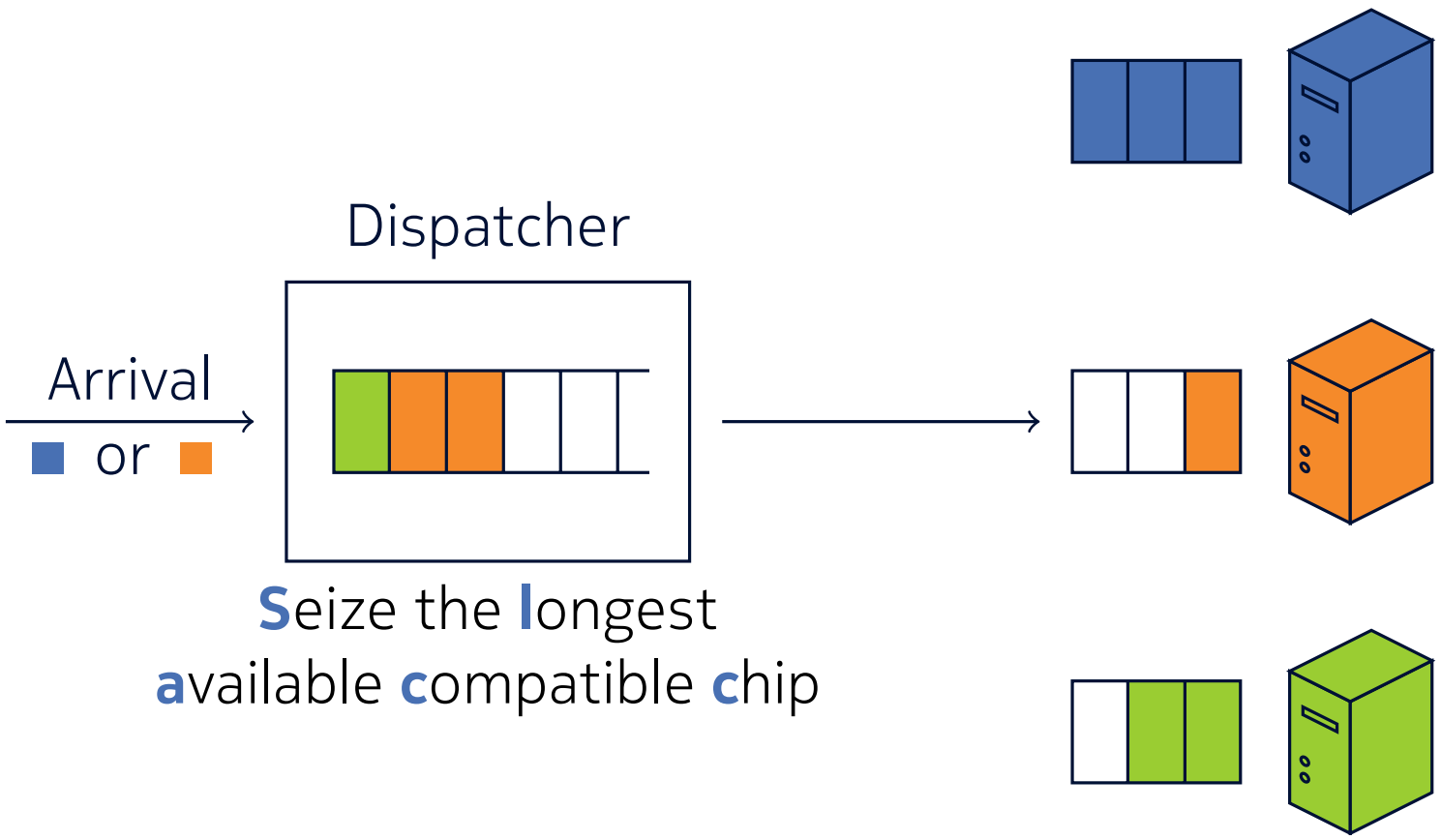

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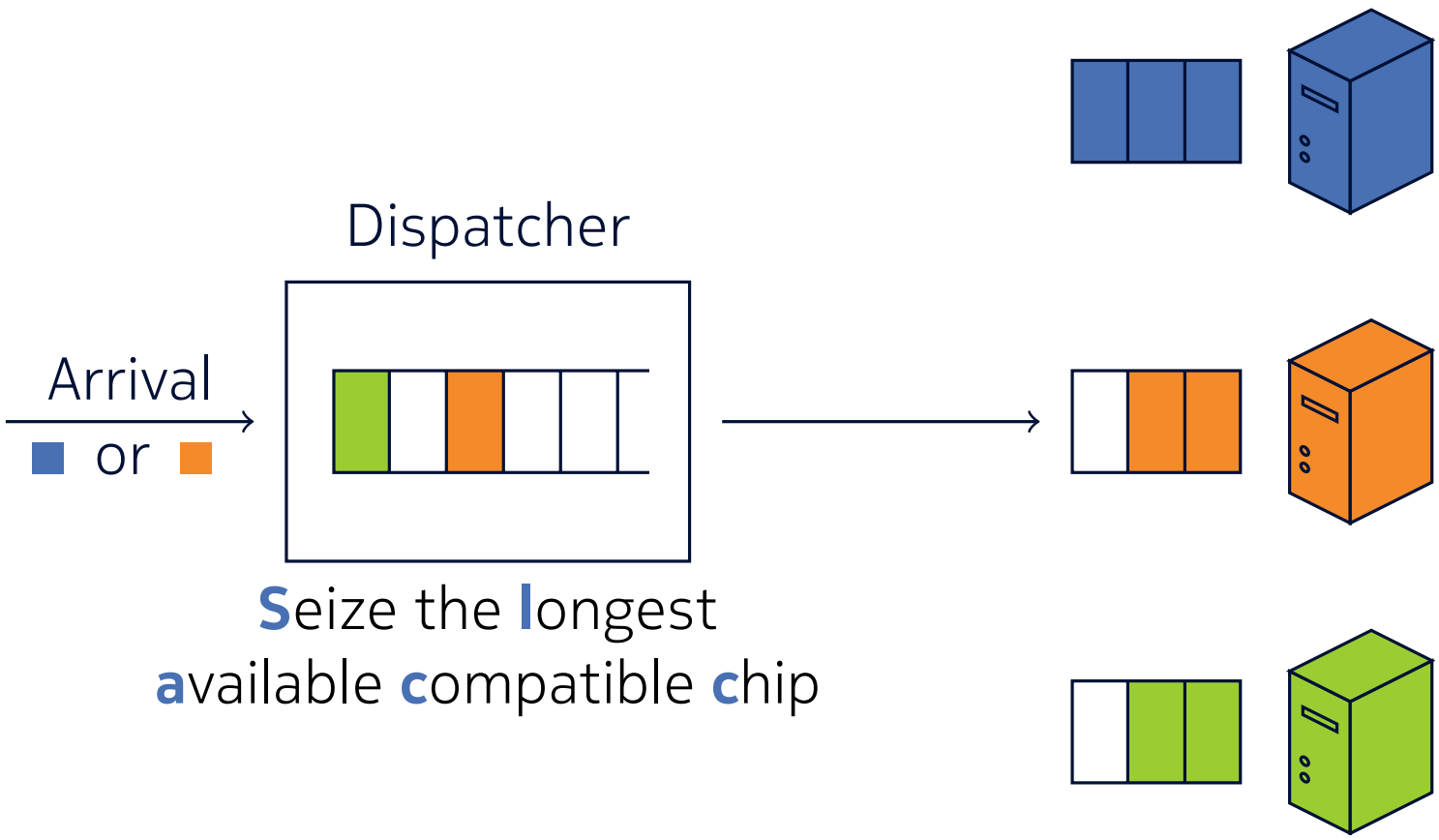

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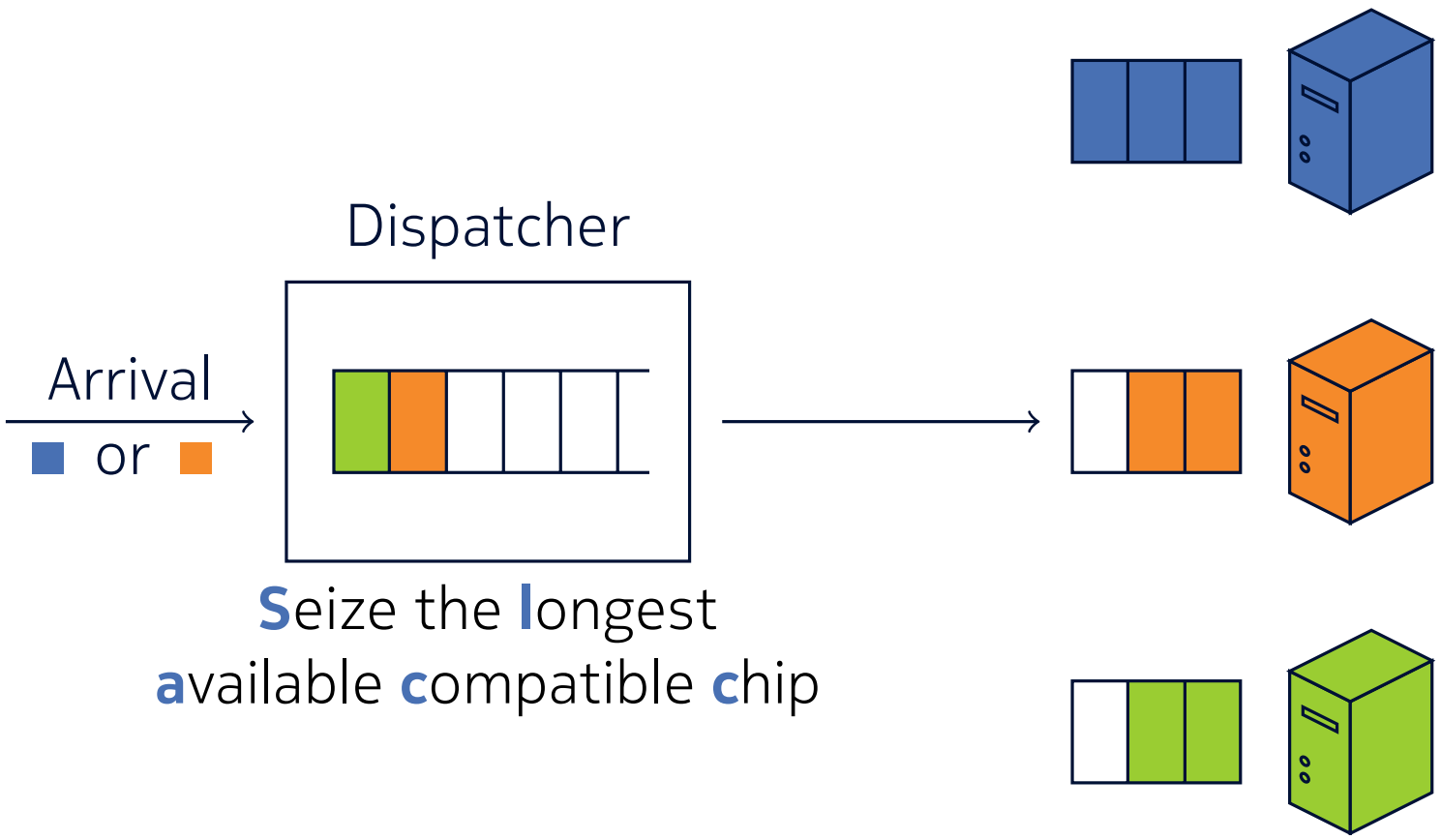

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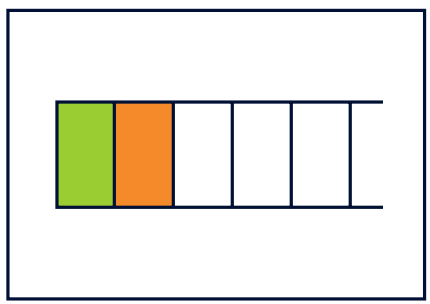

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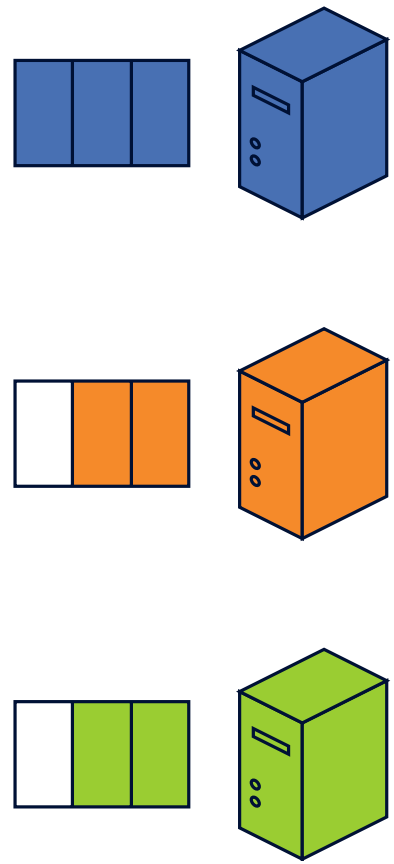

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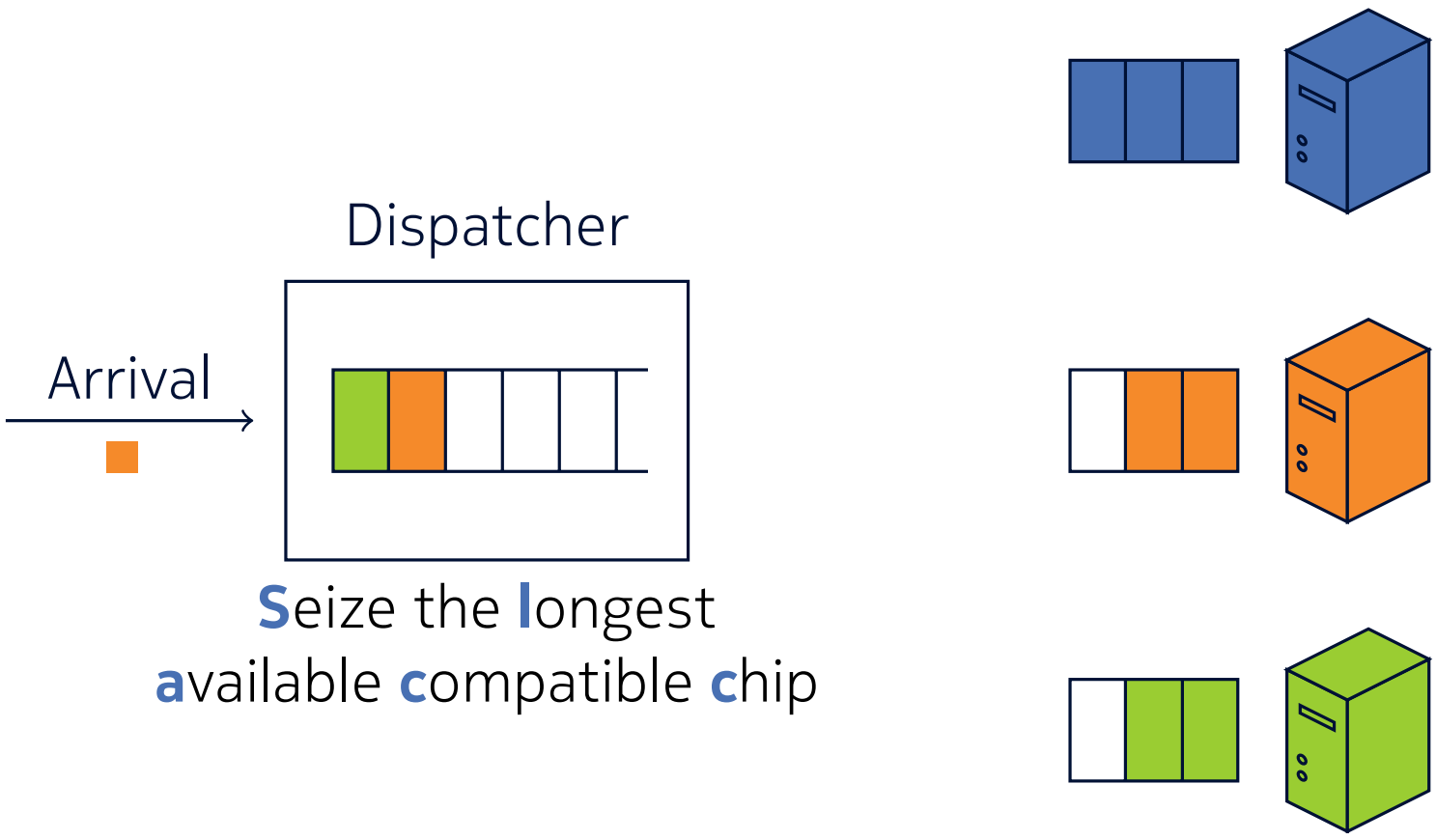

