1 Scientific Research

1.1 State-Constrained Systems

One of the major topics of my research is to study systems with constraints, where we are particularly interested by models where these constraints give rise to nonsmooth behavior. We have studied different mathematical models, existence of solutions, and control related design problems such as estimation and output regulation. My primary references for the material in this section are \[BT20\] \[CIT19\] \[TBP18\] \[TBP16\] \[TBP14\].

1.1.1 Mathematical Modeling

The basic mathematical models that we consider for modeling constrained systems are basically written as time-varying differential inclusions. Roughly speaking, if we consider a time-varying set \( S : \mathbb{R}_+ \rightarrow \mathbb{R}^n \), and assume \( S \) is absolutely continuous (meaning that the difference in the Hausdorff distance between the values of \( S \) is bounded by an absolutely continuous function), then the evolution of constrained state trajectory is given by:

\[
\begin{align*}
\dot{x}(t) &\in f(t, x) - \mathcal{N}_S(t)(x(t)) \\
 x(t) &\in S(t), \quad \text{for all } t \geq 0
\end{align*}
\]

\(\star\)

Figure 1: Graphical description of the vector fields in constrained systems.
where $S(t)$ may be convex or non-convex (e.g. a prox-regular set) for each $t$, and $N_{S(t)}(x(t))$ denotes the normal cone to the set $S(t)$ at $x(t)$. The presence of normal cone in $[1a]$ allows one to write it in the form of a variational inequality as well. A graphical illustration of the trajectories of such systems is given in Figure $[1a]$. One sees that, if $x(t)$ is in the interior of the set $S(t)$, then the normal cone operator vanishes, and $x$ is obtained by solving the differential equation $\dot{x}(t) = f(t, x)$. However, if at some $t \geq 0$, $x(t)$ is at the boundary of the set $S(t)$, and the boundary is regular enough, then we look for an element $\eta \in N_{S(t)}(x(t))$ such that the vector $f(t, x) - \eta$ points inside the set $S(t)$. Thus, by adding this nonsmooth behavior due to the presence of the normal cones in the system description, we can ensure that any solution of $[1a]$ remains inside the set $S(t)$ at all times.

The system class $[1]$ basically considered constraint sets which are parameterized by time only. It can be generalized to model Lagrangian systems with unilateral constraints and impacts which induce velocity discontinuities. The primary difference comes from the fact that certain variables are constrained by sets parameterized, not only by time, but also by the current value of the other state variables. The simplest form of such systems (with unit constant mass matrix) is given by:

$$
\dot{q} = v \tag{2a}
$$

$$
dv \in f(t, q, v)dt - N_{V(q)}(v_e) \tag{2b}
$$

$$
q(t) \in \Phi, \quad v(t) \in \mathcal{V}(q(t)), \text{ for all } t \geq 0 \tag{2c}
$$

where $v_e(t) = \frac{v(t^+)+v(t^-)}{1+e}$, $dv$ is the differential measure of the right-continuous function $v(\cdot)$ (equals Lebesgue almost everywhere to $\dot{q}(\cdot)$, $e \in [0, 1]$ is a restitution coefficient. The dynamics in $[2]$ describe a Measure Differential Inclusion (MDI), i.e., the left-hand side is a measure included in the normal cone. The cone $V(q) = \{v \in \mathbb{R}^n | v^T \nabla h_i(q) \geq 0, i \in I(q)\}$ is the tangent cone linearization of the configuration space $\Phi = \{q \in \mathbb{R}^n | h_i(q) \geq 0\}$, which is the finitely represented admissible domain, $I(q)$ is the set of indices of active constraints. When $q(t) \in \text{int}(\Phi)$ then $V(q(t)) = \mathbb{R}^n \Rightarrow N_{V(q)}(\cdot) = \{0\}$, when $q(t)$ is at the boundary of $\Phi$, and $v_e(t) \in \text{int}(V(q(t))$ then $N_{V(q)}(v_e(t)) = \{0\}$. Outside discontinuity times, $v_e(t) = v(t^+) = v(t)$ (solutions are right-continuous). The major difference between $[1]$ and $[2]$ is that the convex set is the tangent cone $V(q)$, which depends on the position $q$, while the normal cone is calculated at the velocity variable $v_e$. Solutions of $[2]$ therefore possess an intrinsic non-regularity (vel jumps at impact times), while solutions of $[1]$ are continuous if $S(\cdot)$ is continuous. In other words, the jump instants are determined by the position variable $q$ and are not known a priori.

Using these fundamental ideas to describe evolution of trajectories with constraints and impacts, we have drawn connections with several other mathematical models of nonsmooth dynamical systems appearing in the literature $[BT20]$. Below, we describe certain results which we obtained using these models or their variants.

