Uplink dynamic discrete power control in cellular networks

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Abstract— **We consider an uplink power control where each mobile wishes to maximize its throughput (which depends on the transmission powers of all mobiles) but has a constraint on the average power consumption. A finite number of power levels are available to each mobile. The decision of a mobile on which power level to choose may depend on its channel state. We consider two frameworks concerning the state information of the channels of other mobiles: (i) the case of full state information and (ii) the case of local state information. We consider both a cooperative as well as a non-cooperative power control. We manage to characterize the structure of equilibria policies and more generally of best-response policies in the non-cooperative case: (i) For each mobile, the best response policy consists of randomization between at most two adjacent power levels, (ii) the optimal power levels are non-decreasing functions of the channel state, and (iii) if two power levels are both optimal for a given channel state then they cannot be jointly optimal for another channel state. We present an algorithm to compute equilibria policies in the case of two non-cooperative players. We finally study the case where a malicious mobile, which also has average power constraints, tries to jam the communications of another mobile. Our results are illustrated and validated through various numerical examples.**

I. Introduction

We consider in this paper *dynamic* power control in cellular networks: mobiles choose their transmission power level from a discrete set in a *dynamic* way, i.e., the transmission power level is chosen based on the available channel state information. In terms of decision making, we consider two cases:

- **Decentralized case:** Each mobile chooses its own power level based on the condition of its own radio channel to the base station.
- **Centralized case:** The transmission power levels for all the mobiles are chosen by the base station that has full information on all channel states.

Power control is used so as to maximize the throughputs of the mobiles. We assume that there are upper bound constraints on the average power that a mobile can use. Thus in very bad channel conditions, one can expect a mobile to avoid transmission and save its power for more favorable channel conditions.

We consider in this paper both cooperative as well as non-cooperative decision making.

A. Related work

There has been an intensive research effort on non-cooperative power control in cellular networks [3], [8], [11], [17], [20], [21]. In all these work, however, the set of available transmission powers has been assumed to be a whole interval or the whole set of nonnegative real numbers. In this paper we consider the case of a discrete set of available power levels, which is in line with standardized cellular technologies. Very little work on power control has been done on discrete power control [23] and to our knowledge, there has not been any work on non-cooperative power control with discrete available power levels.

The mathematical formulation of the power control problem shows much similarity with a well studied problem of assigning transmission powers to parallel channels between a mobile and a base station with a constraint on the sum of assigned powers, see e.g. [9, p. 161]. This problem is often known as the "water filling" (which is in fact the structure of the optimal policy). The difference between the models is that in our case we split powers over time, where as in the water filling problem the powers are split over space. Our results are therefore quite relevant to the water filling problem as well. All the references we know of concerning the water filling problem assumed that the transmission power in any channel could be any nonnegative real number (as long as the total power constraint is met). Moreover, we know of almost no work that considered the non-cooperative case, with [18] being an exception (where a special case of two players is studied).

Our setting is different also in the following. In our model, in a given channel state, each mobile can either choose a fixed power level or can make randomized decisions, i.e. it can base the choice of power levels in a state on some (state dependent) randomization.

B. Main results.

We obtain results for both the cooperative setting in which the mobiles' objective is to maximize the global throughput, as well as the non-cooperative case, where each mobile has its own objective: to maximize its own transmission rate.

We identify the structure of equilibria policies for the decentralized non-cooperative case. We show in fact, that the following structure holds for any mobile i, given any set of policies u_{-i} chosen by mobiles other than i. Any best response policy (i.e. an optimal policy for player i for a given policy u_{-i} other mobiles) has the following properties:

1) It needs randomization between at most two adjacent power levels,
2) the optimal power levels are non-decreasing functions of the channel state, and
3) if two power levels are both optimal at a given channel state then they cannot be jointly optimal for another channel state.

We present an algorithm to compute equilibria policies in the case of two non-cooperative players.

For the cooperative centralized problem with two mobiles, we obtain insight on the structure of optimal policies through a numerical study. An interesting property that we obtain is the fact that the optimal policy has a TDMA structure: in each combined state (x_1, x_2) there is only one mobile that will transmit information. This will of course eliminate the interference. We also show that unlike the decentralized case, the average power level constraints may hold with strict inequality when using the optimal policy.

We finally study the case where a malicious mobile, which also has average power constraints, tries to jam the communications of another mobile. Our results are illustrated and validated through various numerical examples.

C. The structure of the paper.

We first present the model (Section II) as well as the mathematical formulation of both the case of centralized information (Section III) as well as the one of decentralized information (Section IV). In Section V we derive the structure of best-response policies and thus of equilibria for the decentralized case. Power control in the presence of a malicious mobile is studied in Section VI. In Section VII we present numerical examples that illustrate the structural properties that we had obtained and which allow us further to obtain insight in cases for which we the structure of optimal policies remains open. After a concluding section and the bibliography, we present some proofs in the appendix as well as a computation methodology for computing some equilibria.

II. THE MODEL

A. Preliminaries

Consider a set of N mobiles and a single base station. Time is slotted. At each time slot t, each mobile i transmits data with power level $A_i(t)$ chosen among a finite set $\mathbf{A}_i = (1, 2, 3, \dots, \alpha_i)$ containing α_i power levels. The actual power corresponding to the ath power level where $a \in \mathbf{A}_i$ is given by $h_i(a)$. Denote $\mathbf{A} = \prod_{i=1}^N \mathbf{A}_i$.

The channel state model: We assume that the channel between mobile i and the base station can be modeled as an ergodic finite Markov chain $X_i(t)$ taking values in a set $\mathbf{X}_i = (1, 2, , ..., m_i)$ of m_i states with transition probabilities $\mathbf{P}^i_{xy}$. The Markov chains $X_i(t)$, $i = 1 \dots N$, are assumed to be independent. Let π_i be the row vector of steady state probabilities of Markov chain $X_i(t)$; let $\pi_i(x)$ be its entry corresponding to the state $x \in \mathbf{X}_i$. It is the unique solution of

$$\pi_i \mathbf{P}^i = \pi_i, \qquad \pi_i(x) \geq 0, \ \forall x \in \mathbf{X}_i, \qquad \sum_{x \in \mathbf{X}_i} \pi_i(x) = 1.$$

We also denote by $\pi(\mathbf{x})$ the probability of state $\mathbf{x} = (x_1, \dots x_N)$. Since the Markov chains that describe the channel states are independent, $\pi(\mathbf{x}) = \prod_{i=1}^N \pi_i(x_i)$.

The power received at the base station from mobile i is given by $g_i(t) h_i(A_i(t))$ where the attenuation $g_i(t) = g_i(X_i(t))$ is a function of the channel state $X_i(t)$. We shall denote the global state space of the system by $\mathbf{X} = \prod_{i=1}^N \mathbf{X}_i$.

Performance measures: The signal to interference ratio SIR_i at the base station related to mobile i when the power level choices of the mobiles are $\mathbf{a} = (a_1, ..., a_N)$ and the channel states are $\mathbf{x} = (x_1, ..., x_N)$ is given by

$$SIR_i(\mathbf{x}, \mathbf{a}) = \frac{g_i(x_i) h_i(a_i)}{N_o + \sum_{j \neq i} c_{ij} g_j(x_j) h_j(a_j)},$$

where c_{ij} are the coding orthogonality coefficients.

We consider the following instantaneous utility of mobile i:

$$r_i(\mathbf{x}, \mathbf{a}) = \log_2 \left(1 + SIR_i(\mathbf{x}, \mathbf{a})\right). \qquad (1)$$

$r_i(\mathbf{x}, \mathbf{a})$ is known as the Shannon capacity and can thus be interpreted as the throughput that mobile i can achieve at the uplink when the channel conditions are given by $\mathbf{x}$ and the power level used by all mobiles are $\mathbf{a}$.