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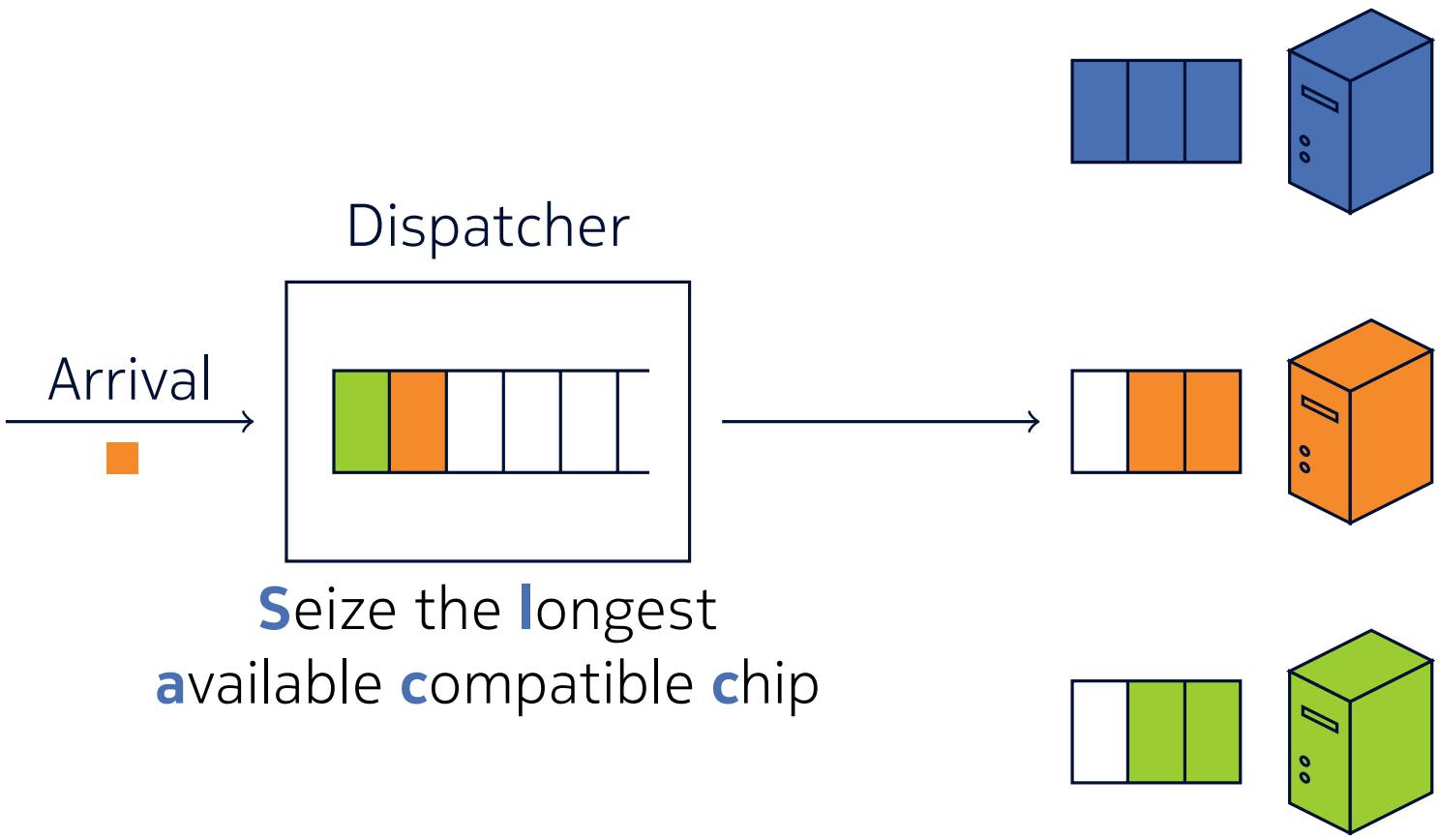

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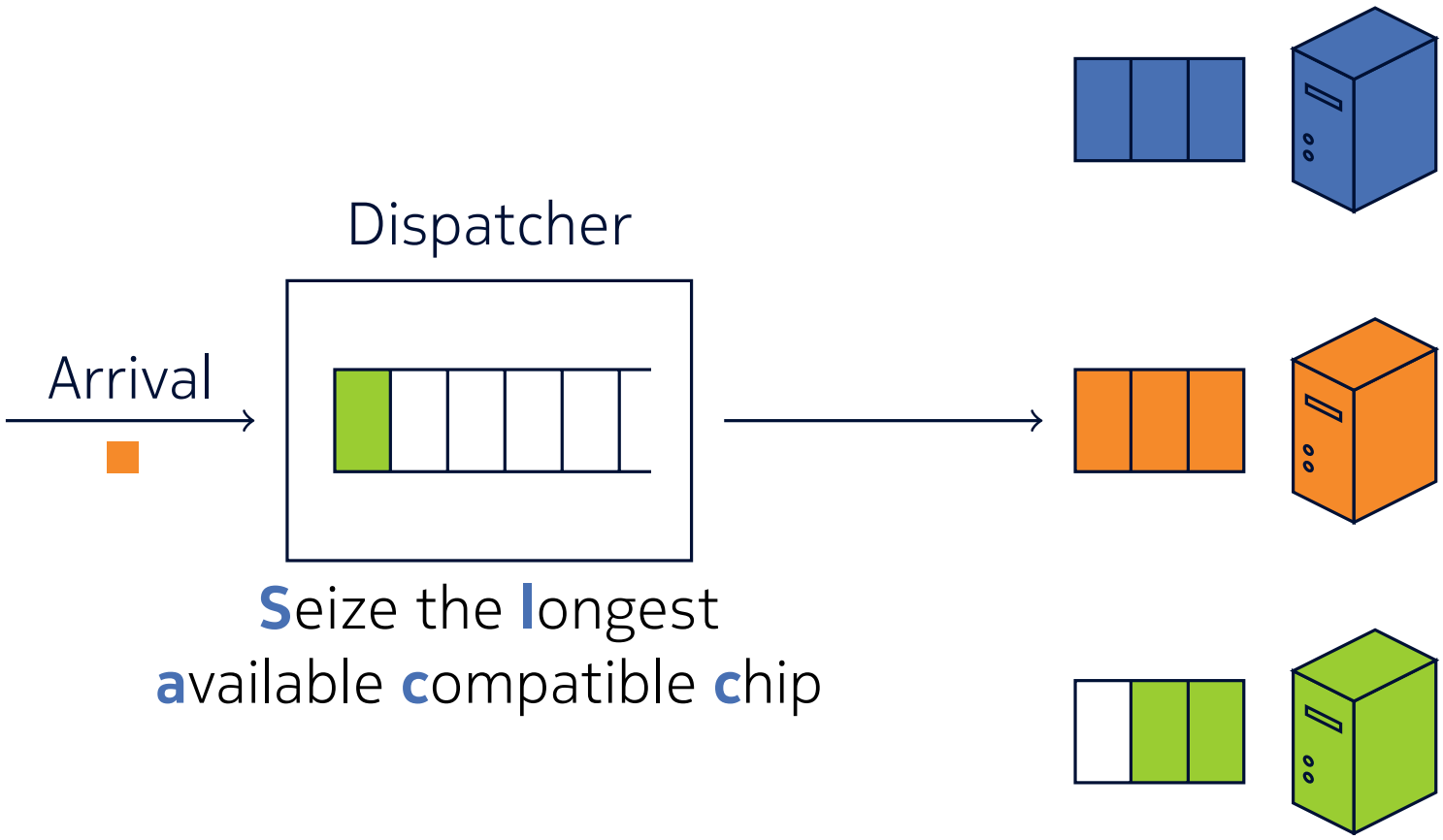

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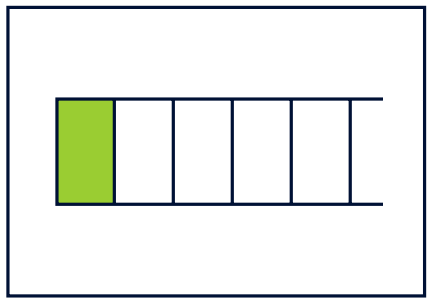

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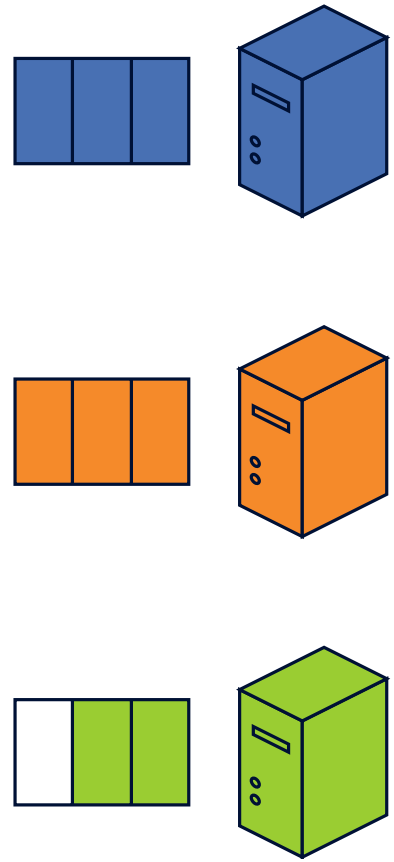

Intuition

Release order of the tokens ≃ Relative load of the servers

A token that has been available for a long time is likely to identify a server that is less loaded than others

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A token that has been available for a long time is likely to identify a server that is less loaded than others

→ Better to seize the longest available compatible token

Number of tokens ≃ Freshness of the information

→ When the number of tokens increases:

- ☺ Higher tolerance to momentary mistakes
- ☹ The information at the head of the queue is “less fresh”

Insensitivity to the job size distribution

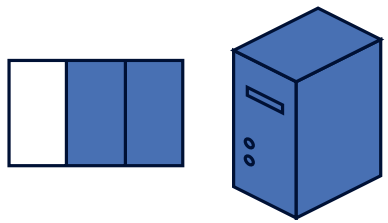
Under a Poisson arrival process

Insensitivity to the job size distribution

Under a Poisson arrival process

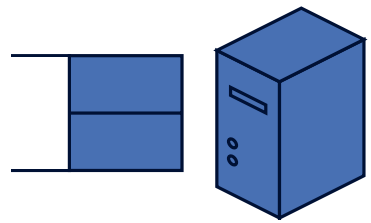
A single server:

First-come, first-served



Sensitive

Processor-sharing



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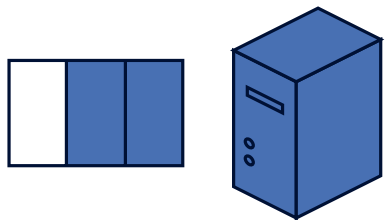
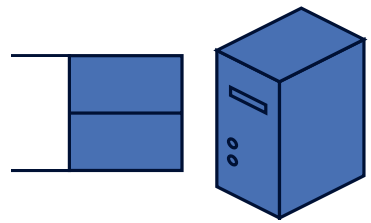


Sensitive

Round-robin

Processor-sharing



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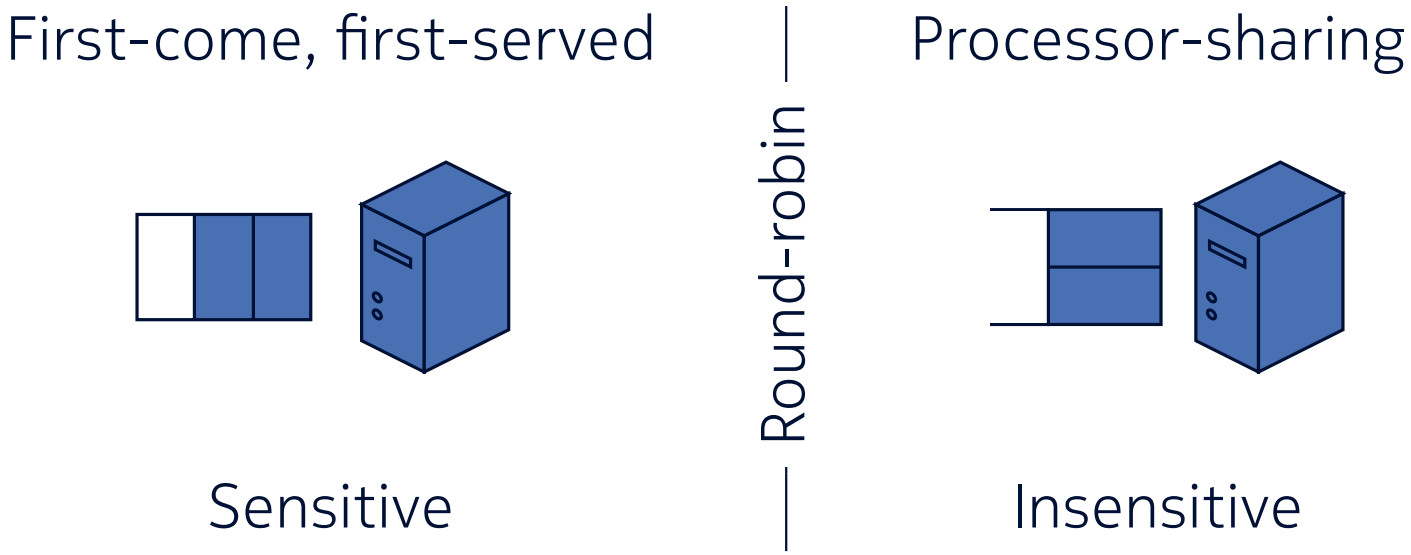


Many servers: Our queueing model shows that the performance is insensitive if each server applies processor-sharing policy.

