Postdoc Proposal

Title:	**Nonlinear Filtering under Randomly Sampled Observations: Analysis and Computational Methods**
Supervisors:	Aneel Tanwani : `aneel.tanwani@laas.fr`
Location:	LAAS–CNRS, Toulouse, France
Duration:	12 months (with possible extension)
Start date:	as soon as possible
Salary:	2900 – 4081 euros/ month (based on experience)

Filtering is one of the fundamental problems in the theory of stochastic dynamical systems which has attracted the attention of researchers in applied mathematics, control theory, and computational methods. A recently compiled book [1] provides an overview of the developments carried out in the area of filtering theory since its inception. While the problem is relatively well-studied for linear dynamical systems with closed form solutions, the analysis and implementation of nonlinear filters has proven to be a rather challenging problem.

In this project, we are concerned with stochastic dynamical systems of the following form:

$$\mathrm{d}x_t = f(x_t)\,\mathrm{d}t + G(x_t)\;\mathrm{d}\omega_t \tag{1a}$$

$$\mathrm{d}z_t = h(x_t)\;\mathrm{d}t + \mathrm{d}\eta_t, \tag{1b}$$

where $(x_t)_{t\geqslant 0}$ is an $\mathbb{R}^n$-valued diffusion process describing the state, and $(z_t)_{t\geqslant 0}$ describes an $\mathbb{R}^p$-valued continuous-time observation process. It is assumed that, for each $t \geqslant 0$, ω_t and η_t are standard Wiener processes of appropriate dimension. For many applications, it is of interest to consider the discretized, and noisy, observation process:

$$y_{\tau_{N_t}} = h(x(\tau_{N_t})) + \nu_{\tau_{N_t}}, \qquad t \geqslant 0.$$

Our goal is to construct the estimate $\widehat{x}_t$, which minimizes the mean square estimation error, using the observations $\mathcal{Z}_t := \{\mathrm{d}z(s) \,|\, s \leqslant t\}$, or $\mathcal{Y}_t := \{y_{\tau_k} \,|\, \tau_k \leqslant t\}$. Some recent work in our group provides filtering algorithms with discrete observations, along with stability analysis, for a class of nonlinear systems [5], for distributed observation process with switching graph topology [4], and more recently particle filters for efficient computational purposes [6].

Motivated by the current research trends at the intersection of machine learning, systems theory, and data-driven design methods, the objective of this project is to study the filtering problem for nonlinear stochastic systems in similar settings. Our previous work on filtering with discrete observations relies on knowledge of the model, but some recent work has initiated filter design by using techniques based on neural networks without system data [2]. On the other hand, some algorithms in machine learning can also be shown equivalent to filtering algorithms for a certain class of dynamics [3]. In the same spirit, the project aims at exploring other possible techniques for filter design which require minimal knowledge of system dynamics, are computationally efficient, and provide reliable state estimates using discrete observations only.

The candidate applying for the postdoc is required to have a PhD with speciality in systems and control theory. The knowledge of stochastic systems is also very important for this project. Some familiarity with optimization methods and machine learning techniques would also be useful for this project.

References

[1] D. Crisan and B. Rozovskii. *The Oxford handbook of nonlinear filtering.* Oxford University Press, 2011.

[2] H. Qian, G. Yin and Q. Zhang. Deep filtering with adaptive learning rates. *IEEE Transactions on Automatic Control*, 68 (6) : 3285-3299, 2023,

[3] F. Steinke and B. Schölkopf. Kernels, regularization, and differential equations. *Pattern Recognition*, 41: 3271-3286, 2008.

[4] A. Tanwani. Suboptimal filtering over sensor networks with random communication. *IEEE Transactions on Automatic Control*, 67(10):1–8, 2022.

[5] A. Tanwani and O. Yufereva. Error covariance bounds for suboptimal filters with Lipschitzian drift and Poisson-sampled measurements. *Automatica*, 122, 2020.

[6] O. Yufereva and A. Tanwani. Transport inspired particle filters with Poisson-sampled observations in Gaussian setting. *IEEE Conf. Decision & Control*, 2023.