Notation: In the rest of the paper, we shall use the following notation. We shall denote an element of the set $\mathbf{X}$ by $\mathbf{x}$. The ith component of $\mathbf{x}$ will be denoted by x_i, i.e., $\mathbf{x} = (x_1, x_2, \dots, x_N)$, where $x_i \in \mathbf{X}_i$ for $i = 1, 2, \dots, N$. We define $\mathbf{a}$ and a_i in a similar manner. Let $\mathbf{X}_{-i}$ and $\mathbf{A}_{-i}$ denote the set of channel states and the set of actions, respectively, corresponding to all the players other than player i. For an element $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$, let $\mathbf{x}^j_{-i}$ denote the jth component of $\mathbf{x}_{-i}$. We define $\mathbf{a}_{-i}$ and $\mathbf{a}^j_{-i}$ in a similar way.

B. Policy types

A Mobile's choice of successive transmission power levels is made based on the information it has. The latter could be local, in which case the policy is said to be distributed. We shall also consider centralized policies in which all decisions are taken at the base station. We have the following definitions.

- A **Centralized policy**, $u(\mathbf{a} \,|\, \mathbf{x})$, is the probability that the base station assigns the transmission power levels $\mathbf{a} = (a_1, ..., a_N)$ to the different mobiles if the current channel's states are given by the vector $\mathbf{x} = (x_1, ..., x_N)$. This is equivalent to the situation where all system information is available to all mobiles, and moreover, all mobiles can *coordinate* their actions. This situation describes central decision making by the base station. The class of centralized policies is denoted by U_{ce}.
- A **Decentralized policy**, $u_i(a \,|\, x)$, is the probability that player i chooses the transmission power level $a \in \mathbf{A}_i$ if

its channel state is $x \in \mathbf{X}_i$. Thus, only local information is available to each mobile, and there is no coordination in the random actions. This situation describes individual decision making by each mobile without any involvement of the base station. The class of decentralized policies for player i is denoted by U^i_{dc}. Define $U_{dc} = \prod_{i=1}^N U^i_{dc}$.

Along with policies we shall use also the occupation measures. For a given $\mathbf{x} \in \mathbf{X}$ and $\mathbf{a} \in \mathbf{A}$, the global occupation measure, $\rho^u(\mathbf{x}, \mathbf{a})$, will be used in the context of a **centralized** policy, $u \in U_{ce}$, and is defined as

$$\rho^u(\mathbf{x}, \mathbf{a}) = \prod_{i=1}^N \pi_i(x_i) u(\mathbf{a}|\mathbf{x}).$$

Note that given a global occupation measure, ρ^u, the corresponding u can be obtained by

$$u(\mathbf{a} \,|\, \mathbf{x}) = \frac{\rho^u(\mathbf{x}, \mathbf{a})}{\sum_{\mathbf{b} \in \mathbf{A}} \rho^u(\mathbf{x}, \mathbf{b})}. \tag{2}$$

(It is chosen arbitrarily if the denominator is zero.). For a given $x \in \mathbf{X}_i$ and $a \in \mathbf{A}_i$, the local occupation measure, $\rho_i^{u_i}(x, a)$, is defined with respect to a **decentralized** policy, $u_i \in U^i_{dc}$, and is given by

$$\rho_i^{u_i}(x, a) = \pi_i(x) u_i(a|x).$$

For a given global occupation measure, $\rho_i^{u_i}$, the corresponding u_i can be obtained by

$$u_i(a \,|\, x) = \frac{\rho_i^{u_i}(x, a)}{\sum_{b \in \mathbf{A}_i} \rho_i^{u_i}(x, b)}. \tag{3}$$

(It is chosen arbitrarily if the denominator is zero.) In case of **decentralized** decision making, we define $\rho^u(\mathbf{x}, \mathbf{a})$ as

$$\rho^u(\mathbf{x}, \mathbf{a}) = \prod_{i=1}^N \rho_i^{u_i}(x_i, a_i), \tag{4}$$

for a given $(u_1, u_2, \ldots, u_N)$.

C. Utility, constraints and optimization

For any given policy*, u, and the corresponding occupation measure, $\rho^u(\mathbf{x}, \mathbf{a})$†, we now define the utility function, the constraints, and the optimization problem.

The utility functions: We define the utility for player i as

$$R_i(u) := \textstyle\sum_{\mathbf{a} \in \mathbf{A}} r_i(\mathbf{x}, \mathbf{a}) \rho^u(\mathbf{x}, \mathbf{a}). \tag{5}$$

Power constraints: Player i is assumed to have the following constraint on the power level it can choose:

$$D_i(u) \le V_i, \tag{6}$$

where

$$D_i(u) = \sum_{\mathbf{x} \in \mathbf{X}} \sum_{\mathbf{a} \in \mathbf{A}} \rho^u(\mathbf{x}, \mathbf{a}) h_i(a_i), \tag{7}$$

where a_i is the ith component of $\mathbf{a}$. Note that unlike the utility, the power constraint of a mobile does not depend on the decisions of other mobiles.

1) Cooperative optimization: We consider here the problem of maximizing a *common objective* subject to individual side constraints. Namely, we define for any policy u

$$R_\gamma(u) := \sum_{i=1}^N \gamma_i R_i(u),$$

where γ_i are some nonnegative constants. For an arbitrary set of policies $\overline{U}$ we consider the problem:

$$\mathbf{COOP}(\overline{U}) : \qquad \max_{u \in \overline{U}} R_\gamma(u), \qquad \text{s.t. (6)}, \quad \forall i = 1, ..., N$$

2) Non-cooperative optimization: Here each mobile is considered as an individual non-cooperative decision maker, which we then call "player". We naturally restrict to decentralized policies U_{dc}.

For a policy $u = (u_1, ..., u_N) \in \overline{U}$ we define u_{-i} to be the set of components of u other than the ith component. For a policy $v_i \in \overline{U}_i$ we then define the policy $[v_i, u_{-i}]$ as one in which player $j \neq i$ uses the element u_j of u whereas player i uses v_i.

We say that $u^* \in \overline{U}$ is a constrained Nash equilibrium [19] if it satisfies (6) for all players, and if for any i and any $v_i \in \overline{U}$ the following holds. If the constraint (6) holds for player i when it uses the policy $[v_i, u_{-i}]$ then

$$R_i(u^*) \ge R_i([v_i, u^*_{-i}]).$$

III. Centralized cooperative optimization

When the cooperative optimization is considered over the set of *centralized policies*, then the problem is in fact of a single controller (the base station) which has all the information. Let $r_\gamma(\mathbf{x}, \mathbf{a}) := \sum_{i=1}^N \gamma_i r_i(\mathbf{x}, \mathbf{a}),\ \gamma_i \ge 0,\ i = 1, 2, \ldots, N$, denote the common instantaneous utility when power level $\mathbf{x}$ is chosen in channel state $\mathbf{a}$. The problem is then a special case of constrained MDPs (Markov Decision Processes) for which we know the following:

Theorem 1: Consider the cooperative optimization problem $\mathbf{COOP}(U_{ce})$ over the set of centralized policies. Assume that there exists a policy u under which the power constraints (6) hold for all the mobiles. Then

- (i) there exists an optimal centralized policy $u \in U_{ce}$.
- (ii) Consider the following linear program whose decision variables are $u(\mathbf{a} \,|\, \mathbf{x}), \mathbf{a} \in \mathbf{A}, \mathbf{x} \in \mathbf{X}$:

$$\text{maximize}_{\ u} \quad R_\gamma(u) := \sum_{\mathbf{x} \in \mathbf{X}} \sum_{\mathbf{a} \in \mathbf{A}} \pi(\mathbf{x}) u(\mathbf{a} \,|\, \mathbf{x}) r_\gamma(\mathbf{x}, \mathbf{a}) \tag{8}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{x \in \mathbf{X}_i} \sum_{a \in \mathbf{A}_i} \pi_i(x) u_i(a \,|\, x) h_i(a) \le V_i, \\ & u_i(a \,|\, x) \ge 0, \quad \forall x \in \mathbf{X}_i, a \in \mathbf{A}_i, \\ & \sum_{a \in \mathbf{A}_i} u_i(a \,|\, x) = 1, \quad \forall x \in \mathbf{X}_i, \end{aligned} \tag{9}$$

*With slight abuse of notation, we shall denote both centralized and decentralized policies by u. In the **centralized** case, $u(\mathbf{a}|\mathbf{x})$ will denote a probability measure over $\mathbf{a}$ for a given $\mathbf{x}$. In the **decentralized** case, u will denote the vector $u = (u_1, u_2, \ldots, u_N)$, where u_i is the decentralized policy for player i, for $i = 1, 2, \ldots, N$.