Outline

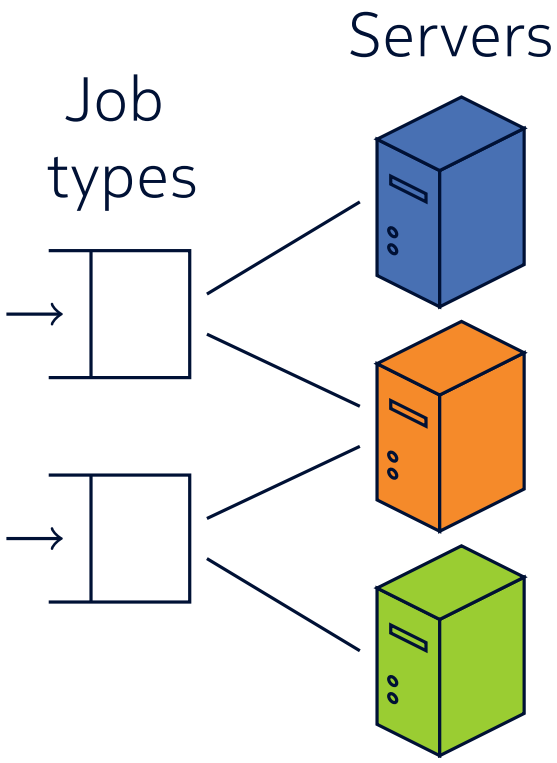
Algorithm

Queueing analysis

Numerical results

Extensions

Server pool model



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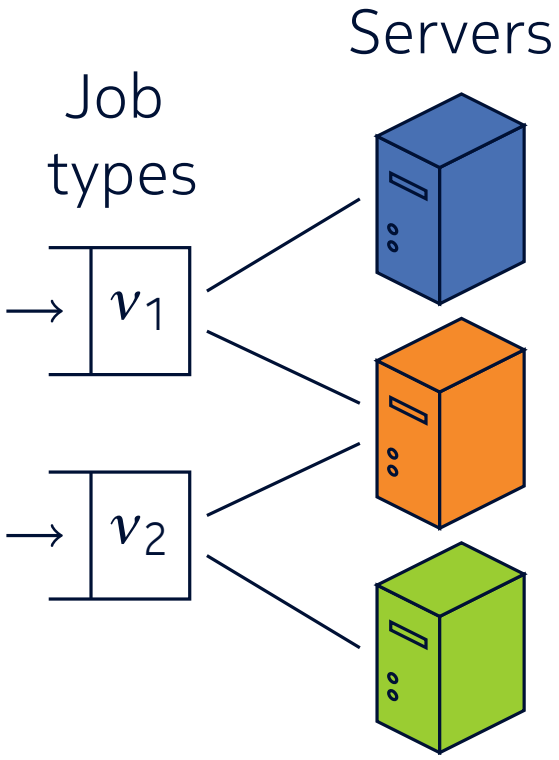
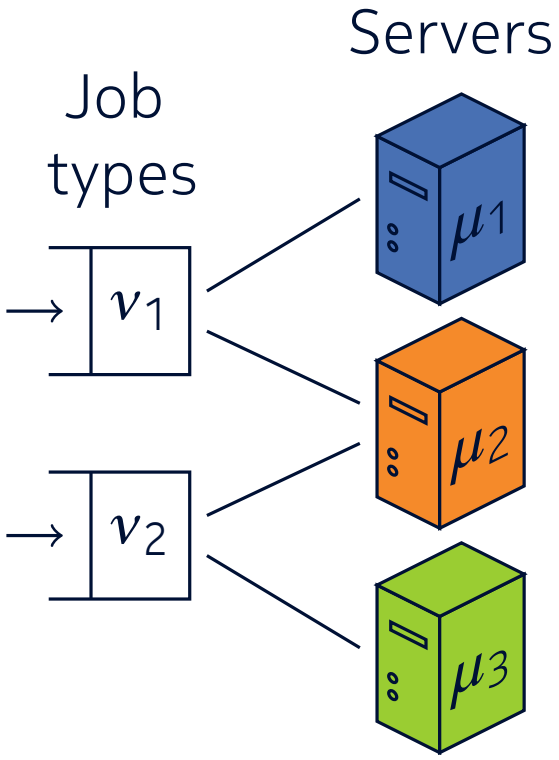


Arrivals:

- Type-k jobs arrive at rate ν_k
- Poisson arrival process

Server pool model



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- Server s has capacity μ_s
- Exponentially distributed job sizes with unit mean

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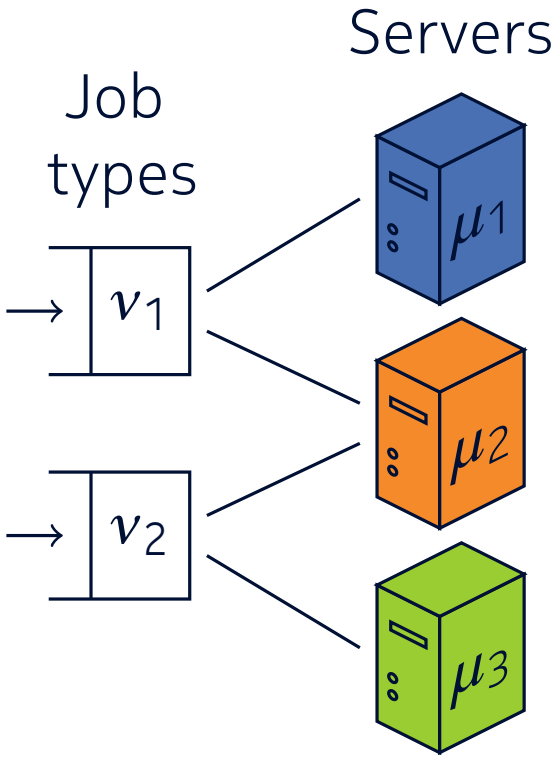

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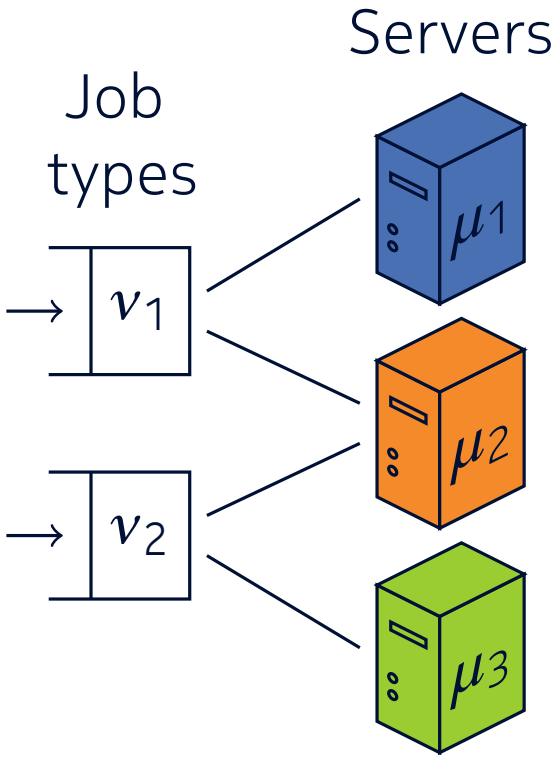

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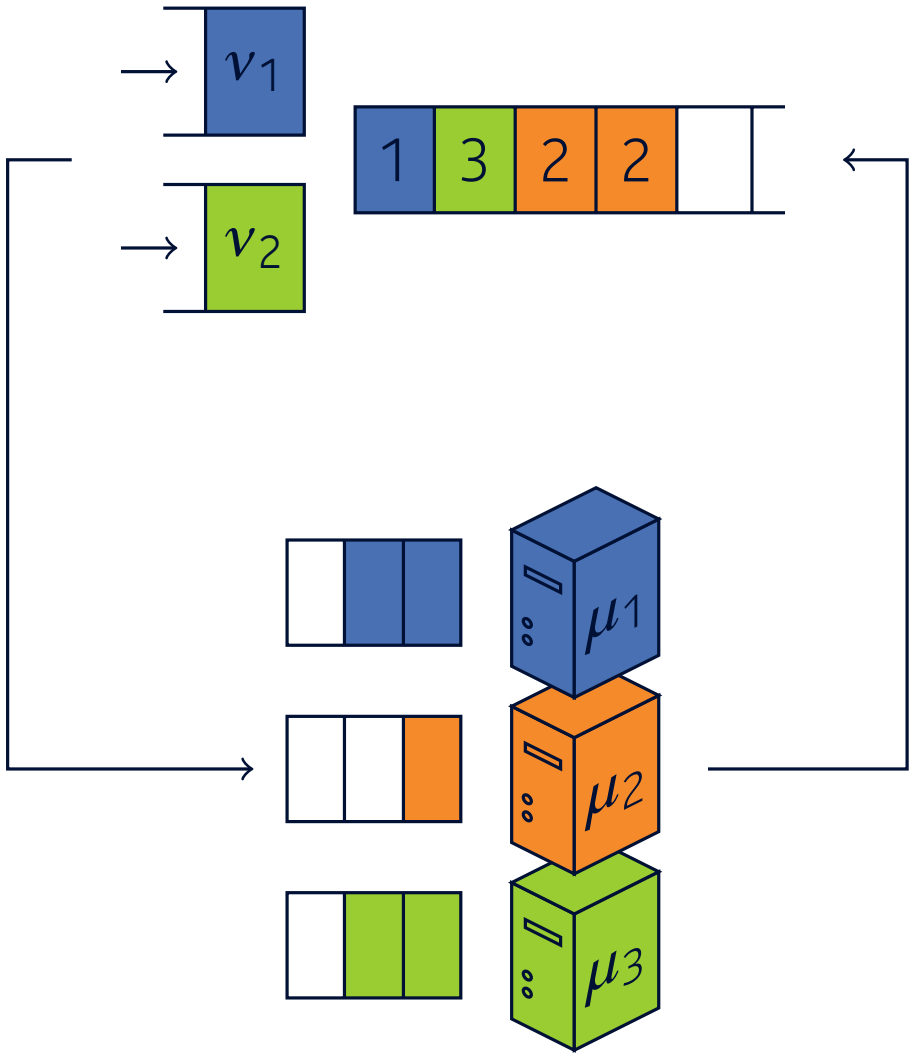
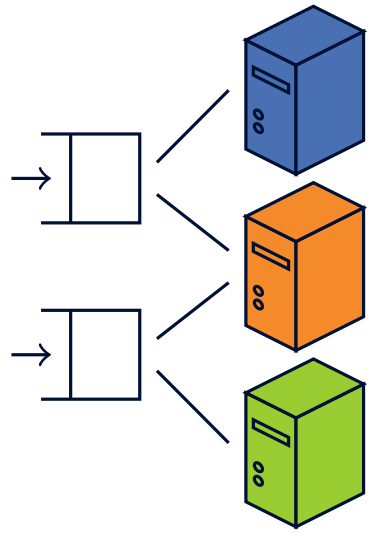
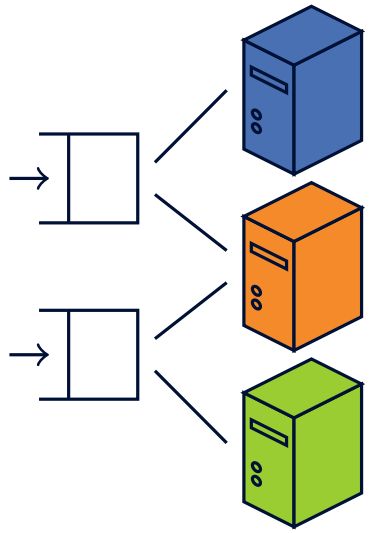
Remark: We only need to know the graph to analyze the algorithm

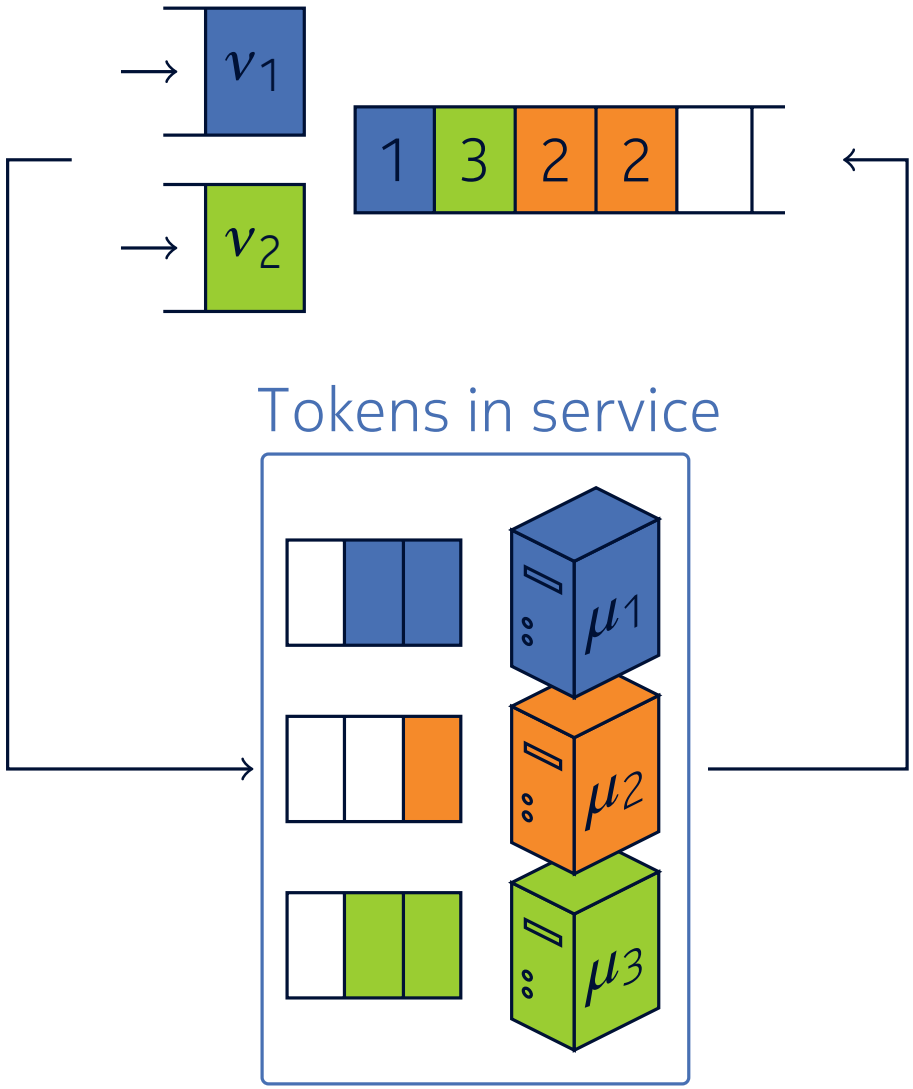
Queueing model
Describe the token state





Queueing model
Describe the token state





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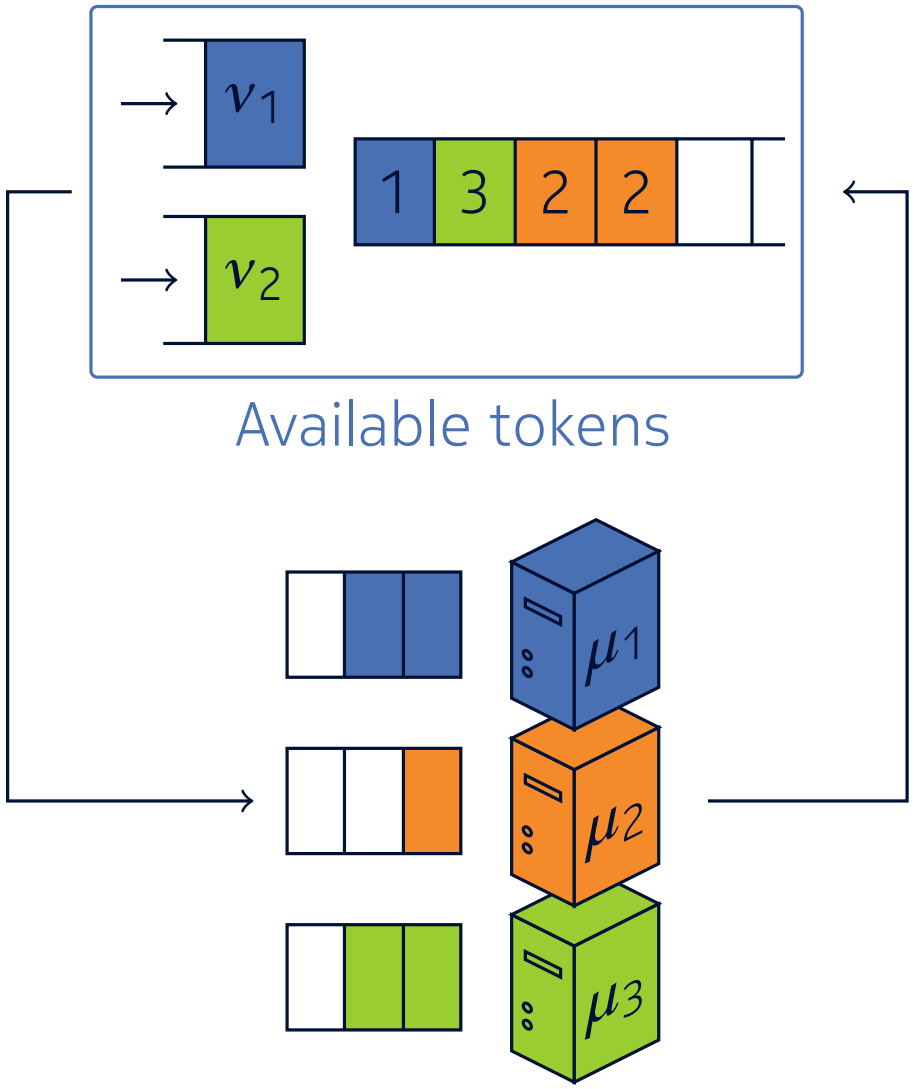