### 1.1.2 Well-posedness Results

Finding a solution to $[1]$ corresponds to finding a solution to time-varying variational inequality with differential equations. For systems of type $[1]$, we find some results in the literature. However, if we consider the a slightly general system class

$$
\dot{x} \in f(t, x) + B\eta, \quad \eta \in N_{S(t)}(Cx + D\eta)
$$

then existence of solutions for such models has not been much studied. The motivation to consider such generalization comes from studying certain electrical and mechanical systems. In our work $[TBP18]$, we provide conditions on the system data which allow us to prove well-posedness of such differential inclusions. Going one step further, it can be seen that the aforementioned class of inclusions can be more abstractly written as

$$
\dot{x} \in -\mathcal{M}(t, x)
$$
where $\mathcal{M}(t, \cdot)$ is a maximal monotone operator for each $t \geq 0$. The results on existence of solutions for such systems have certain limitations, and in our recent work [CIT19], we provide generic conditions for existence of solutions to such abstract differential inclusions. We also apply our results to the so-called complementarity systems as well.

### 1.1.3 Design of State Estimators

Beyond the fundamental question of studying existence of solutions, we have studied the problem of designing state estimators for systems with constraints. The challenging aspect of addressing the estimation problem is that the vector field, or even the state trajectory, is discontinuous. In our initial work [TBP14], we first consider the dynamical systems with discontinuous vector fields and prox-regular constrained sets (which are a generalization of convex sets). Two types of state estimation algorithms are proposed: a generalization of the Luenberger observer which results in local asymptotic convergence, and estimators based on high-gain observers which results in semiglobal practically convergent state estimators. For the case of dynamical systems with discontinuities in state trajectories, we have focused on designing estimators for mechanical systems with unilateral constraints and impacts [TBP16]. From technical standpoint, systems with impacts, in general, do not exhibit continuity of solutions with respect to initial conditions. The reason being that the state trajectories starting from different initial conditions are not contained in the same set at all times because of which, the monotonicity argument cannot be invoked. When the system is governed by a time-dependent constrained evolution, the trajectories of the estimator can be constrained within the same set as the plant, and then under appropriate passivity assumption on system data, the convergence of the estimate is obtained due to monotonicity of the normal cone operators of convex sets.

### 1.1.4 Output Regulation Control under Constraints

Continuing with control related problems for constrained systems, we have also studied the design of controllers for systems with constraints [TBP18]. The system dynamics are driven by a control input which can be chosen as a function of measured states to drive the output of the system to track a prescribed function asymptotically, see Figure 2. In the literature on output regulation, the controlled plant is driven by the output of an exosystem that models the dynamics of the reference trajectories and/or disturbances. Intuitively speaking, the proposed control input that achieves the output regulation comprises a feedback component to make the closed-loop dynamics stable and an additional open-loop component that shapes the steady state of the plant. The derivation of the open-loop component of the control input requires the exact knowledge of the exosystem dynamics, and hence the approach is termed as internal model principle. In our results on output regulation, we restrict ourselves to vector fields which are linear in state and input. The sets considered in the description of variational inequalities for the exosystem and the plant are assumed to be the same but the mappings used to describe the relations could be different. We derive sufficient conditions under which there exists a control input that achieves output regulation while maintaining state constraints. In addition to the classical regulator synthesis equations, additional conditions are needed in our work to generate a dissipative relation between the multivalued part and the output regulation error. These additional conditions also guarantee that the

![Figure 2: Architecture for output regulation control](image-url)
A closed-loop system is well-posed, that is, it admits a unique solution which is an important consideration for designing controllers for such class of systems. We study two cases for control synthesis depending on how much information is available to the controller. In the first case, it is assumed that the entire states of the plant and the exosystem are available and thus, a static controller is designed to achieve output regulation. In the second case, it is assumed that only the regulation error (which needs to converge to zero) is available and in that case a dynamic compensator is designed.

1.2 Switched Dynamical Systems

Another class of discontinuous dynamical system that I have studied in my research, are the so-called, switched systems which are modeled as:

\[ \dot{x} = f_\sigma(x, u) \]  

where \( \sigma : [0, \infty) \rightarrow \mathcal{P} \) is a right-continuous switching signal, for some discrete index set \( \mathcal{P} \). Over the past two decades, switched systems form an active topic of research in control community. In our work, we have either borrowed tools based on analysis of switched systems to solve certain control problems with switching measurements, or developed some tools for stabilization of switched systems. The main references for the material in this section are [ZT19, TPF16, TTP15].