†For the **decentralized** case, we note that $\rho^u(\mathbf{x}, \mathbf{a})$ is given by (4).

where $u_i(a\,|\,x) =$

$$\sum_{\mathbf{x}_{-i}\in\mathbf{X}_{-i}} \sum_{\mathbf{a}_{-i}\in\mathbf{A}_{-i}} \prod_{j\neq i} \pi_j(\mathbf{x}^j_{-i}) u((a,\mathbf{a}_{-i})\,|\,(x,\mathbf{x}_{-i})).$$

The optimal solution u^* for (8) is an optimal policy for the cooperative centralized problem.

- (iii) An optimal u^* can be chosen with no more than $2N$ nonzero elements.

Note that in the centralized framework it does not make sense to speak about a non-cooperative game, since there is a single decision maker.

IV. Decentralized Information

A. The cooperative case

We briefly discuss here the situation where, even though there is a common goal that is optimized, the power level choices are not done by the base stations but by the mobiles themselves who have only their local information available to take decisions. Coordination is thus not possible.

We shall show in Theorem 2 that there exists an optimal decentralized policy. In section VII we provide an example of solving the decentralized power control problem in the case of two mobiles.

Theorem 2: Consider the decentralized framework within the **cooperative** framework (where all players have the common objective function $R_\gamma(u)$). Then there exists an optimal policy.
The proof is delayed to Appendix A.

B. Non-cooperative equilibrium

Theorem 3: There exists a constrained Nash equilibrium.

Proof: The set of policies for a player i can be identified by a set of m_i probability measures over the $\mathbf{A}_i$. The subset of policies of mobile i that furthermore meet the power constraints can thus be identified by the set $\Big(u_i(a|x)\Big)$, $x \in \mathbf{X}_i$, $a \in \mathbf{A}_i$, satisfying

$$\textstyle\sum_{x\in\mathbf{X}_i}\sum_{a\in\mathbf{A}_i} \pi_i(x) u_i(a|x) h_i(a) \leq V_i,$$

$$u_i(a|x) \geq 0, \quad \forall a\in\mathbf{A}_i,\ \forall x\in\mathbf{X}_i,$$
$$\textstyle\sum_{a\in\mathbf{A}_i} u_i(a|x) = 1, \quad \forall x\in\mathbf{X}_i.$$

This is a closed convex set for each player. Moreover, for each mobile i, the utility $R_i(u)$ is concave in u_i and continuous in $u_j,\ j\neq i$. We conclude from Theorem 1 of [19] that a Nash equilibrium exists. ■

V. Structure of non-cooperative equilibrium

We study the structure of best response policies of any given user when the policies of the other users are fixed. We fix throughout the policy v_{-i} of players other than player i, where

$$v_{-i}(\mathbf{a}_{-i}\,|\,\mathbf{x}_{-i}) = \prod_{j\neq i} v_j(\mathbf{a}^j_{-i}\,|\,\mathbf{x}^j_{-i})$$

is the probability that each mobile $j\neq i$ chooses a_j when its local state is x_j. The product form here is due to the decentralized nature of the problem and to the fact that there is no coordination between the mobiles is possible.

We shall make the following assumption on the properties of g_i, h_i, and π_i.

Assumption 1:
1) The function g_i has an increasing interpolation in x.
2) The function h_i has a strictly concave and increasing interpolation in a.
3) The probability measure $\pi_i(x)$ has a non-decreasing interpolation in x.

We shall establish the following main result on the structure of any best response policy:

Theorem 4: Consider the decentralized non-cooperative case. Under Assumption 1, the following hold:

1) In each channel state x_i, the best response policy consists of either the choice of a single action, or in a randomized choice between at most two adjacent power levels.
2) The optimal power levels are non-decreasing functions of the channel state.
3) If two power levels are jointly optimal for a given channel state then they cannot be jointly optimal for another channel state.

The rest of this section is devoted to proving these results. The proof follows the following steps. We first formulate the problem of obtaining a best response as a linear program (Subsection V-A). Using Lagrange relaxation in Subsection V-B we are able to *decouple* the problem to several simpler ones: in each one of the latter, the channel state is fixed. In Subsection V-C we then establish concavity and supermodularity properties of the best response value function corresponding to a fixed channel state.

A. A linear program formulation

With $r_i(\mathbf{x},\mathbf{a})$ as defined in (1), denote

$$r^v_i(x,a) = \sum_{\mathbf{x}_{-i}\in\mathbf{X}_{-i}} \sum_{\mathbf{a}_{-i}\in\mathbf{A}_{-i}} \textstyle\prod_{j\neq i} \pi_j(\mathbf{x}^j_{-i}) v(\mathbf{a}^j_{-i}|\mathbf{x}^j_{-i}) \times r_i\Big((\mathbf{x}_{-i},x),(\mathbf{a}_{-i},a)\Big).$$

For the fixed v_{-i}, player i is faced with the problem

$$\text{maximize over } u_i\in\mathbf{U}_i\ R^v_i(u_i) := R_i(v_{-i},u_i) = = \textstyle\sum_{x\in\mathbf{X}_i}\sum_{a\in\mathbf{A}_i} \pi_i(x) u_i(a\,|\,x) r^v_i(x,a) \quad (10)$$

$$\text{s.t.}\quad D_i(u_i) := \sum_{x\in\mathbf{X}_i}\sum_{a\in\mathbf{A}_i} \pi_i(x) u_i(a\,|\,x) h_i(a) \leq V_i.$$

B. Lagrange relaxation

Consider the following relaxed problem parameterized by some finite real λ_i:

$$\text{maximize over } u_i\in\mathbf{U}_i$$
$$J^v_i(\lambda_i,u_i) = R^v_i(u_i) + \lambda_i(D_i(u_i) - V_i) =$$
$$= \sum_{\substack{x\,\in\,\mathbf{X}_i\\ a\,\in\,\mathbf{A}_i}} \pi_i(x) u_i(a\,|\,x)\Big(r^v_i(x,a) + \lambda_i h_i(a)\Big) - \lambda_i V_i \quad (11)$$

$$J^*_i(\lambda_i,v) = \max_{u_i} J^v_i(\lambda_i,u_i)$$

Problem (10) faced by player i can be viewed as a special degenerate case of constrained Markov decision processes (it is degenerate since the transition probabilities of the radio channel of mobile i are not influenced by the actions. The latter only have an impact on the immediate payoff r_i^v and on h_i). We know from [4] that a policy u_i^* is optimal for (10) only if it is optimal for the relaxed problem (11) for some finite λ_i. By characterizing the structure of the policies that are optimal for (11) we shall obtain the structure of optimal policies for (10). In the sequel, we shall omit the constant $-\lambda_i V_i$ from the objective function in (11) since it has no influence on the structure of the optimal policies.

Observation. We now make the following key observation on (11). The relaxed problem can be solved separately for each channel state $x \in \mathbf{X}_i$. A policy $u_i = \{u_i(a \,|\, x)\}_{a\in\mathbf{A}_i,\, x\in\mathbf{X}_i}$ is optimal for (11) if and only if for each fixed $x \in \mathbf{X}_i$, $u_i(\cdot \,|\, x)$ maximizes

$$J_i^v(x,\lambda_i,u_i) := \sum_{a\in\mathbf{A}_i} \pi_i(x) u_i(a \,|\, x)\Big(r_i^v(x,a) + \lambda_i h_i(a)\Big). \tag{12}$$

Due to linearity, for each $x \in \mathbf{X}_i$ there is a non-randomized decision $a \in \mathbf{A}_i$ such that

$$J_i^*(v,x,\lambda_i) = \max_{u_i} J_i^v(x,\lambda_i,u_i) = \max_{a\in\mathbf{A}_i} \nu(x,a)$$

where $\nu(x,a) := \pi_i(x)(r_i^v(x,a) + \lambda_i h_i(a))$.