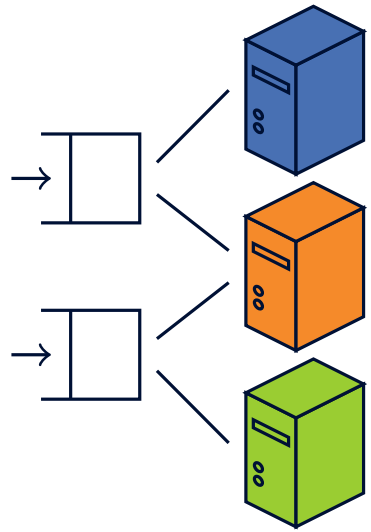

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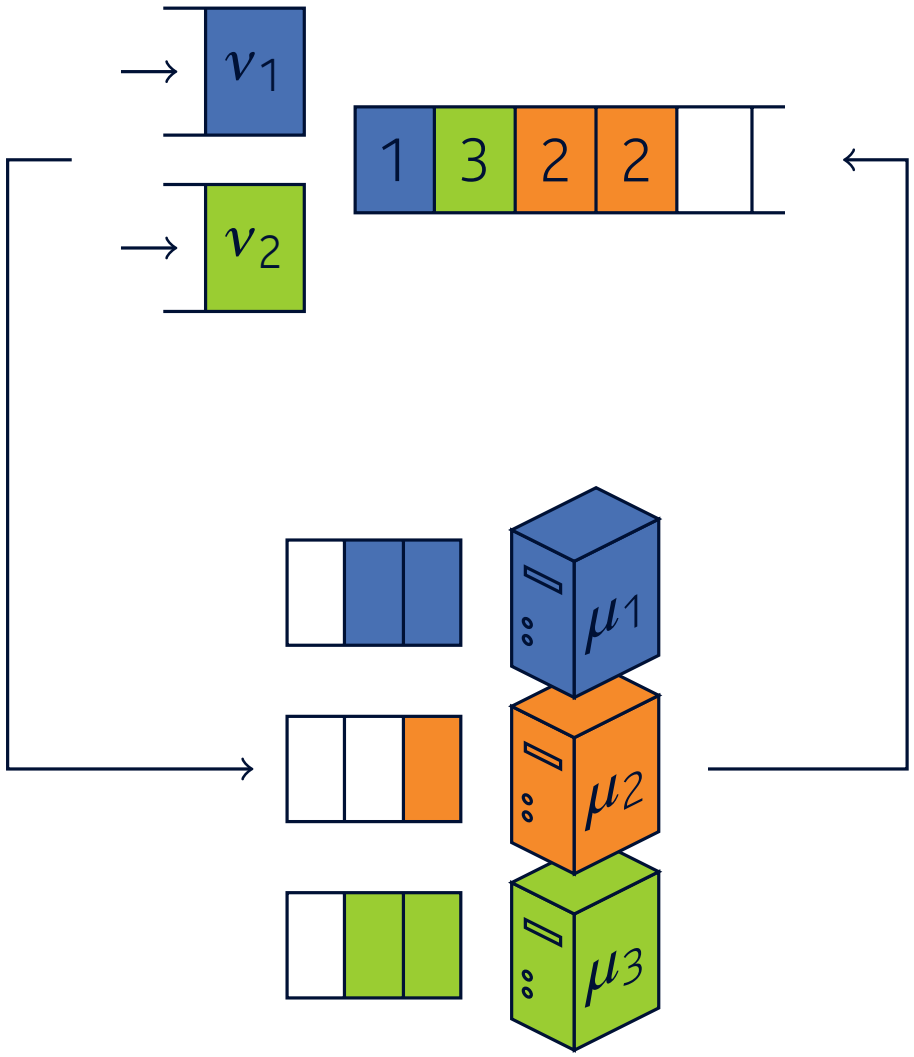

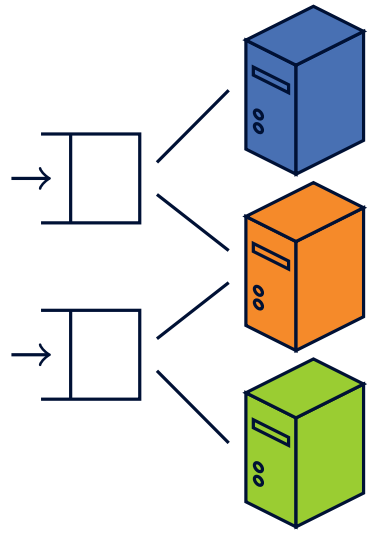

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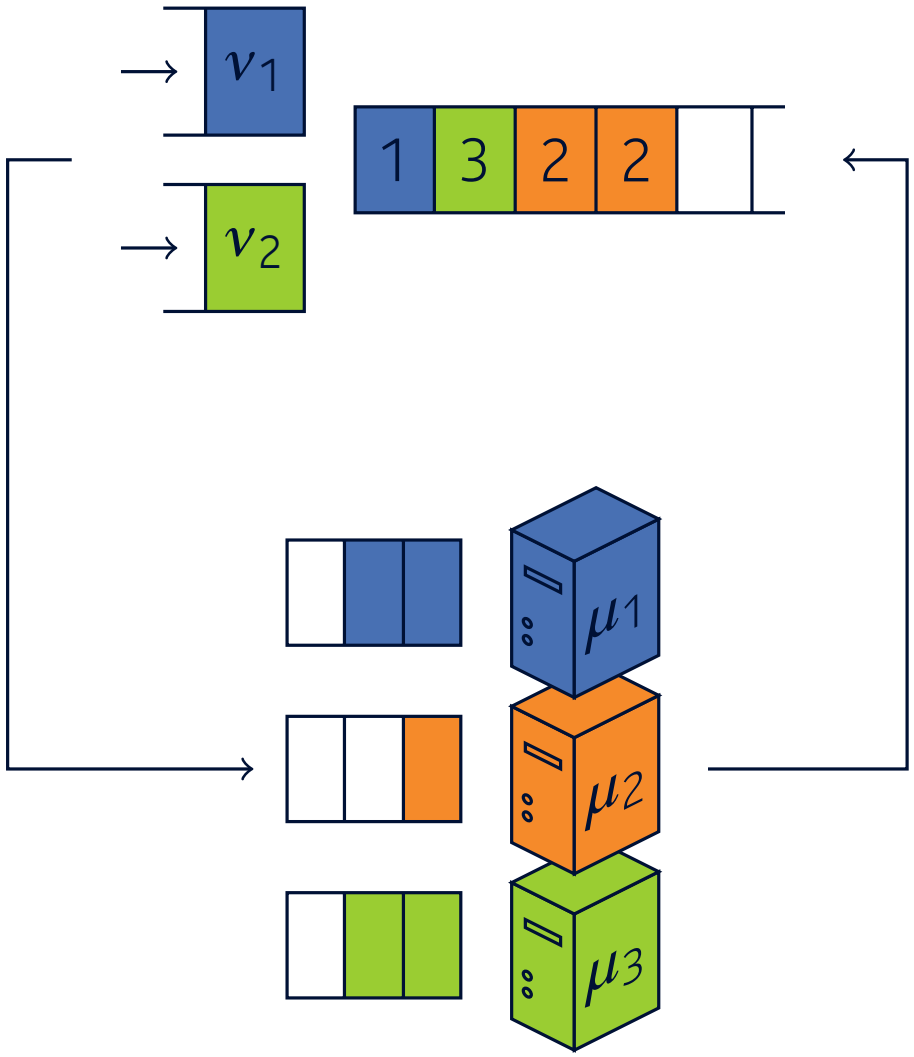

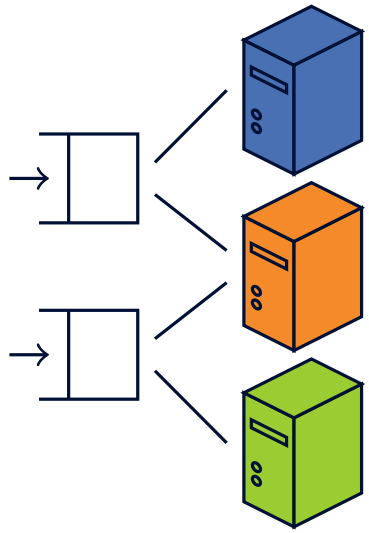

The job types play the part of the servers in the queue of available tokens

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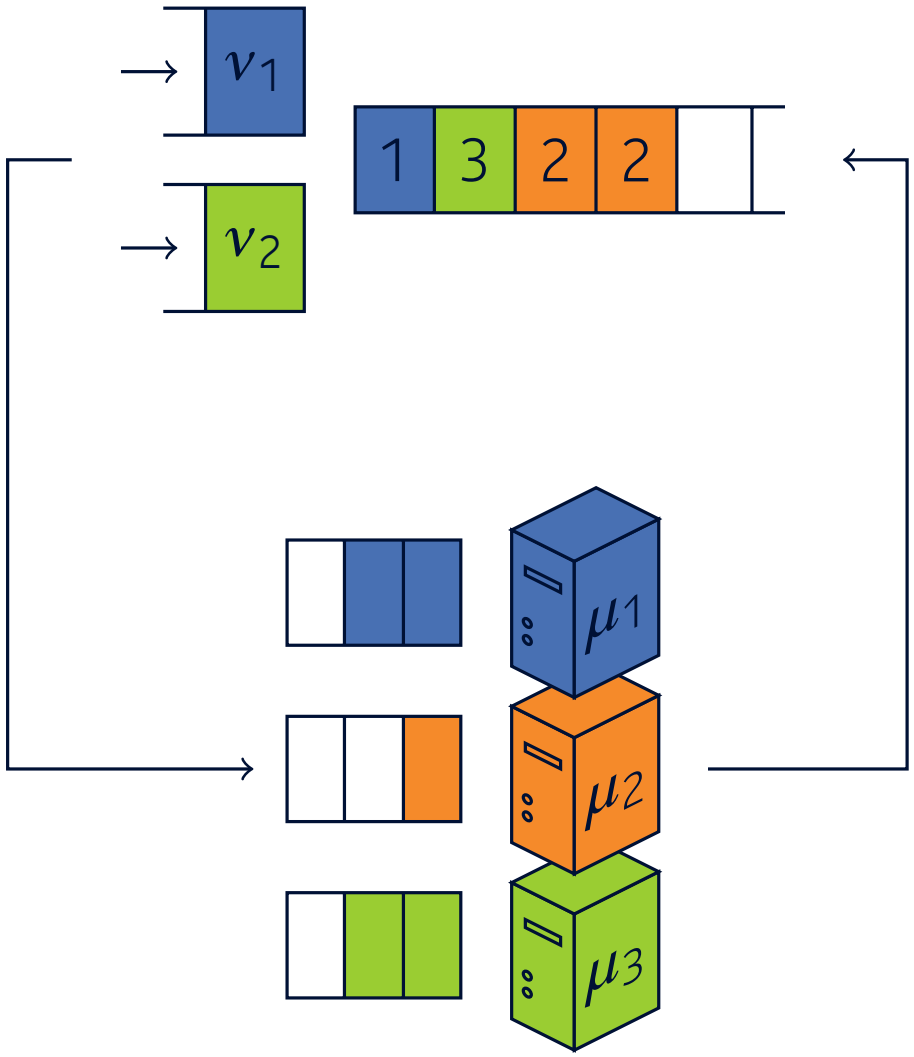

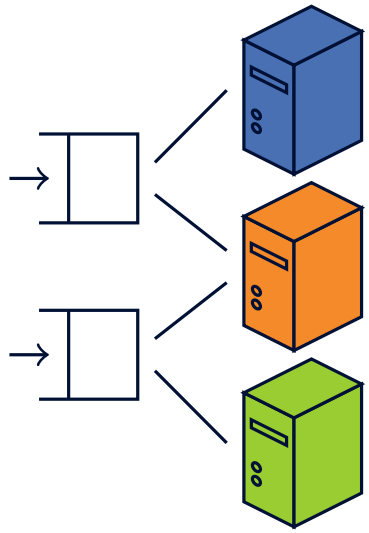

The job types play the part of the servers in the queue of available tokens

Network state:
The sequence of available tokens
t = (■, ■, ■, ■)

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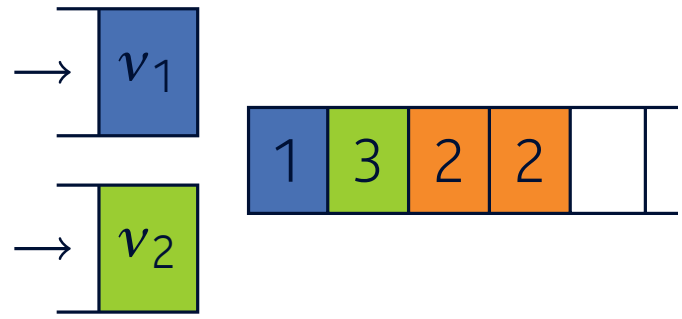
Order-independent (OI) queues

(Berezner, Kriel, and Krzesinski, 1995)



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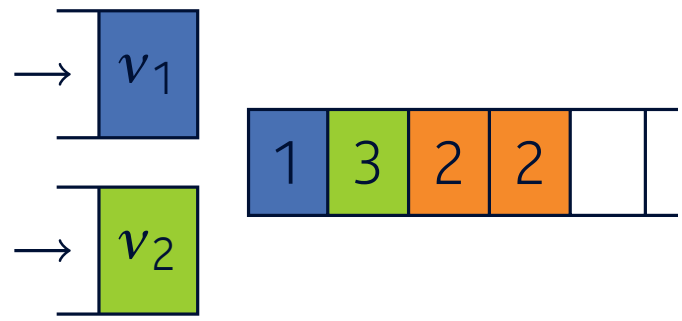

Definition:

- The total service rate only depends on the **number** of tokens of each server in the queue
- The service rate received by a token doesn't depend on the subsequent tokens

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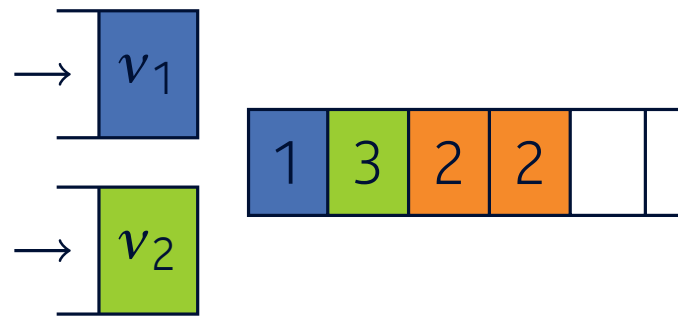

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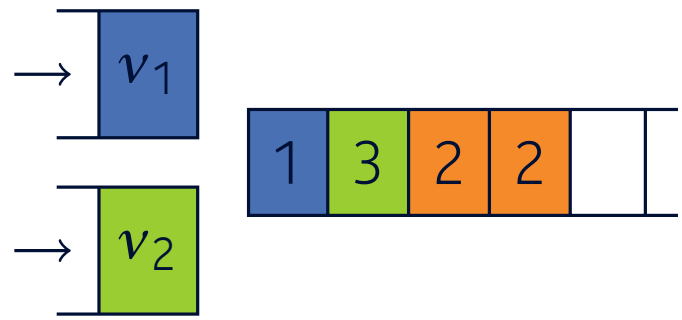

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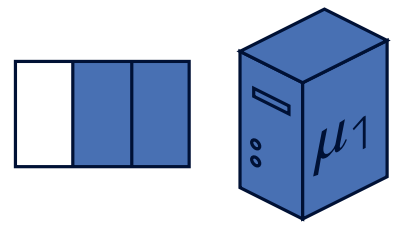