1.2.1 Control with Quantized and Sampled Measurements

Based on the work reported in [TPF16], we have consider the problem of output feedback stabilization in linear systems when the measured outputs and control inputs are subject to event-triggered sampling and dynamic quantization. The architecture is depicted in Figure 3 and we see that due the presence of digital communication channel, the measurements and control values are discretized during transmission which can be regarded as switching in the dynamics. A new sampling algorithm is proposed for outputs which does not lead to accumulation of sampling times and results in asymptotic stabilization of the system. The approach for output sampling is based on defining an event function that compares the difference between the current output and the most recently transmitted output sample not only with the current value of the output, but also takes into account a certain number of previously transmitted output samples. This allows us to reconstruct the state using an observer with sample-and-hold measurements. The estimated states are used to generate a control input, which is subjected to a different event-triggered sampling routine; hence the sampling times of inputs and outputs are asynchronous. Using Lyapunov-based approach, we prove the asymptotic stabilization of the closed-loop system and show that there exists a minimum inter-sampling time for control inputs and for outputs. To show that these sampling routines are robust with respect to transmission errors, only the quantized (in space) values of outputs

\[
\mathcal{P} : \begin{cases} 
\dot{x} = Ax + Bq_\mu(u_{\text{nom}}(\tau_j)) \\
y = Cx 
\end{cases}
\]

\[
\mathcal{C} : \begin{cases} 
\dot{z} = Az + Bq_\mu(u_{\text{nom}}(\tau_j)) + f(q_\nu(y(t_k)), Cz(t_k)) \\
u_{\text{nom}}(t) = Kz(t)
\end{cases}
\]

Figure 3: Feedback loop where the inputs and outputs are time-sampled and quantized.
and inputs are transmitted to the controller and the plant, respectively. A dynamic quantizer is adopted for this purpose, and an algorithm is proposed to update the range and the center of the quantizer that results in an asymptotically stable closed-loop system.

1.2.2 Input-to-State Stability of Interconnections of Switched Systems

It is also possible to address more general classes of switched systems and their interconnections. Our paper [ZT19] addresses stability analysis of interconnected switched systems with continuous and discrete dynamics. The fundamental tool, that we build on, relates to the robustness with respect to external disturbances, formalized by the notion of input-to-state stability (ISS). System (3) is ISS with respect to the input $u$ if there exist functions $\gamma \in K$, and $\beta \in KL$ such that

$$|x(t)| \leq \beta(|x(0)|, t) + \gamma(\|u\|),$$

for every $t \geq 0$. We are particularly interested in studying systems where the interconnection is described by a cascade configuration, see Fig. 4a. Using the Lyapunov function construction in [TTP15], we construct the Lyapunov functions for this cascade interconnection. We then use the framework of hybrid systems to describe the overall system with jump maps, and switching signal with average dwell-time constraints. A novel Lyapunov function is constructed for this hybrid system and the corresponding analysis provides the lower bounds on average dwell-time which yield global asymptotic stability of a certain set. In our approach, we do not require the decay rates of the individual Lyapunov functions to be linear, and the upper bounds on the value of individual Lyapunov function at jump instants may be nonlinear functions of other Lyapunov functions. When studying linear systems as an example, even though we associate quadratic Lyapunov functions to individual subsystems, the Lyapunov function for the overall hybrid system involves a product of the exponential function with a non-quadratic function.

1.2.3 Sampled-data control of switched systems

We then use these constructions to study the feedback stabilization of switched nonlinear systems when the output measurements and control inputs are time-sampled. The resulting control system to be analyzed is depicted in Figure 4b. Using an observer-based controller, where the estimation error dynamics and the closed-loop system (with static control) are ISS with respect to measurement errors, we rewrite the whole system in cascade configuration where the estimation error drives the state of the controlled plant. The measurement errors are introduced because we only send time-sampled outputs to the controller, and the controller only sends sampled control inputs to the plant. In both cases, the sampled measurements are subjected to a zero-order hold, and thus remain constant until the next sampling instant. Our goal is to derive algorithms to compute sampling algorithms which result in global asymptotic stability of the closed-loop system under the average dwell-time assumptions derived earlier. The event-based sampling strategy that we use is inspired from [TTP15], where the dynamic filters are introduced. The next sampling instant occurs when the difference between the current value of the output (resp. input) and
its last sample is comparatively larger than the value of the dynamic filter’s state. Beyond the realm of periodic sampling, stabilization of dynamical systems has been studied subject to various sampling techniques. Among these methods, event-based control has received attention as an effective means of sampling and various variants of this problem have been studied over the past few years. However, this technique has not yet been studied for switched systems which is the main contribution of our work.