C. Concavity and number of randomizations

For a fixed x, assume that there is an interpolation of $\nu(x,a)$ which is concave in a. In our case r_i is already concave for $a \in [0,\infty)$ when using the definition in (1), and h_i has a concave interpolation from Assumption 1. Thus, in our case, $\nu(x,a)$ has a concave interpolation in a for a fixed x. This means that

1) either there is only one action, say a, which has a non-zero probability to be used by any optimal policy, or
2) except for two adjacent actions, say a and $a+1$, all other actions are not used by any policy which is optimal.

The above structure holds not only for the relaxed problem (11) but also for the original problem (10). This follows since any optimal policy for (10) is necessarily optimal for the relaxed problem (11) for some λ_i, and since we just saw that any optimal policy for the relaxed problem has this structure.

D. Supermodularity and strong monotonicity of policies

Definition 1: $\nu(x,a)$ is said to be strictly supermodular in (x,a) if

$$\nu(x+1,a+1) - \nu(x+1,a) > \nu(x,a+1) - \nu(x,a) \tag{13}$$

for all $x = (1,2,3,...,m_i-1)$ and all $a = (1,2,...,\alpha_i-1)$.

Next we derive conditions for strong monotonicity of the argmax of supermodular functions.

Proposition 1: Assume that $\nu(x,a)$ is supermodular. Let A_x be the set $A_x = \text{argmax}_a \nu(x,a)$. Let $\overline{a}_x$ be the largest element of A_x and let $\underline{a}_x$ be the smallest one. Then for any $x < m_i$, we have $\overline{a}_x \le \underline{a}_{x+1}$.

Proof: The proof is straight forward and is a special case [4, thm. 7.1]. ∎

Theorem 5: Under Assumption 1, the function $\nu(x,a)$ is strictly supermodular.

Proof: ν has the form of a convex combinations of terms of the form $\nu_k = \pi_i(x)\log(1 + g_i(x)h_i(a)/N_k)$. For each k we have

$$\begin{aligned} &\nu_k(x+1,a+1) - \nu_k(x+1,a) = \\ &= \pi_i(x+1)\log\left(\frac{N_k + g_i(x+1)h_i(a+1)}{N_k + g_i(x+1)h_i(a)}\right) = \\ &= \pi_i(x+1)\log\left(1 + \frac{g_i(x+1)\Big(h_i(a+1) - h_i(a)\Big)}{N_k + g_i(x+1)h_i(a)}\right). \end{aligned}$$

Similarly,

$$\begin{aligned} &\nu_k(x,a+1) - \nu_k(x,a) = \\ &= \pi_i(x)\log\left(1 + \frac{g_i(x)\Big(h_i(a+1) - h_i(a)\Big)}{N_k + g_i(x)h_i(a)}\right) \end{aligned}$$

It is now easily seen that when g_i and h_i are strictly increasing, and π_i is non-decreasing then (13) indeed holds for ν_k which then implies that ν_k and, hence, ν are supermodular. ∎

E. Structure and computation of the equilibrium

Corollary 1: Consider the decentralized non-cooperative case. For each mobile i, assume that h_i, g_i, and π_i satisfy Assumption 1. Then there exists at least one equilibrium. Moreover, at any equilibrium u_i^* the following hold for each mobile i:

1) In each channel state $x \in \mathbf{X}_i$, $u_i^*(\cdot|x)$ consists of either a choice of a single power level, or in a randomized choice between at most two adjacent power levels.
2) The power levels used in u_i^* are non-decreasing functions of the channel state.
3) If two power levels are used at a state x by mobile i with positive probability (i.e. $u_i^*(a_j|x) > 0$ and $u_i^*(a_k|x) > 0$ for $a_k \neq a_j$) then under u_i^*, not more than one of them is used with positive probability at any other channel state.

Proof: The structure of best response policies characterizes in particular the structure of the constrained Nash equilibria policies since at equilibrium, each mobile uses a best response policy. Therefore, the structure we derived for the best response policies holds for any Nash equilibrium u_i^* for any of the mobiles. ∎

We have used two approaches for computing the equilibrium. The first is based on iterations: we fix the policy of all mobiles and then update according to a round robin order the strategies of each mobile. Whenever this method converges to some u^*, then u^* is indeed an equilibrium strategy since u^* is an equilibrium if and only if for each mobile i, the policy u_i^* is a best response against the other policies u_{-i}^*. We have tried this approach for the numerical examples in Section VII and have observed that the algorithm converged in three iterations. Nevertheless, we do not have a theoretical proof of this convergence.

An alternative approach for computing an equilibrium in the special case of two mobiles is through Linear Complementarity Approach introduced by Lemke. We have adapted this approach to our setting, see Appendix B.

VI. Power control in the presence of a malicious mobile

In recent years, there has been a growing interest in identifying and studying the behavior of potential intruders to networks or of malicious users, and in studying how to best detect these or to best protect the network from their actions (see e.g. [7], [12], [14] and references therein).

We consider in this section a scenario where a malicious player attempts to jam the communications of a mobile to the base station. We consider the distributed case and restrict for simplicity to two mobiles and a base station.

The first mobile (player 1) seeks to maximize the rate of information that it transmits to the base station. In other words it wishes to maximize $R_1(u)$ defined in (5) where r_1 is given in (1) with $c_{12} = 1$.

The second mobile (player 2) has an antagonistic objective: to prevent or to jam the transmissions of the first mobile, with the objective of minimizing the throughput of information that mobile 1 transmits to the base station. It thus seeks to *minimize* $R_1(u)$. We assume that the interference of the second mobile is presented as a Gaussian white noise.

Except for the objective of the jamming mobile, the model, including the average power constraints, defined in Section II holds. In particular, we conclude that Theorem 1 applies to player 1 at equilibrium.

A. Formulation as a zero-sum game

We now specify the objective of the players and some properties of the equilibrium. Denote U_c^i the set of policies for player i, (where i takes the values 1 and 2) that satisfy player i's power constraints, i.e., $u_i \in U_c^i$ if it satisfies $D_i(u_i) \le V_i$. Player 1 seeks to obtain an optimal policy, i.e. a policy $u_1^* \in U_c^1$ such that for any other $u_1 \in U_c^1$,

$$\inf_{u_2 \in U_c^2} R_1(u_1^*, u_2) \ge \inf_{u_2 \in U_c^2} R_1(u_1, u_2).$$

We call this the jamming problem. It consists of identifying a policy for player 1 that guarantees the largest throughput under the worst possible strategy of player 2. In fact, we shall be able not only to identify the optimal policy for player 1 but also the "optimal" policy for player 2 (which is the worst for player 1).

A policy $u^* = (u_1^*, u_2^*)$ is said to be a saddle point if

$$\begin{aligned} &\sup_{u_1 \in U_1^c} \inf_{u_2 \in U_2^c} R_1(u_1, u_2) \\ &= \inf_{u_2 \in U_2^c} R_1(u_1^*, u_2) = R_1(u_1^*, u_2^*) \\ &= \sup_{u_1 \in U_1^c} R_1(u_1, u_2^*) = \inf_{u_2 \in U_2^c} \sup_{u_1 \in U_1^c} R_1(u_1, u_2) \end{aligned}$$

u_1^* and u_2^* are called saddle point policies or optimal policies.

Unlike all problems we considered previously, deriving both u_1^* as well as u_2^* is possible using a linear program. Due to lack of space we do not include the computation, but it can be found in [5]. Below we derive properties of u_2^*.

B. Structure for player 1

Theorem 6: (i) There exists a saddle point policy u^* in the above game.
(ii) Under Assumption 1, any optimal policy for player 1 has the structure identified in Theorem 4.

For the proof of (i) we refer to [5]. Part (ii) is a direct result of Theorem 4.

C. Structure for player 2: Convexity and number of randomizations

For player 1, from Theorem 6 we can infer that the relaxed objective function has a structure similar to that of (11).