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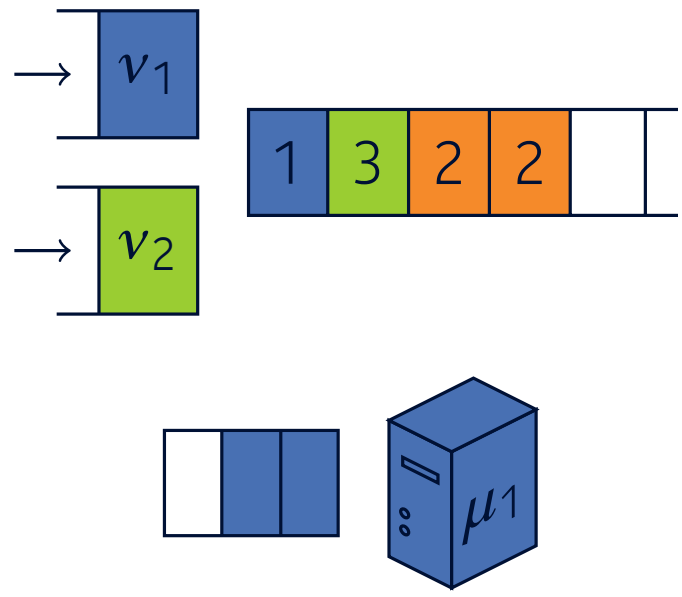

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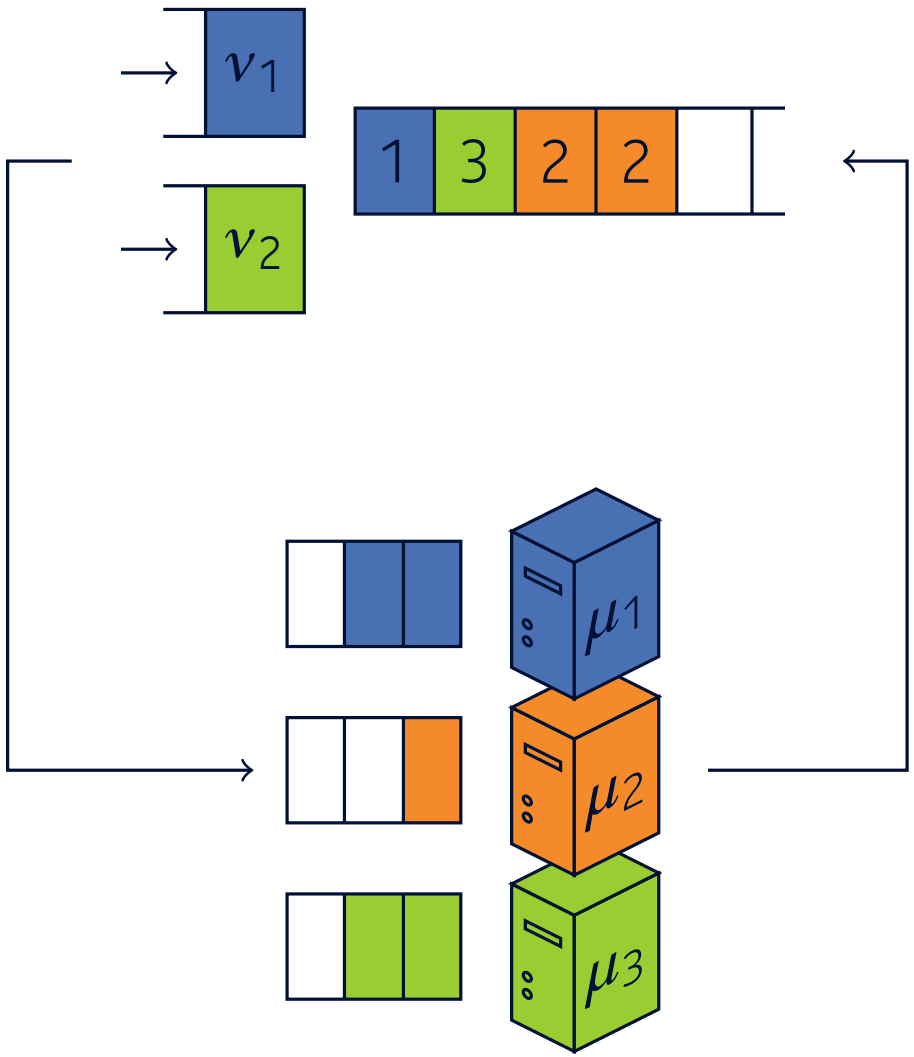
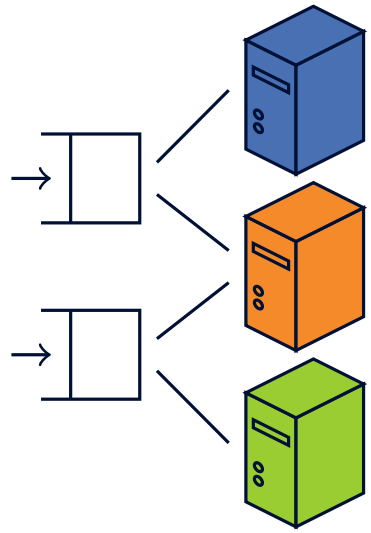
Quasi-reversibility:

- A network of OI queues has a product-form
- The stationary distribution of the network is unchanged by the addition of Markov routing

State aggregation

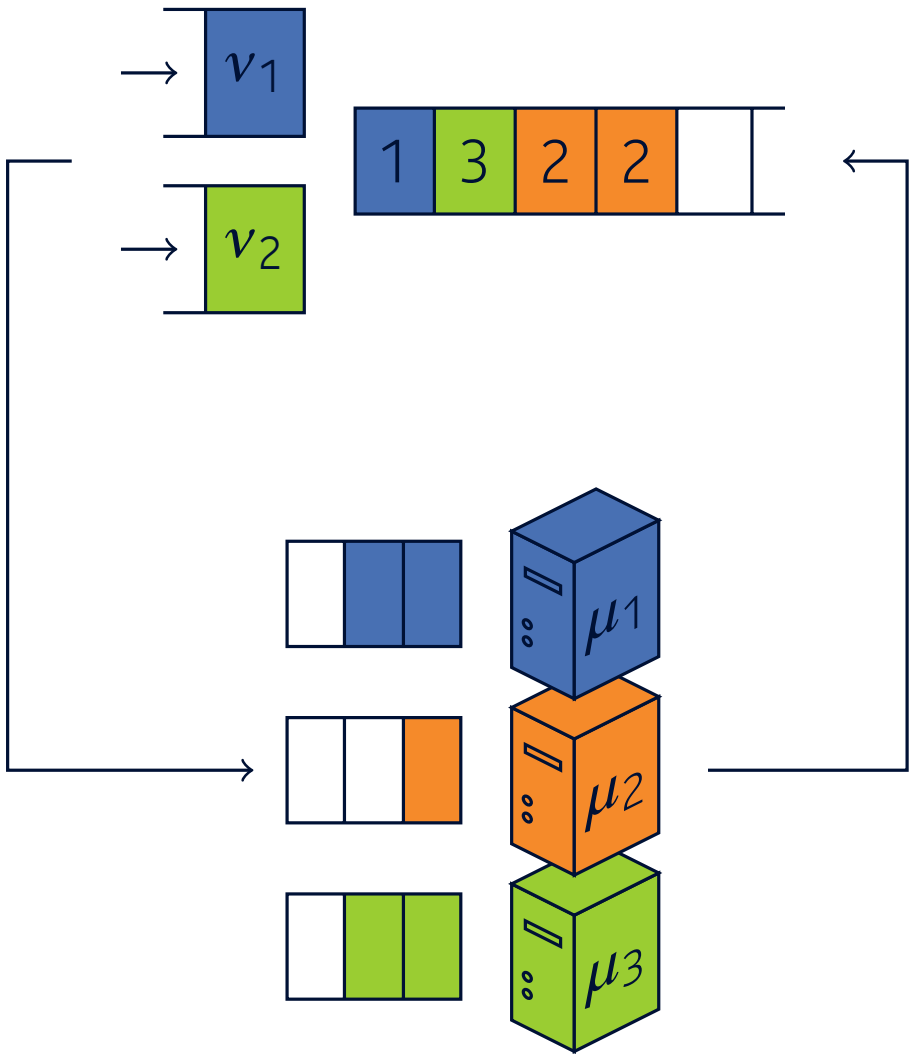


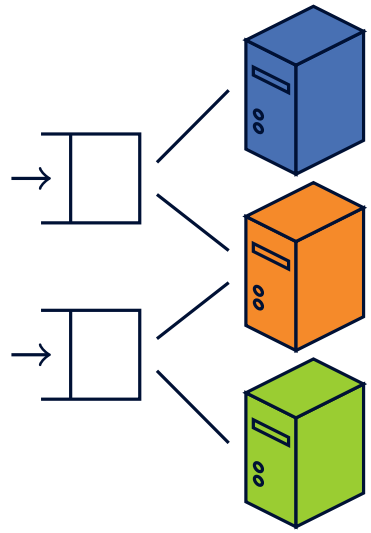


Detailed state:

t = (■, ■, ■, ■)

State aggregation





Detailed state:

t = (■, ■, ■, ■)

Aggregate state:

The number of available tokens

$$y = \begin{pmatrix} 1\blacksquare \\ 2\blacksquare \\ 1\blacksquare \end{pmatrix}$$

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State aggregation

Equivalent Whittle network:
The stationary distribution of the aggregate state is that of a Whittle network (Whittle, 1985)

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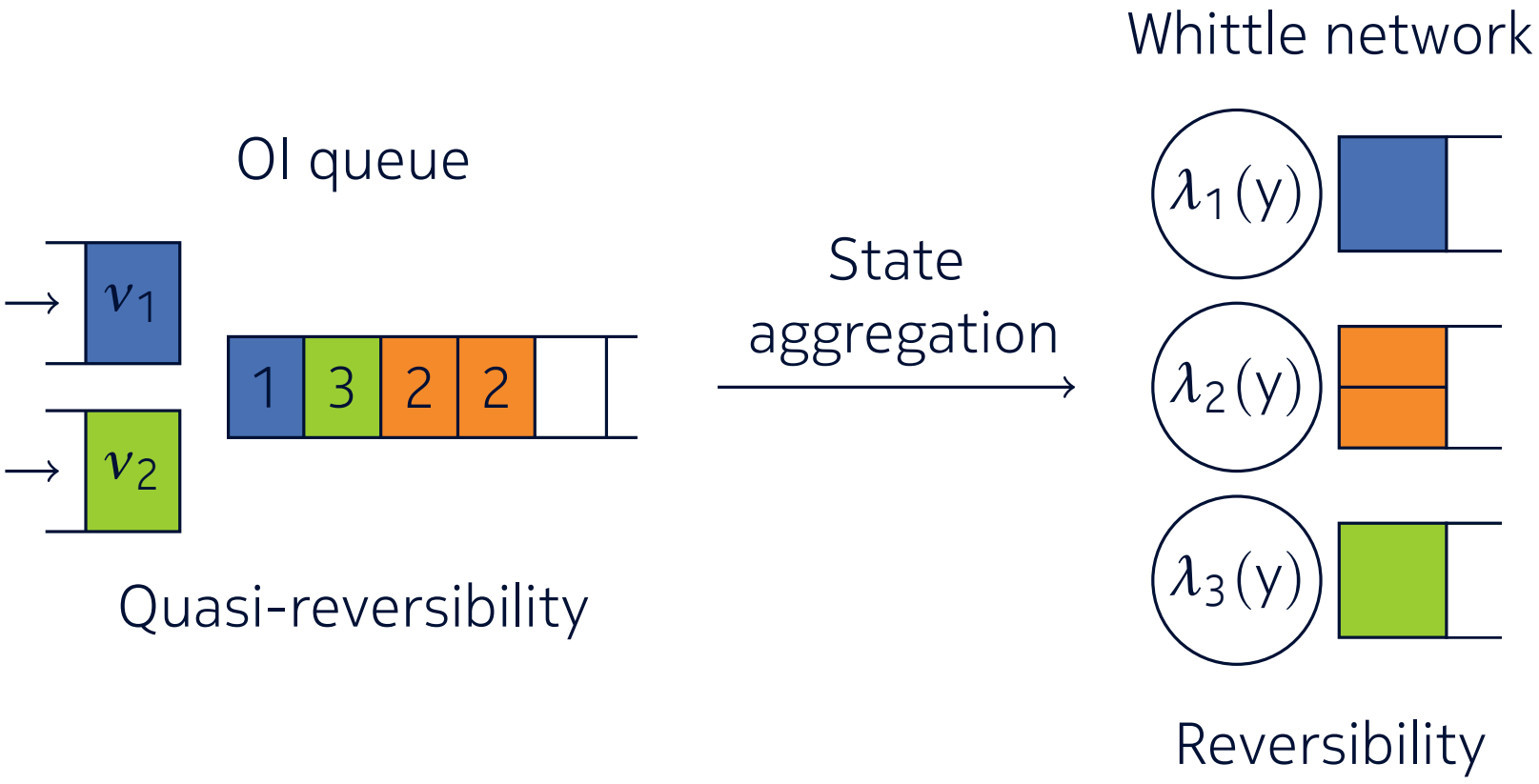

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Performance metrics: Explicit formulas
$O(\ell_1 \times \ell_2 \times \ldots \times \ell_S)$ terms to compute