1.3 Stabilization and Control under Random Sampling

Continuing with my previous work on sampled-data, my recent work considers stabilization of dynamical systems when the arrival of the measurements to the control or estimator is modeled by a random process. Several results have been obtained in this direction which deal with stabilization of nonlinear dynamical systems with smooth feedback laws [TCL18]; stabilization with hybrid controllers [TT17]; and performance bounds for model predictive control of discrete-time systems [TGG19]. Moreover, we have been to use the ideas in this work to address optimization problem under random switching in a game setting when each player wants to optimize its own performance criterion in a noncooperative manner. The details of these results can be found in following references:
2 Research Supervision and Teaching

2.1 Supervision of PhD Students

Currently, I am supervising two PhD students, and some details of their thesis are given below:

2.1.1 Stability of Switched Systems

Student’s Name: Matteo Della Rossa  
Dates: November 2017 – October 2020  
Supervisors: Aneel Tanwani and Luca Zaccarian  
Funding: Bourse par école doctorale  
Tentative Title: Input-Output Stability of Switched Systems  
Prizes: Travel Support Award for IEEE Conf. on Decision and Control (CDC), 2020

Brief description: We have so far studied the problem of stability analysis in switched and hybrid systems. The main emphasis is on constructing nonsmooth Lyapunov functions and derive sufficient conditions using generalized derivatives. This PhD has so far resulted in following articles [DGTZ19, DTZ19, DTZ18].

2.1.2 Analysis of Constrained Interconnected Systems

Student’s Name: Marianne Souaiby  
Dates: November 2018 – October 2021  
Supervisors: Aneel Tanwani and Didier Henrion  
Funding: ANR Project: CONVAN  
Tentative Title: Analysis and Control of Constrained Systems using Optimization Tools  
Prizes: Travel Support Award for American Control Conference, 2018.

Brief description: We have considered the problem of stability analysis of state constrained systems. In particular, we have studied the question of existence of Lyapunov functions which take into account the constraints imposed on the system dynamics. Algorithms based on semidefinite programming hierarchies have also been developed to render feasibility to the computation of Lyapunov functions. The results obtained so far have been submitted for publication [STH19].

2.2 Masters Students

So far, I have supervised two students for Masters internship individually. Some details of the work carried out during their internship are given below:

2.2.1 Sampled-data Control of Switched Systems

Student’s Name: Guang-Xue Zhang  
Dates: February 2017 – July 2017  
Supervisors: Aneel Tanwani  
Funding: Project PEPS by INS2I  
Title: Cascade Interconnections of Switched Systems and Sampled-Data Control  
Prizes: Travel Support Award for American Control Conference, 2018.  
Placement: Graduate Student at University of California, Irvine, USA.

Brief description: We worked on the problem of analyzing interconnections of switched systems. The work from this internship resulted in following publications: [ZT18, ZT19].
2.2.2 Nonlinear Filtering

Student’s Name: Olga Yufereva  
Dates: February 2017 – July 2017  
Supervisors: Aneel Tanwani  
Funding: ANR Project: CONVAN  
Title: Nonlinear Filtering under Random Sampling  
Placement: PhD Student at Ural Federal University, Yeketerinburg, Russia.  

Brief description: Based on my ongoing work in the direction of studying control related problems under random sampling, this internship dealt with filtering problem under random sampling. The results were presented in a conference (see the following item), and an extended version has been submitted to a journal [TY19, TY20].

2.3 Teaching

2.3.1 Doctoral School Course

The basic details of the course that I offered as a part of the doctoral school are:

Title of the Course: Calculus of Variations and Optimal Control  
Dates: January 2018 (1 weeks)  
Instructors: Aneel Tanwani (15 hrs) and Didier Henrion (5 hrs)  
Program: Formation Ecole Doctorale Systemes (EDSYS), Toulouse  
Location: ISAE, SUPAERO, Toulouse

2.3.2 Mini Course: Stability and Control of Nonsmooth Systems

Title of the Course: Stability and Control of Nonsmooth Dynamical Systems  
Dates: October 2019 (1 weeks)  
Instructors: Vincent Acary (3hrs), Samir Adly (3 hrs), and Aneel Tanwani (3 hrs)  
Program: Mini cours, Journées annuelles de GdR MOA  
Location: Université de Rennes, Rennes

2.3.3 Masters Course: Nonsmooth Dynamical Systems

Title of the Course: Nonsmooth Dynamical Systems  
Dates: November 2019  
Instructors: Aneel Tanwani (18 hrs)  
Program: Masters in Applied Mathematics (ACSYON), University of Limoges  
Location: University of Limoges, Limoges
References