We now identify a structural property of the optimal policy of player 2, i.e., of the jamming mobile. Let h_2 have a convex interpolation in a, and g_2 have an increasing interpolation in x. Therefore, for a given x, the relaxed objective function would have a convex interpolation in a. This means that

1) there is only one action, say a, which has a non-zero probability to be used by any optimal policy, or
2) except for two adjacent actions, say a and $a+1$, all other actions are not used by any policy which is optimal.

Using arguments similar to those in Subsection V-C, we can conclude that the above structure holds not only for the relaxed problem but also for the original problem.

We finally note that the monotonicity property enjoyed by the saddle point policy of mobile 1, *need not hold* for mobile 2. This will be illustrated in Section VII-D (see Figure 6).

VII. Numerical Examples

In this section we provide examples of power control problem for two mobiles that interact with the same base station. The decentralized policies are provided both for the cooperative and non-cooperative case. Moreover, the single controller problem for centralized cooperative framework is also solved. All three problems are considered in the same settings, so one has an opportunity to compare the obtained strategies and the objective value functions for different approaches.

We assume, that the radio channel between mobile $i = 1, 2$ and the base station is characterized by a Markov chain X_i with states $x_i \in \mathbf{X}_i = \{1, \ldots, M\}$, $M = 11$, and a uniform vector of steady state probabilities. One of the transition probability matrices which has a uniform steady state probability vector is given by $\mathbf{P}^i_{xy} = \frac{1}{M}$.

The power attenuation for each state of the Markov chain X_i is defined by the following:

x_i	1	2	3	…	11
$g_i(x_i)$	0.0	0.1	0.2	…	1.0.

Let mobile i's action set $\mathbf{A}_i$ be given by $\mathbf{A}_i = (0, ..., 10)$. The actual power corresponding to the a_ith power level, where $a_i \in \mathbf{A}_i$, is

a_i	0	1	2	...	10
$h_i(a_i)$	0	W_0	$2\mathrm{W}_0$	...	$10\mathrm{W}_0$,

where W_0 is some base value of the power. We assume that the total noise power at the base station, N_0, is given by $N_0 = \mathrm{W}_0 n_0$, with $n_0 = 1$. Since (1) depends only on the ratio between the power of signal received from a certain mobile and the total power received from other mobiles and the thermal noise power at the receiver, we do not specify the exact value of the base power W_0.

We note that, with the above definitions, g_i, h_i and π_i satisfy the properties in Assumption 1.

The power consumption constraints for players are the following:

$$D_1(u_1) \le 2.9\mathrm{W}_0,$$
$$D_2(u_2) \le 5.2\mathrm{W}_0.$$

where $D_i(u_i)$ is defined by (7). Note, that both right and left hand sides of these constraints have the multiplier W_0, which can be canceled.

A. Decentralized policies

First we consider the decentralized problems that arise in cooperative and non-cooperative case. Both problems are formulated in terms of occupation measures $\rho_i(x_i, a_i)$. In order to compute the strategies one can use (3).

1) Cooperative optimization: Let $\mathbf{x} = (x_1, x_2)$ and $\mathbf{a} = (a_1, a_2)$. Here we consider the following cost function

$$r(\mathbf{x}, \mathbf{a}) = r_1(\mathbf{x}, \mathbf{a}) + r_2(\mathbf{x}, \mathbf{a}), \tag{14}$$

where $r_i(\mathbf{x}, \mathbf{a})$ are defined by (1) with $c_{ij} = 1$.

Consider the following bilinear problem

$$\text{maximize over } \rho_1, \rho_2 \quad \sum_{\substack{x_1 \in \mathbf{X}_1 \\ a_1 \in \mathbf{A}_1}} \sum_{\substack{x_2 \in \mathbf{X}_2 \\ a_2 \in \mathbf{A}_2}} \rho_1(x_1, a_1) r(\mathbf{x}, \mathbf{a}) \rho_2(x_2, a_2), \tag{15}$$

where

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) h_i(a_i) \le V_i, \tag{16}$$

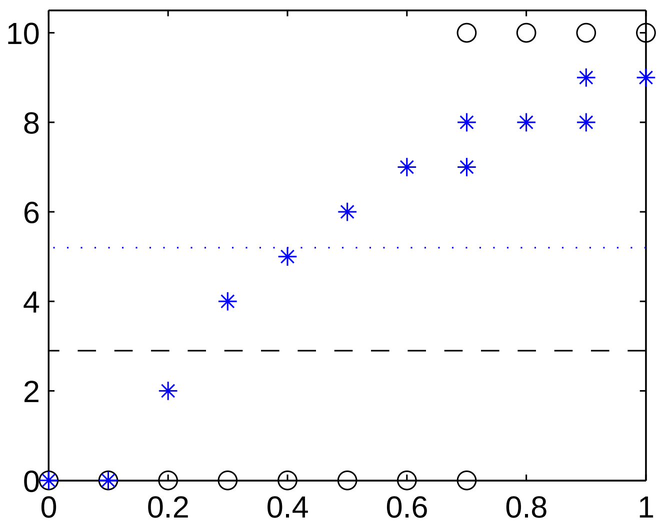
and

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i)(\delta(x_i, y_i) - \mathbf{P}^i_{x_i y_i}) = 0, \quad \forall y_i \in \mathbf{X}_i,$$
$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) = 1,$$
$$\rho_i(x_i, a_i) \ge 0, \quad \forall x_i \in \mathbf{X}_i, a_i \in \mathbf{A}_i. \tag{17}$$

Here $\mathbf{P}^i$ is the transition matrix of the Markov chain, which describes the radio channel between the mobile i and the base station, and $\delta(x, y)$ is equal to one if $x = y$ and is zero otherwise.

The problem (15) could be solved using the quadratic programming technique.

In Fig. 1, the supports of the optimal policies for both players are shown as a function of the channel state.



- Circles — support of the optimal policy of the first mobile;
- Stars — support of the optimal policy of the second mobile;
- Dashed line — bound on the average power consumption of the first player;
- Dotted line — bound on the average power consumption of the second player.

Fig. 1. Supports of the optimal policies in cooperative case.

The exact values of the policies $u_i(a_i \,|\, g_i(x_i))$ are follows:

$$u_1(0\,|\,0.0) = u_1(0\,|\,0.1) = u_1(0\,|\,0.2) = u_1(0\,|\,0.3) =$$
$$= u_1(0\,|\,0.4) = u_1(0\,|\,0.5) = u_1(0\,|\,0.6) = u_1(10\,|\,0.8) =$$
$$= u_1(10\,|\,0.9) = u_1(10\,|\,1.0) = 1;$$
$$u_1(0\,|\,0.7) = 0.81, \quad u_1(10\,|\,0.7) = 0.19;$$

$$u_2(0\,|\,0.0) = u_2(0\,|\,0.1) = u_2(2\,|\,0.2) =$$
$$= u_2(4\,|\,0.3) = u_2(5\,|\,0.4) = u_2(6\,|\,0.5) =$$
$$= u_2(7\,|\,0.6) = u_2(8\,|\,0.8) = u_2(9\,|\,1.0) = 1;$$
$$u_2(7\,|\,0.7) = 0.0622, \quad u_2(8\,|\,0.7) = 0.9378;$$
$$u_2(8\,|\,0.9) = 0.7378, \quad u_2(9\,|\,0.9) = 0.2622;$$

The value of the objective function in this problem is $R(u^*) = 1.9423$.

2) Non-cooperative equilibrium: Now, in the same setting as in the cooperative case, we consider an example of non-cooperative optimization. This problem is described in Appendix B. Each mobile needs to maximize it's own objective function:

$$\max_{\rho_1, \rho_2} \sum_{\substack{x_1 \in \mathbf{X}_1 \\ a_1 \in \mathbf{A}_1}} \sum_{\substack{x_2 \in \mathbf{X}_2 \\ a_2 \in \mathbf{A}_2}} \rho_1(x_1, a_1) r_1(\mathbf{x}, \mathbf{a}) \rho_2(x_2, a_2),$$

$$\max_{\rho_1, \rho_2} \sum_{\substack{x_1 \in \mathbf{X}_1 \\ a_1 \in \mathbf{A}_1}} \sum_{\substack{x_2 \in \mathbf{X}_2 \\ a_2 \in \mathbf{A}_2}} \rho_1(x_1, a_1) r_2(\mathbf{x}, \mathbf{a}) \rho_2(x_2, a_2),$$

subject to the constraints (22)-(25) (in the Appendix B).