Outline

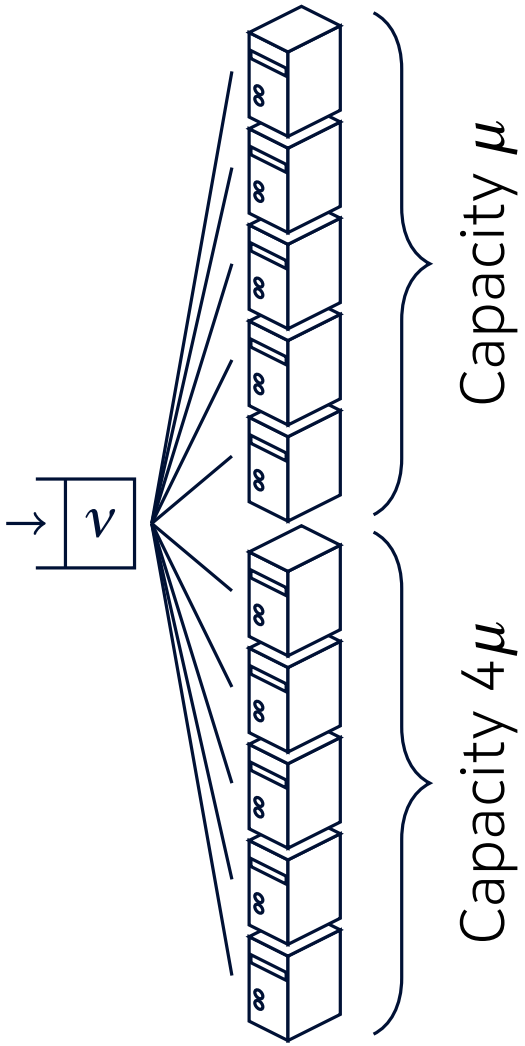
Algorithm

Queueing analysis

Numerical results

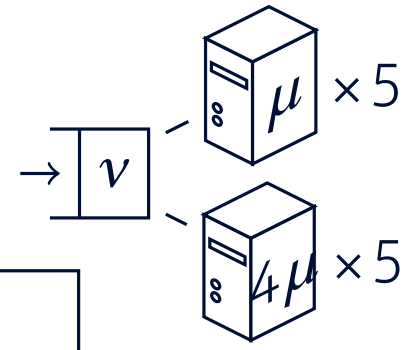
Extensions

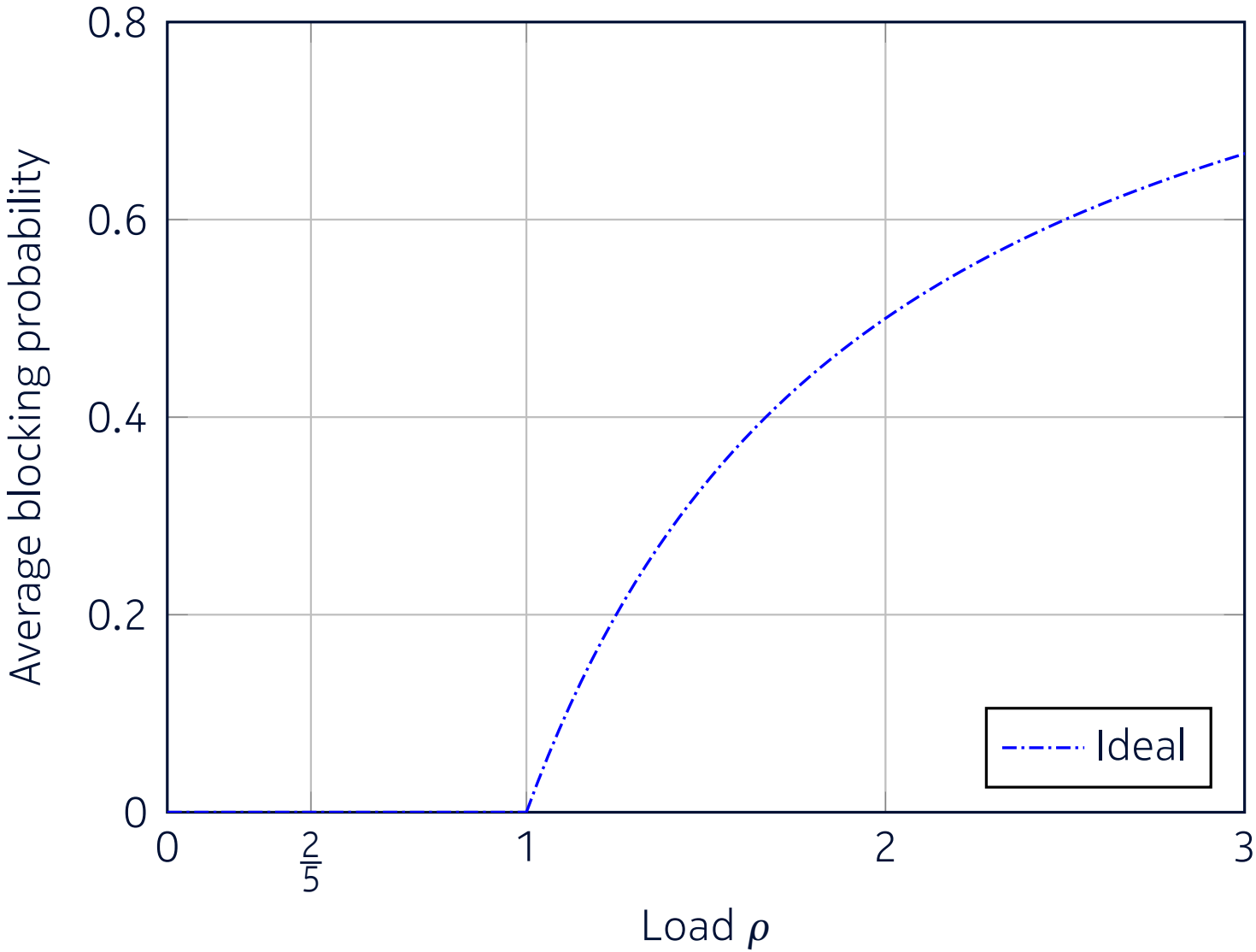
Servers with different service rates



- No compatibility constraints:
  All jobs can be assigned to all servers
- Poisson arrival process with rate ν
- Half of the servers have capacity μ,
  the other have capacity 4μ
- Each server has 6 tokens
- Load $\rho = \frac{\nu}{25\mu}$

Servers with different service rates





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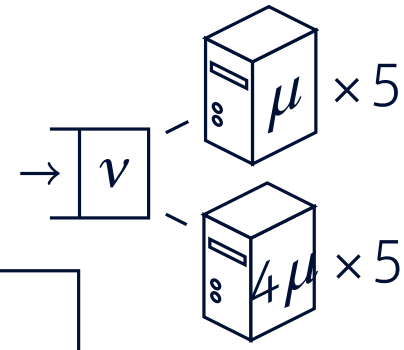

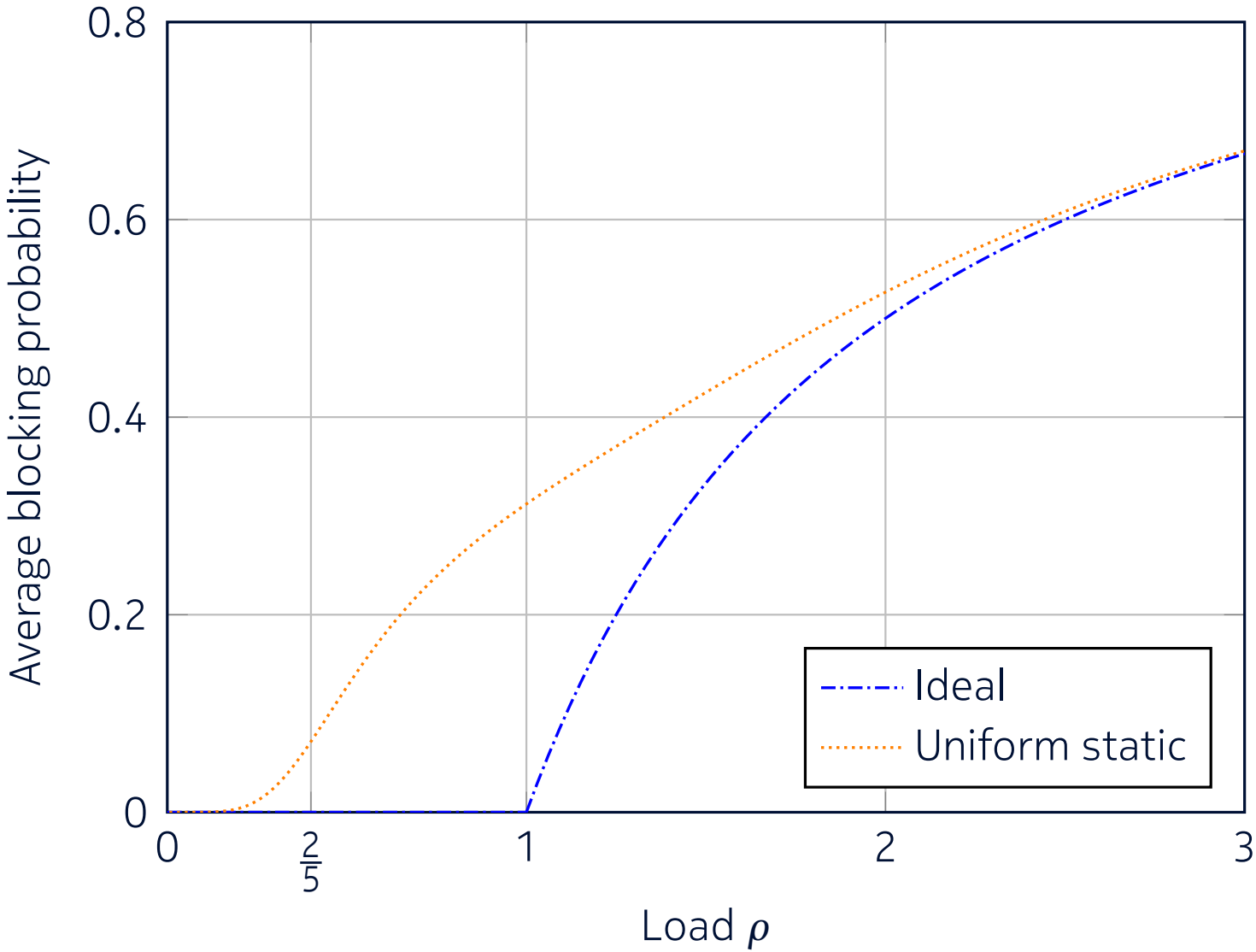

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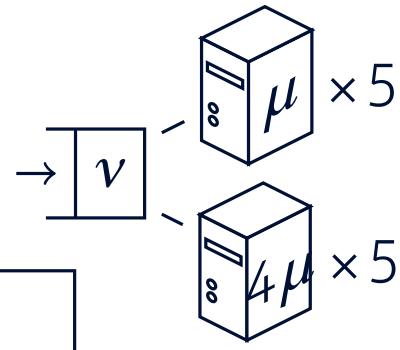

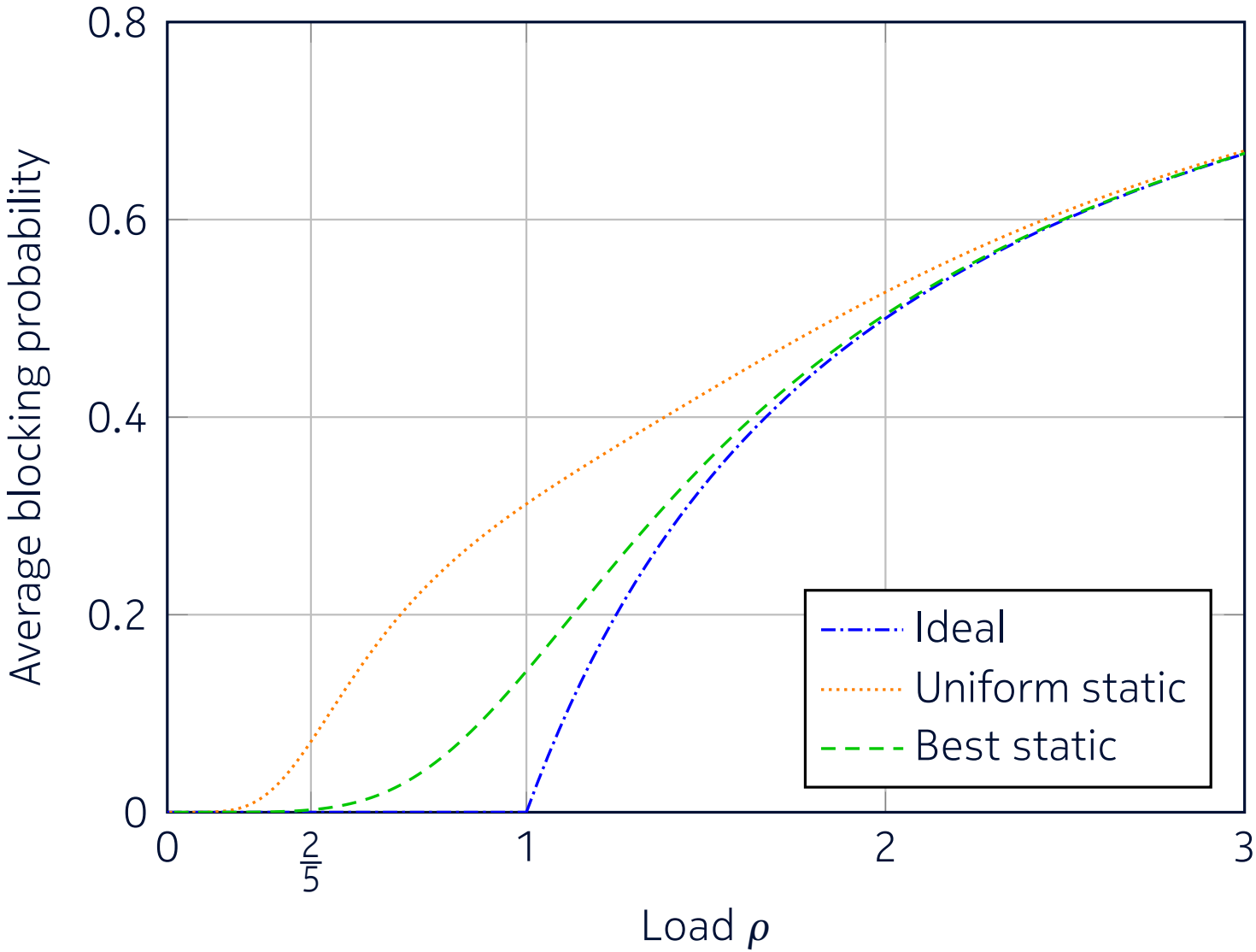

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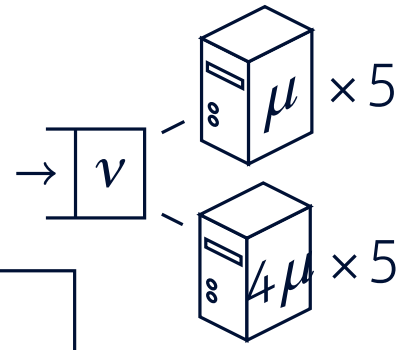

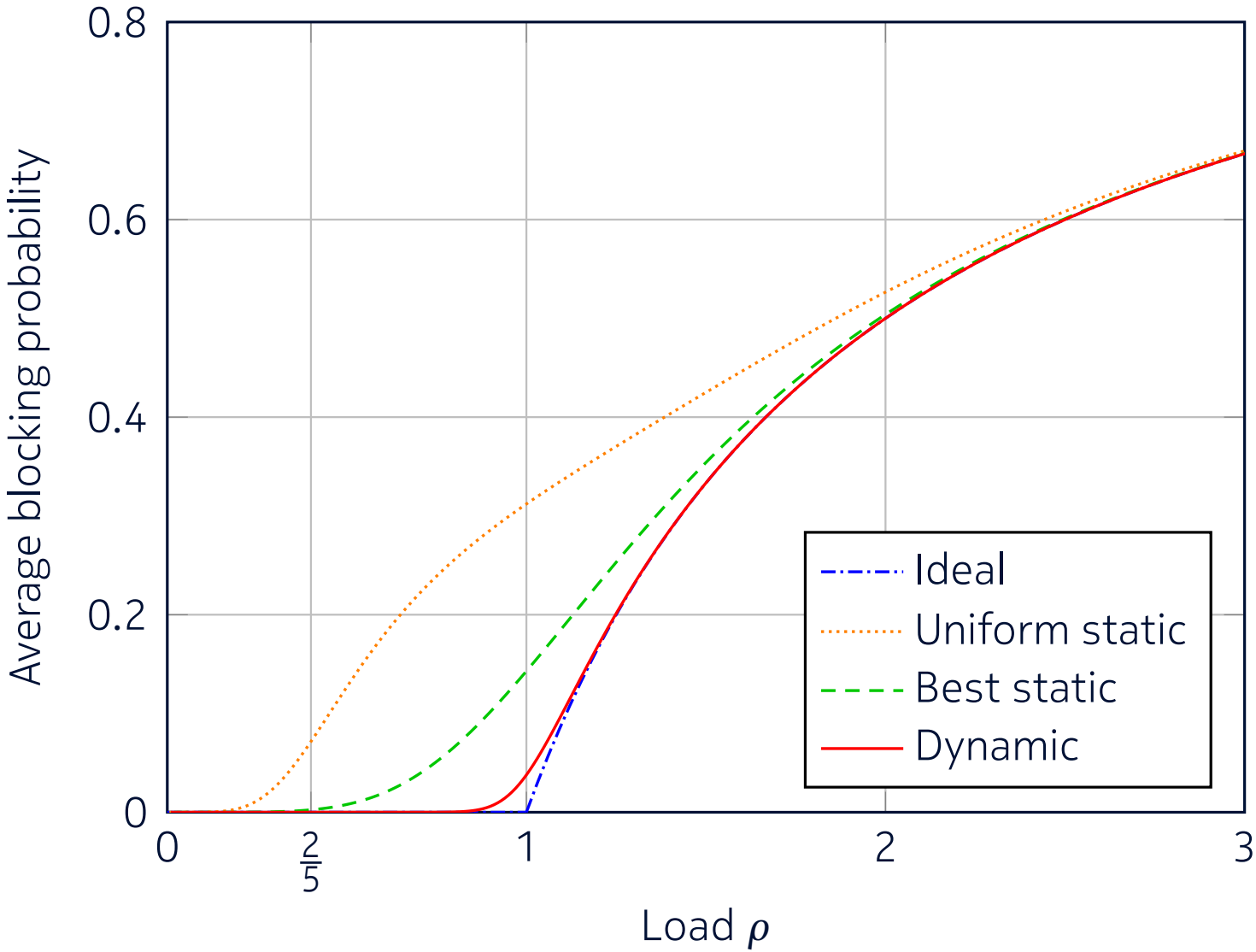

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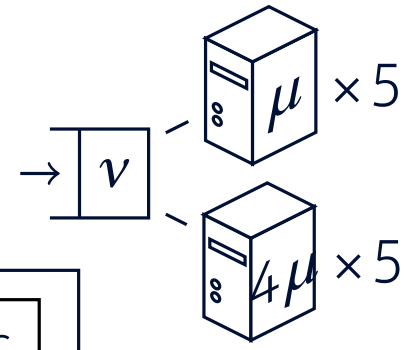