By means of the linear complementarity problem (30) one can obtain the following optimal strategies:

$$\begin{gathered} u_1(0\,|\,0.0) = u_1(0\,|\,0.1) = u_1(0\,|\,0.2) = u_1(1\,|\,0.3) = \\ = u_1(2\,|\,0.4) = u_1(3\,|\,0.5) = u_1(4\,|\,0.6) = u_1(5\,|\,0.7) = \\ = u_1(5\,|\,0.8) = u_1(6\,|\,1.0) = 1; \\ u_1(5\,|\,0.9) = 0.1, u_1(6\,|\,0.9) = 0.9; \end{gathered}$$

$$\begin{gathered} u_2(0\,|\,0.0) = u_2(0\,|\,0.1) = u_2(1\,|\,0.2) = u_2(4\,|\,0.3) = \\ = u_2(6\,|\,0.4) = u_2(7\,|\,0.6) = u_2(8\,|\,0.7) = u_2(8\,|\,0.8) = \\ = u_2(8\,|\,0.9) = u_2(9\,|\,1.0) = 1; \\ u_2(6\,|\,0.5) = 0.8, u_1(7\,|\,0.5) = 0.2. \end{gathered} \tag{18}$$

In Fig. 2 the supports of the optimal policies for both players are depicted. We note that the structure obtained in Theorem 4 holds for both the players.

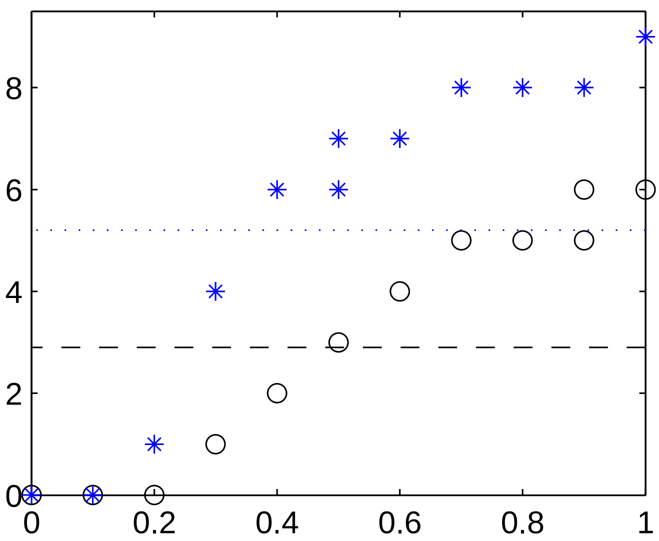


Fig. 2. Supports of the optimal policies in non-cooperative case.

The values of the costs in this problem are $R_1(u^*) = 0.6768$, $R_2(u^*) = 1.1565$. As it was expected the summary throughput value $R(u^*) = R_1(u^*) + R_2(u^*) = 1.8334$ is smaller than in cooperative case.

B. Centralized optimization

Now let us consider the single controller problem, that arises in the case of centralized optimization. As in the decentralized framework, we operate here in terms of occupation measures. Thus, the problem (8) for the case of two players can be rewritten as follows:

$$\max_{\rho} \sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x},\mathbf{a}) r(\mathbf{x},\mathbf{a}), \tag{19}$$

where $r(\mathbf{x},\mathbf{a})$ is defined by (14). The maximization is performed subject to the following constraints:

$$\sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x},\mathbf{a}) h_i(a_i) \le V_i, \quad i = 1,2; \tag{20}$$

$$\sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x},\mathbf{a}) = \pi(\mathbf{x}) = \pi_1(x_1)\pi_2(x_2);$$

$$\rho(\mathbf{x},\mathbf{a}) \ge 0, \quad \forall x \in \mathbf{X}, \ \forall a \in \mathbf{A};$$

$$\sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x},\mathbf{a}) = 1.$$

Once the occupation measures are obtained, the strategies can be computed by means of (2).

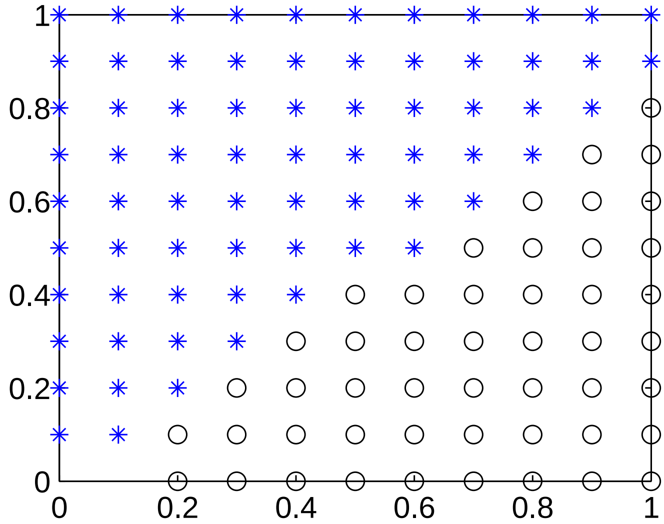


Fig. 3. The sets Ψ_1 and Ψ_2.

Define the following sets:

- Ψ_1: pairs (x_1, x_2): $\exists a_1^*$ such that $h_1(a_1) > 0$ and $u(a_1^*, a_2\,|\,x_1, x_2) > 0$ for some $a_2 \in \mathbf{A}_2$;
- Ψ_2: pairs (x_1, x_2): $\exists a_2^*$ such that $h_2(a_2) > 0$ and $u(a_1, a_2^*\,|\,x_1, x_2) > 0$ for some $a_1 \in \mathbf{A}_1$.

Note, that the set Ψ_i is the set of states in which ith player should transmit with nonzero probability according to the optimal strategy.

In Fig. 3 these sets are provided for the centralized optimization problem (19). The set Ψ_1 is depicted by circles, and the set Ψ_2 —by stars. One can see, that the sets have no mutual points. It means, that the mobiles never transmit at the same time.

Moreover, it turns out that the power level, that mobile should choose for the transmission do not depend on the state of the other player.

In Fig. 4 one can see the supports of the optimal strategies.

A circle on the place $(g_1(x_1^*), a_1^*)$ means that the first mobile should transmit on the power level a_1^* with nonzero probability in all states $(x_1^*, x_2) \in \Psi_1$.

A star on the place $(g_2(x_2^*), a_2^*)$ means that the second mobile should transmit on the power level a_2^* with nonzero probability in all states $(x_1, x_2^*) \in \Psi_2$.

If there are two or more power levels a_i^* for some particular state $g_i(x_i^*)$, then the player should randomize. In the other case (single power level a_i^* for the state $g_i(x_i^*)$), the player should always transmit with power level a_i^*.

One can see that for both players there are states of randomization. We provide here the strategies

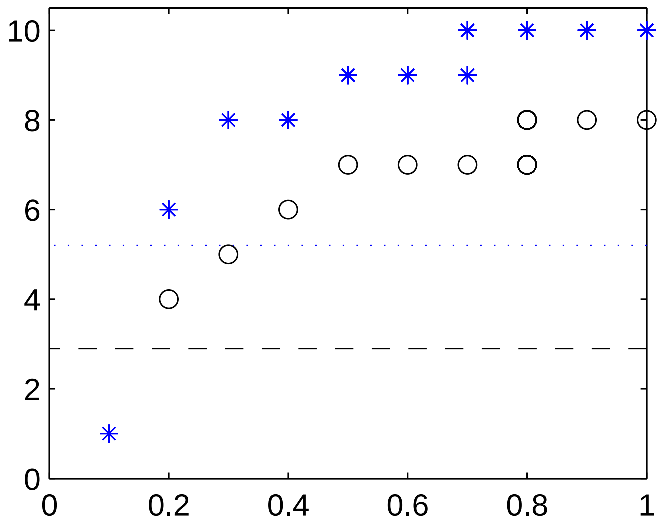


Fig. 4. Supports of the optimal policies in cooperative case.