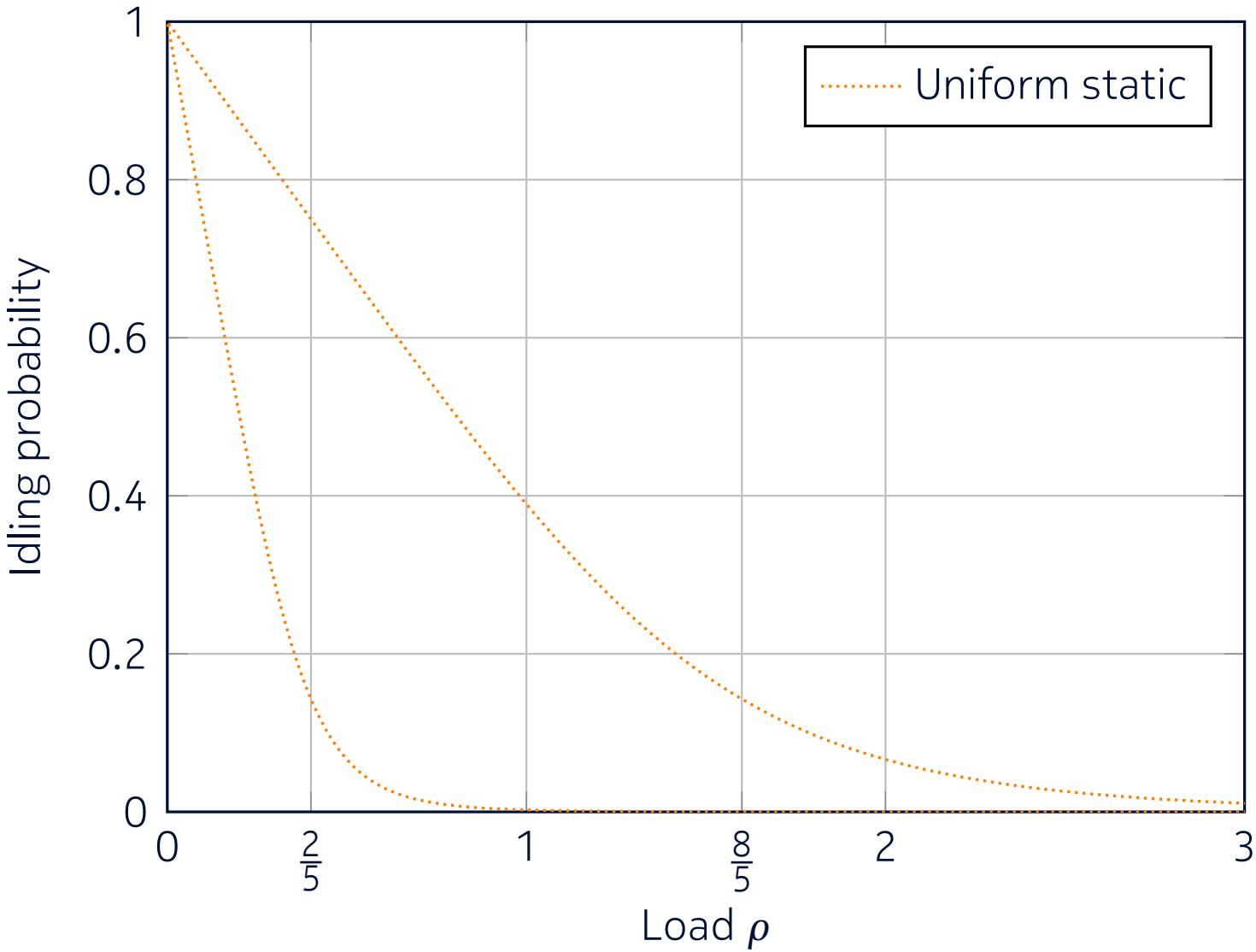

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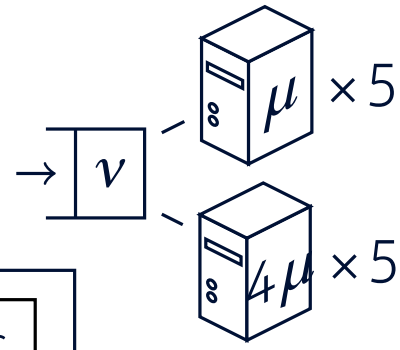

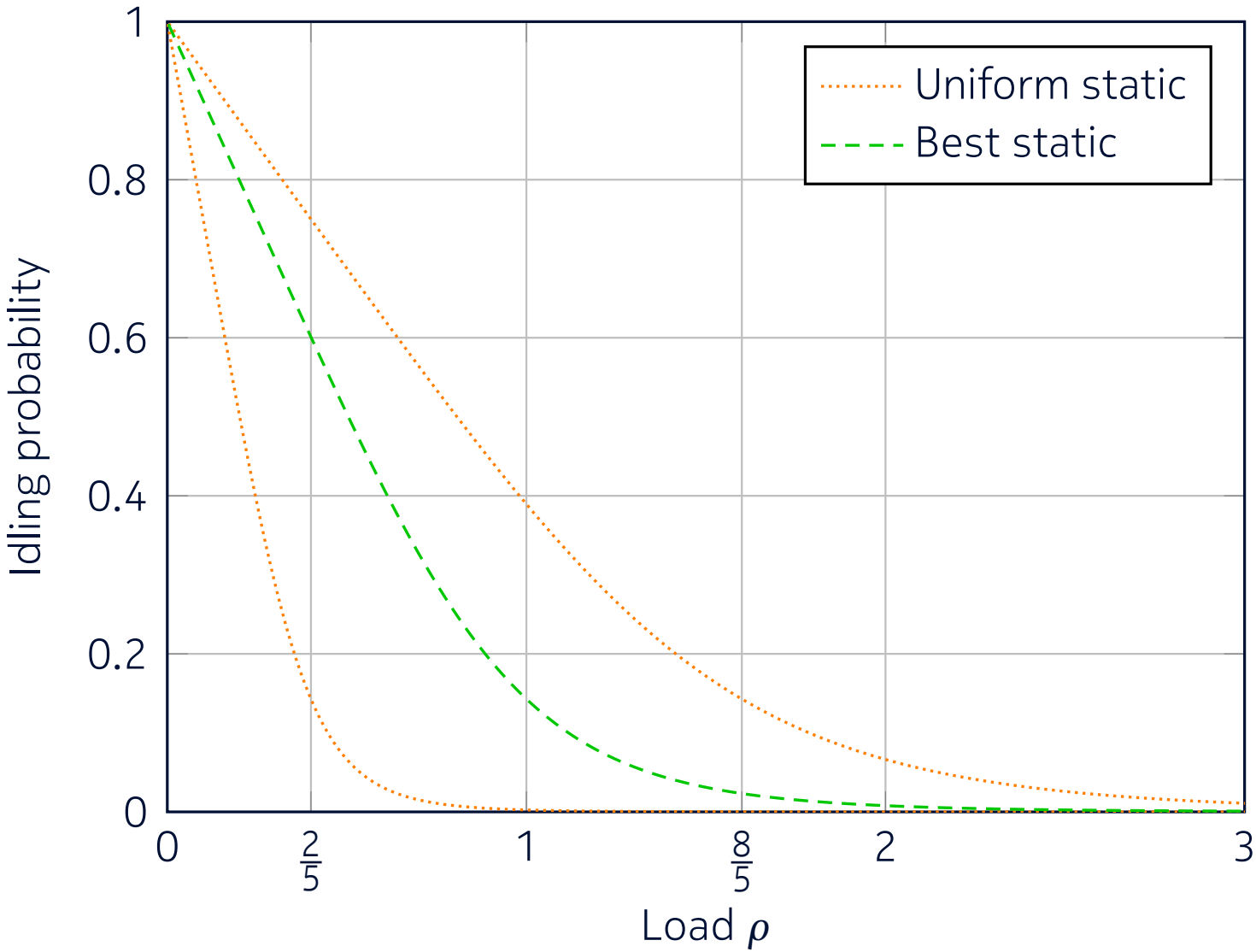

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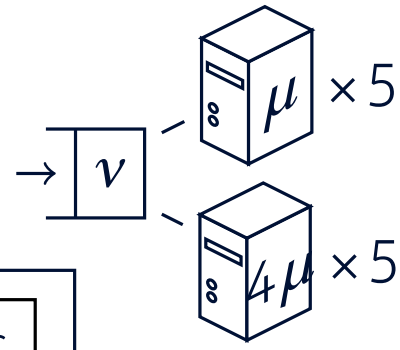

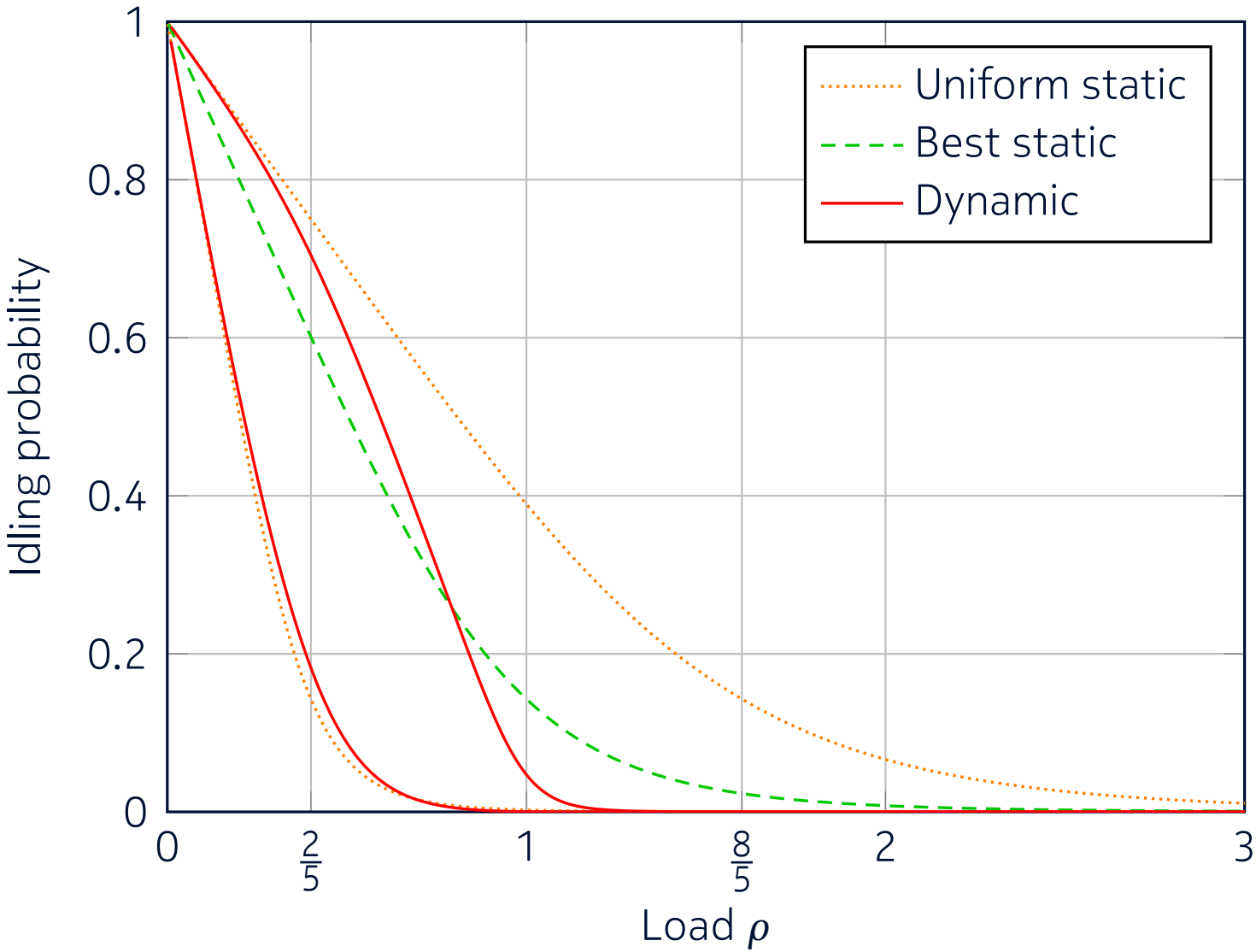

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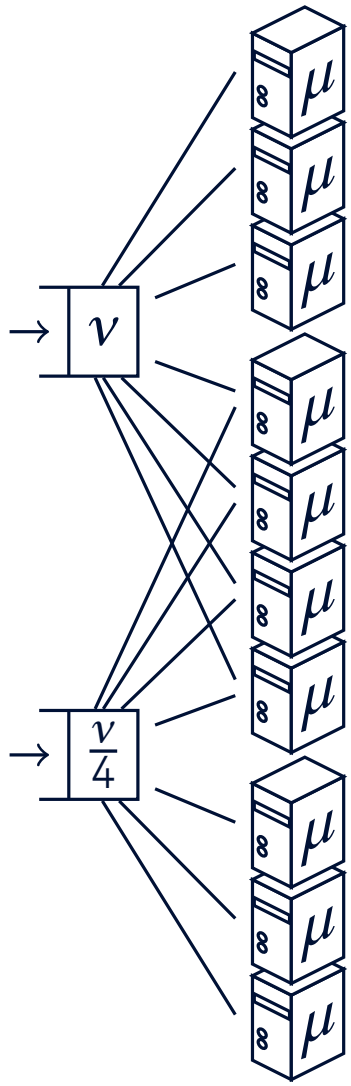
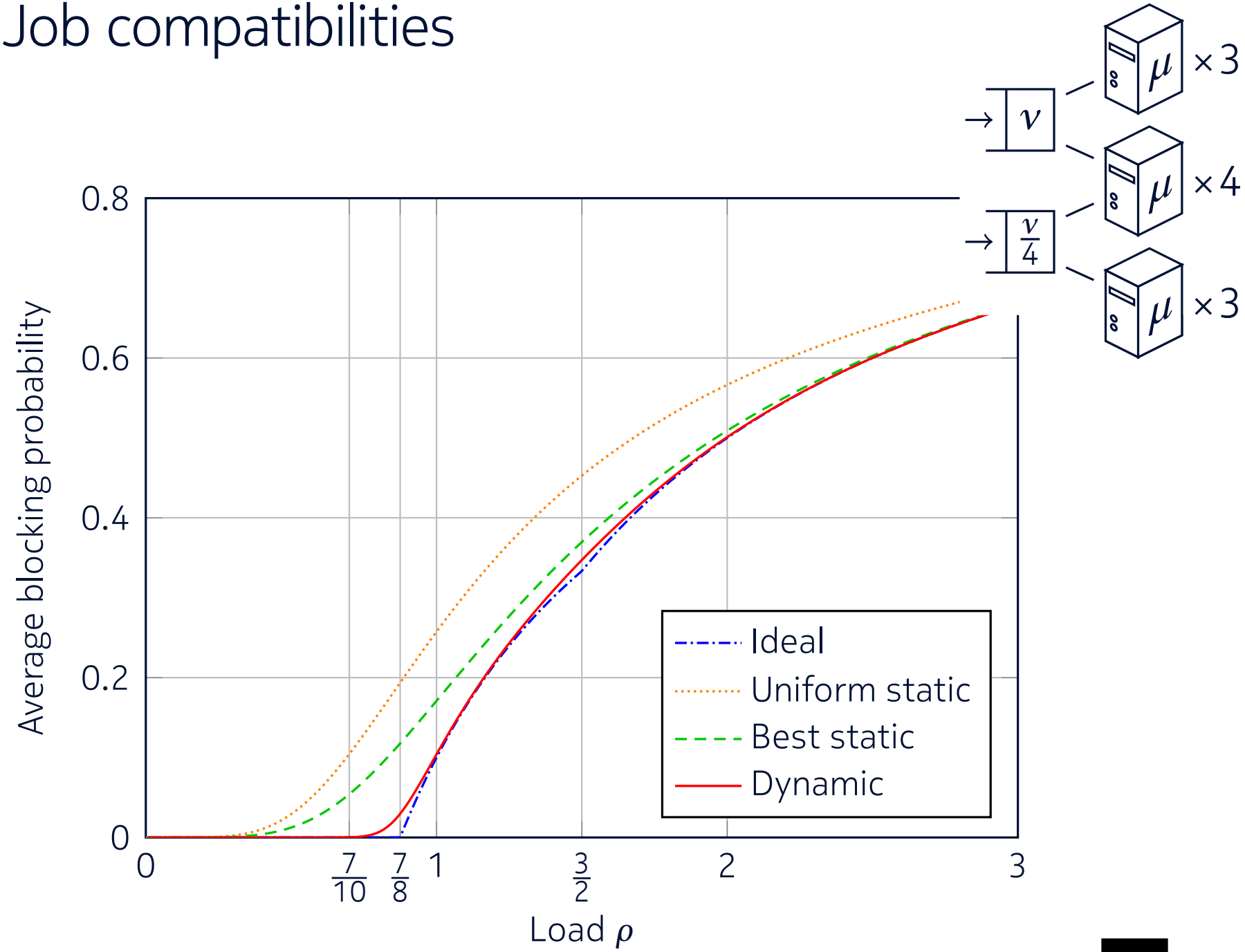
Job compatibilities