$u(a_1, a_2 \,|\, h_1(x_1), h_2(x_2))$ for these states:

$$\begin{array}{ll} u(7,0\,|\,0.8,0.6) = 0.0159, & u(8,0\,|\,0.8,0.6) = 0.9841; \\ u(7,0\,|\,0.8,0.5) = 0.0187, & u(8,0\,|\,0.8,0.5) = 0.9813; \\ u(7,0\,|\,0.8,0.4) = 0.0106, & u(8,0\,|\,0.8,0.4) = 0.9894; \\ u(7,0\,|\,0.8,0.3) = 0.0122, & u(8,0\,|\,0.8,0.3) = 0.9878; \\ u(7,0\,|\,0.8,0.2) = 0.0167, & u(8,0\,|\,0.8,0.2) = 0.9833; \\ u(7,0\,|\,0.8,0.1) = 0.0133, & u(8,0\,|\,0.8,0.1) = 0.9867; \\ u(7,0\,|\,0.8,0.0) = 0.0126, & u(8,0\,|\,0.8,0.0) = 0.9874; \end{array}$$

$$\begin{array}{ll} u(0,9\,|\,0.8,0.7) = 0.8686, & u(0,10\,|\,0.8,0.7) = 0.1314; \\ u(0,9\,|\,0.7,0.7) = 0.8686, & u(0,10\,|\,0.7,0.7) = 0.1314; \\ u(0,9\,|\,0.6,0.7) = 0.8688, & u(0,10\,|\,0.6,0.7) = 0.1312; \\ u(0,9\,|\,0.5,0.7) = 0.8692, & u(0,10\,|\,0.5,0.7) = 0.1308; \\ u(0,9\,|\,0.4,0.7) = 0.8692, & u(0,10\,|\,0.4,0.7) = 0.1308; \\ u(0,9\,|\,0.3,0.7) = 0.8685, & u(0,10\,|\,0.3,0.7) = 0.1315; \\ u(0,9\,|\,0.2,0.7) = 0.8665, & u(0,10\,|\,0.2,0.7) = 0.1335; \\ u(0,9\,|\,0.1,0.7) = 0.8617, & u(0,10\,|\,0.1,0.7) = 0.1383; \\ u(0,9\,|\,0.0,0.7) = 0.8589, & u(0,10\,|\,0.0,0.7) = 0.1411. \end{array}$$

Note, that the centralized power management provides better throughput in comparison with other considered controls, the value of the cost function is $R(u^*) = 2.6103$.

Another interesting point that we want to discuss is the attainability of the power constraints.

Consider the problem (19) without power constraints. The optimal policies for this problem are as follows:

- Player 1 should transmit at the top power level if $g_1(x_1) \geq g_2(x_2)$;
- Player 2 should transmit at the top power level if $g_2(x_2) \geq g_1(x_1)$.

The value of the cost function for this policy is $R(u^*) = 2.8560$. The experiments show, that at the optimal point for problem with constraints (20), where the bounds V_i are both greater then 5, the power constraints are not attained, and the optimal strategy and the value of the cost function are the same as in unconstrained case.

C. Orthogonal coding

In this section we consider the same example as above, but with another cost functions. Let $r_i(\mathbf{x}, \mathbf{a}), i = 1, 2$, be defined by (1) with $c_{ij} = 0$. Zero values of the coefficients correspond to the case of perfectly orthogonal coding. The interesting feature of this example is that the problems of cooperative (centralized and decentralized) optimization and the non-cooperative game provide the same optimal policies. In Fig. 5, the supports of the optimal policies are shown as a function of the channel state.

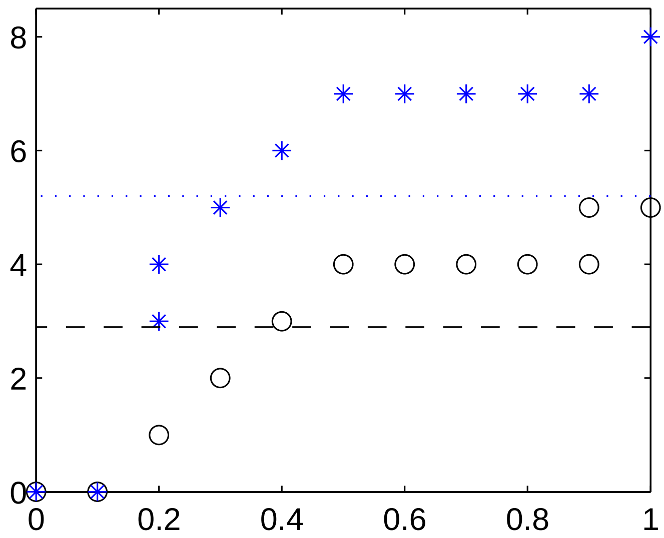


Fig. 5. Supports of the optimal policies in case of orthogonal coding.

Both players have strategies, that are randomized only at one point:

$$\begin{array}{ll} u_1(4\,|\,0.9) = 0.1, & u_1(5\,|\,0.9) = 0.9; \\ u_2(3\,|\,0.2) = 0.8, & u_2(4\,|\,0.2) = 0.2. \end{array}$$

The values of the cost functions are as follows: $R_1(u^*) = 1.3131$, $R_2(u^*) = 1.7881$, $R(u^*) = R_1(u^*) + R_2(u^*) = 3.1012$. As one could expect, the orthogonal coding allows to achieve even higher total throughput.

It should be noted, that in this particular case the solution is not unique: for example, for the second player there exists another strategy, that delivers the same values to the objective function and satisfies the power consumption constraints. This strategy differs from the one that is depicted in Fig. 5 at three channel states:

$$\begin{array}{ll} u_2(3\,|\,0.2) = 0.1893, & u_2(4\,|\,0.2) = 0.8107; \\ u_2(6\,|\,0.5) = 0.1964, & u_2(7\,|\,0.5) = 0.8036; \\ u_2(7\,|\,1.0) = 0.4142, & u_2(8\,|\,1.0) = 0.5858; \end{array}$$

According to this strategy the second player has to randomize at three channel states, and therefore this strategy is less preferable than one that has a single randomization point.

D. Jamming

The average power bounds are the same as in all previous examples: for the transmitter $V_1 = 2.9$, and for the jammer $V_2 = 5.2$.

The supports of the optimal strategies in this problem are depicted in Fig. 6. We note that the structure obtained in Theorem 4 holds for player 1, whereas the structure obtained in Subsection VI-C holds for player 2. Both players have

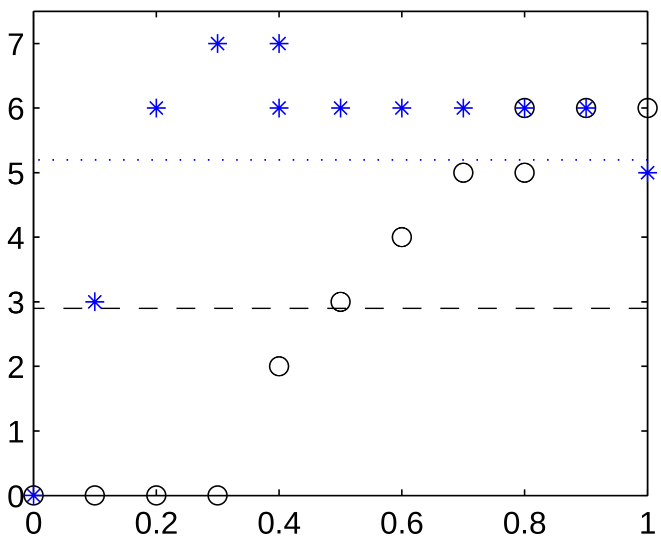


Fig. 6. Supports of the optimal policies in case of jamming.

optimal strategies that are randomized only at one point:

$$u_1(5\,|\,0.8) = 0.1, \quad u_1(6\,|\,0.8) = 0.9;$$
$$u_2(7\,|\,0.4) = 0.8, \quad u_2(6\,|\,0.4) = 0.2.$$

The value of the objective function is $R_1(u^*) = 0.6494$ which is less then the same value for the decentralized non-cooperative case.

It should be noted, that the structure of the optimal strategy for the jammer is not the same as for the transmitter: it is not monotonic.

VIII. Conclusion and further work

We have studied power control in both cooperative and non-cooperative setting. Both centralized and decentralized information patterns have been considered. We have derived the structure of optimal decentralized policies of selfish mobiles having discrete power levels. We further stuied the structure of power control policies when a malicious mobiles tries to jam the communication of another mobile. We have illustrated these results via several numerical examples, which also allowed us to get insight on the structure in the cooperative framework.

The modeling and results open many exciting research problems. Our setting, which could be viewed as a temporal scheduling problem, is quite similar to the ”space scheduling” (i.e. the water-filling) problems discussed in Subsection I-A, for which the context of discrete power levels along with the non-cooperative setting have not yet been explored. It is interesting not only to study the water-filling problem in the discrete noncoooperative context but also to study the combined space and temporal scheduling problem, where we can split the transmission power both in time and in space (different parallel channels).

From both a game theoretic point of view as well as from the wireless engineering point of view, it is interesting to study possibilities for coordination between mobiles in the decentralized case (in both cooperative as well as non-cooperative contexts). This can be done using the concepts from *correlated equilibria* [1], [2], [10], [16], which is known to allow for better performance even in the selfish non-cooperative cases. We note however, that existing literature on correlated equilibria do not include side constraints, which makes the investigation novel also in terms of fundamentals of game theory.

Acknowledgment

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APPENDIX

A. Global optimization with decentralized information

Considering the decentralized framework, we make the following observation concerning the relation between the cooperative and the non-cooperative cases.

Theorem 7: Any policy u that maximizes the common objective $R_\gamma(u)$ while satisfying the constraints is necessarily a constrained Nash equilibrium in the game where each mobile maximizes the common objective $R_\gamma(u)$.

Proof: Let $\mathbf{v}$ be a globally-optimal policy among the decentralized policies. Assume that it is not an equilibrium. Then there is some mobile, say i, that can deviate from v_i to some u_i such that (6) holds and such that its utility, which coincides with the other mobile's utility, satisfies $R_\gamma((\mathbf{v}_{-i}, u_i)) > R_\gamma(\mathbf{v})$. Moreover, for all other players $j \neq i$ as well, the constraint (6) still holds since it does not depend on mobile i's policy. But this implies that $\mathbf{v}$ is not a globally optimal policy which is a contradiction. So we conclude that $\mathbf{v}$ is indeed a constrained Nash equilibrium. ■

Proof of Theorem 2: Consider the non-cooperative setting but with the common objective $R_\gamma(u)$ to all mobiles. There exists at least one such equilibrium due to Theorem 3. If there is a dominating constrained equilibrium (which is the case when there are finitely many constrained equilibria) then it is a globally optimal policy due to Theorem 7. Assume next that there is a set $\mathbf{U}^*$ of infinitely many constrained equilibria. Let $R_\gamma^* = \sup_{u\in\mathbf{U}^*} R_\gamma(u)$ and let $u_n \in \mathbf{U}^*$ be a sequence of constrained equilibria such that $\lim_{n\to\infty} R_\gamma(u_n) = R_\gamma^*$. Then it follows (from an adaptation of [6] and [22]) that there exists a constrained equilibrium u^* such that $R_\gamma(u^*) = R_\gamma^*$. It is thus a dominating equilibrium and hence a globally optimal policy. ■

B. Linear complementarity approach for the decentralized case

In this section we show how the non-cooperative equilibrium can be obtained in the case of two players by means of linear complementarity problem (LCP). Consider the following problem, where each player wants to maximize his own payoff R_i:

$$\text{maximize over } \rho_1, \rho_2 \quad R_i(u) := \sum_{\substack{x_1\in\mathbf{X}_1\\ a_1\in\mathbf{A}_1}} \sum_{\substack{x_2\in\mathbf{X}_2\\ a_2\in\mathbf{A}_2}} \rho_1(x_1,a_1) r_i(x_1,a_1,x_2,a_2)\rho_2(x_2,a_2), \tag{21}$$

where $i = 1, 2$ and

$$\rho_i(x_i,a_i) \geq 0, \qquad \forall x_i \in \mathbf{X}_i, a_i \in \mathbf{A}_i, \tag{22}$$

$$\sum_{\substack{x_i\in\mathbf{X}_i\\ a_i\in\mathbf{A}_i}} \rho_i(x_i,a_i) = 1, \tag{23}$$

$$\sum_{a_i\in\mathbf{A}_i} \rho_i(x_i,a_i) = \pi_i, \quad \forall x_i \in \mathbf{X}_i, \tag{24}$$

$$\sum_{\substack{x_i\in\mathbf{X}_i\\ a_i\in\mathbf{A}_i}} \rho_i(x_i,a_i) h_i(a_i) \leq V_i, \quad i = 1, 2. \tag{25}$$

Here ρ_i: $\mathbf{X}_i \times \mathbf{A}_i \longrightarrow [0,1]$ is the occupation measure for player $i = 1, 2$.

First, assume, that at the equilibrium point the power consumption constraints (25) are active:

$$\sum_{\substack{x_i\in\mathbf{X}_i\\ a_i\in\mathbf{A}_i}} \rho_i(x_i,a_i) h_i(a_i) = V_i, \quad i = 1, 2. \tag{26}$$

This assumption is not restrictive, because if one or both of these constraints are not active, they can be omitted. So we suggest first to solve the problem without constraints (25), then calculate the values of cost functions in RHS of (25). If both calculated values are less then V_i, then the constraints (25) are satisfied and the solution of (21)—(24) is a solution of (21)—(25) as well. If one of the constraints (25) is violated, the problem should be solved again subject to the corresponding constraint of equality type form (26). If in the obtained equilibrium point the second constraint is still violated, then the problem should be solved again subject to both constraints of (26).

Now let ξ be the vector, containing all the $\rho_1(x_1,a_1)$, $\forall x_1 \in \mathbf{X}_1, a_1 \in \mathbf{A}_1$, and ζ —the same vector for $\rho_2(x_2,a_2)$.

Indeed, the problem (21) with constraints (22), (23), (24) and (26) can be represented in the form of the bimatrix game with linear constraints:

$$\begin{array}{c} \max_{\xi,\zeta} \xi^* A\zeta, \\ \max_{\xi,\zeta} \xi^* B\zeta, \end{array} \tag{27}$$

s.t.

$$\xi \geq 0, \ \zeta \geq 0; \tag{28}$$

and

$$\begin{array}{c} C^*\xi = c, \\ D^*\zeta = d. \end{array} \tag{29}$$

Following [13] we introduce the linear complementarity problem whose solution characterizes the equilibrium point of (27), (28), (29):

$$\begin{array}{l} z = (\xi, \zeta, z_1, z_2, z_3, z_4)^* \geq 0, \\ q + Mz \geq 0, \\ z^*(q + Mz) = 0, \end{array} \tag{30}$$

where

$$M = \begin{pmatrix} & -A & C^* & -C^* & & \\ -B^* & & & & D^* & -D^* \\ -C & & & & & \\ C & & & & & \\ & -D & & & & \\ & D & & & & \end{pmatrix},$$

$$q = (0, 0, c^*, -c^*, d^*, -d^*)^*.$$

It is also shown in [13], that under the conditions $A \leq 0$ and $B \leq 0$ Lemke's algorithm [15] computes a solution of the LCP (30).

It should be noted, that in order to satisfy the conditions $A \leq 0$, $B \leq 0$ we can always replace cost matrices A and B with $A - kE$ and $B - kE$, where E is a matrix of unities, and k is the maximal positive entry of A and B.

Once the solution of LCP (30) (ξ_o, ζ_o) is found, the equilibrium point (ξ', ζ') of the bimatrix game (27) could be computed using the following formulas:

$$\begin{array}{l} \xi' = \frac{\xi_o}{e_1^* \xi_o}, \\ \zeta' = \frac{\zeta_o}{e_2^* \zeta_o}, \end{array} \tag{31}$$

where e_1 and e_2 are vectors of appropriate dimension, whose components are all ones.