- Two job types
- Poisson arrival processes with rates ν and $\frac{\nu}{4}$
- All servers have capacity μ
- Each server has 6 tokens
- Load $\rho = \frac{\nu}{8\mu}$

Job compatibilities



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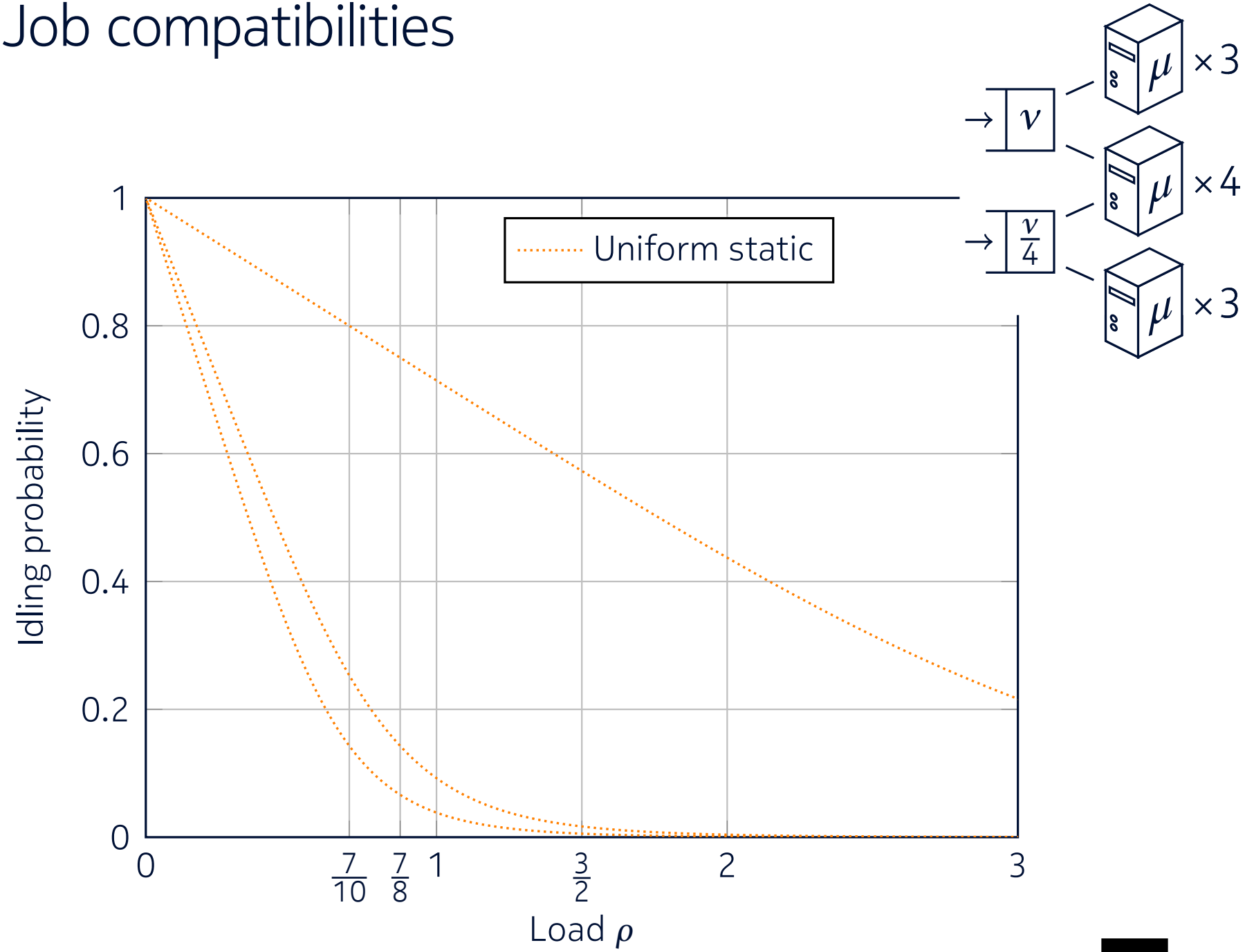

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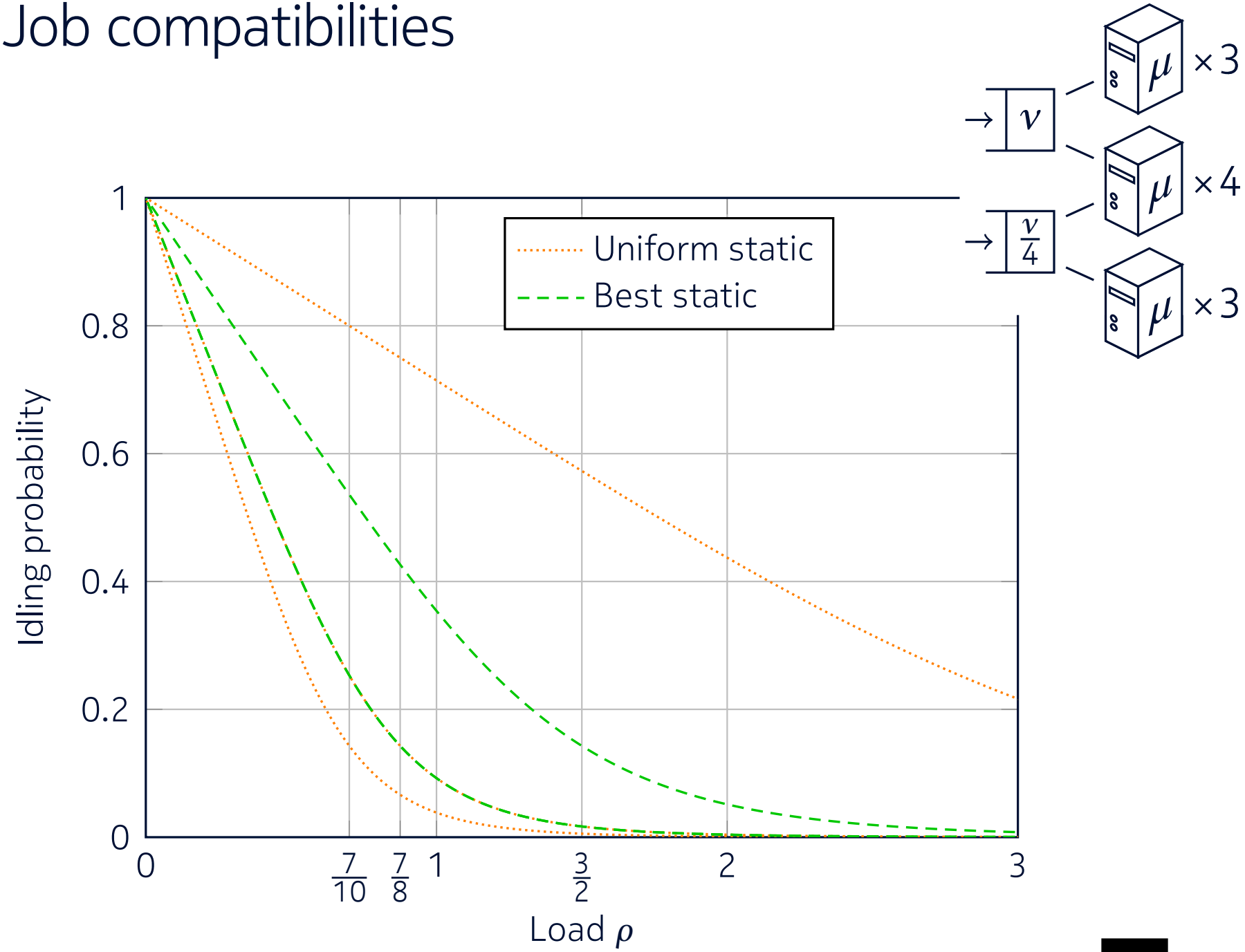

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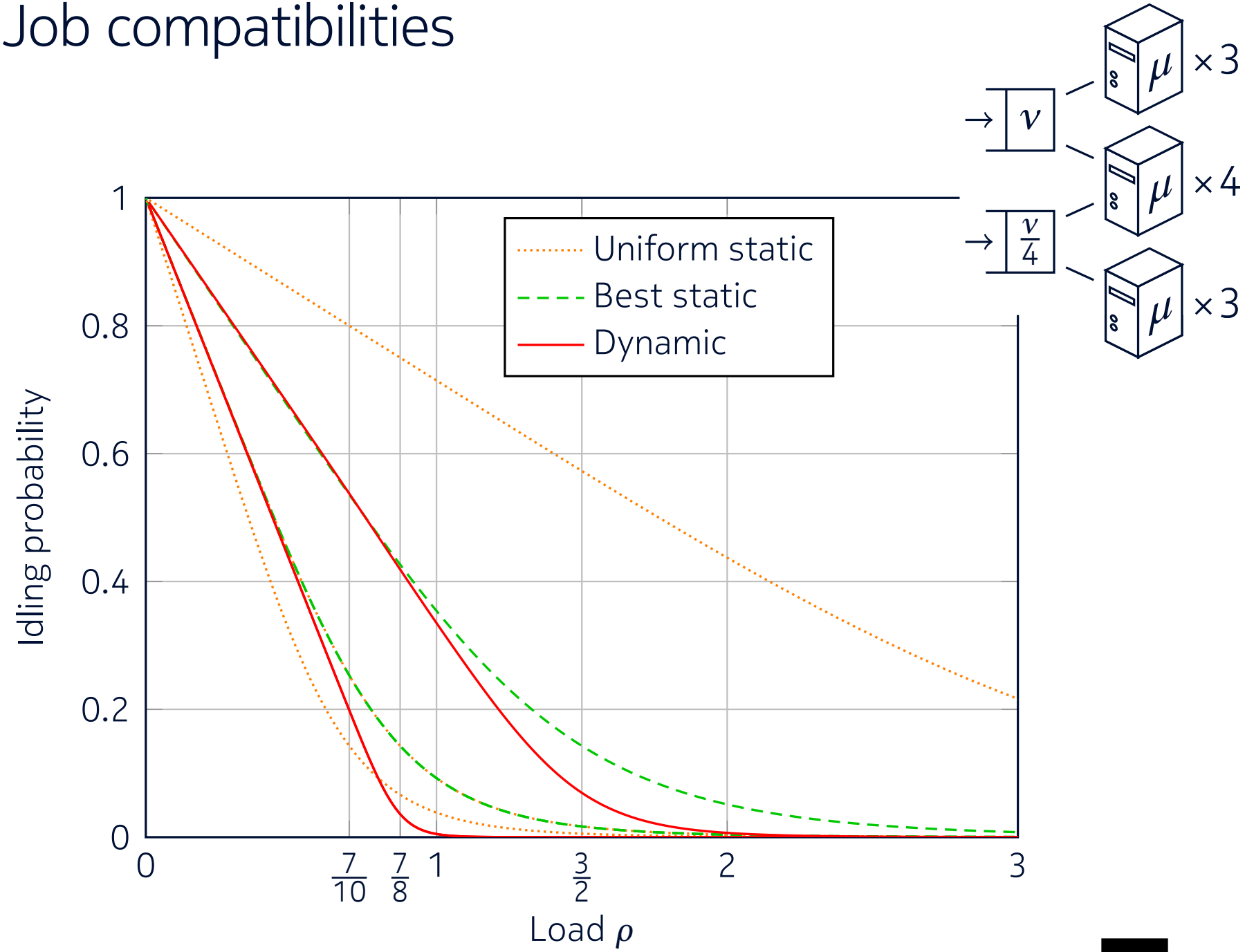

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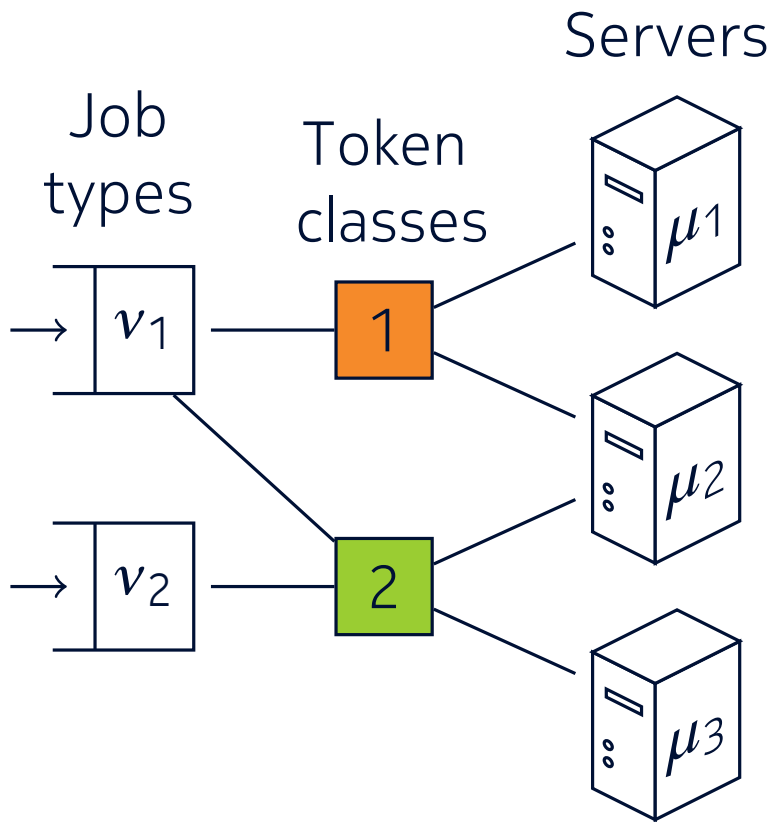
Distributed processing



Each token identifies a **set of servers**

Non-blocking extensions
Future works

Instead of rejecting an incoming job when there is no available compatible token, we could …

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Choose a compatible server uniformly at random

→ Join-Idle-Queue (Lu et al., 2011)

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Future works

Instead of rejecting an incoming job when there is no available compatible token, we could ...

Choose a compatible server uniformly at random

→ Join-Idle-Queue (Lu et al., 2011)

Wait for the release of a compatible token

→ FCFS and assign to the longest idle server (ALIS) (Adan and Weiss, 2012)

Conclusion

Our contributions

- Insensitive and adaptative load balancing algorithm
- Queueing analysis based on order-independent queues under a Poisson arrival process
- Relate several existing works

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- Queueing analysis based on order-independent queues under a Poisson arrival process
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Future works

- Derive simpler formulas for the performance prediction
- Evaluate the non-blocking versions of the algorithm

