Output Stabilization of Time-Varying Input Delay System using Interval Observer Technique

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Motivation

I. Output-based control is usual for practice; challenging problem in many cases.

II. Input Delay arises in models of a control system due to a physical nature of a system (transport delays, computational delay, etc); "artificially", for example, in order to model a sampling effect.

III. In practice, delay may be time-varying; unknown.
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Existing results

I. Stabilizing output-based control is designed for linear systems with known and constant input delay (Olbrot 1972, Watanabe 1986); with known and time-varying input delay (Artstein 1982; Witrant, Canudas-de-Wit, Georges & Alamir 2007); with unknown and constant input delay the possible ways are: finite-time delay identification (Belkoura, Richard & Fliess 2009); adaptive control approach (Bresch-Pierti, Krstic 2009).

II. For linear systems with an unknown time-varying input delay are designed the full state feedback control (Fridman, Seuret & Richard 2004); observer for systems with unknown time-varying input and state delays (Seuret, Floquet, Richard, Spurgeon 2007).

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Consider the input delay control system of the form

$$\dot{x} = Ax + Bu(t - h(t)), \quad y = Cx,$$  \hspace{1cm} (1)

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the vector of control inputs, $y \in \mathbb{R}^k$ is the measured output, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{k \times n}$ are known matrices and input delay $h(t)$ is assumed to be unknown but within the bounded interval:

$$0 \leq h \leq h(t) \leq \bar{h},$$  \hspace{1cm} (2)

where the minimum delay $h$ and the maximum delay $\bar{h}$ are known.
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where the minimum delay \( \underline{h} \) and the maximum delay \( \bar{h} \) are known.

The system (1) is studied with the initial conditions:

\[ x(0) = x_0, \]
\[ u(t) = v(t) \text{ for } t \in [-\bar{h}, 0), \quad (3) \]

where \( v(t) \) is some bounded function. For simplicity we may assume \( v(t) = 0 \).
Basic Assumptions

Assumption (1)

The set $\Omega \subset \mathbb{R}^n$ of admissible initial conditions $x_0 \in \Omega$ of the system (1) is assumed to be bounded and known.
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Assumption (2)

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Assumption (3)

The information on the control signal $u(t)$ on the time interval $[t - \bar{h}, t)$ can be stored and used for control design.
Problem statement

Problem

The main objective of this research is to design a control algorithm for exponential stabilization of the system (1), i.e. for some numbers $c, r > 0$ any solution of the closed-loop system (1) has to satisfy the inequality

$$\|x(t)\| \leq ce^{-rt}, \forall t > 0,$$

where $x(0) \in \Omega$. 
\[ \dot{x}(t) = \tilde{A}\tilde{x}(t) + f(t), \quad y = \tilde{C}\tilde{x} \]

where \( \tilde{x} \in \mathbb{R}^n \), \( y \in \mathbb{R}^k \), \( \tilde{A} \in \mathbb{R}^{n \times n} \), \( \tilde{C} \in \mathbb{R}^{k \times n} \), and \( f : \mathbb{R} \rightarrow \mathbb{R}^n : \)

\[ \underline{f}(t) \leq f(t) \leq \bar{f}(t), \]

where \( \underline{f}(t) \) and \( \bar{f}(t) \) are known.
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where \( \tilde{x} \in \mathbb{R}^n, \ y \in \mathbb{R}^k, \ \tilde{A} \in \mathbb{R}^{n \times n}, \ \tilde{C} \in \mathbb{R}^{k \times n}, \ \) and \( f : \mathbb{R} \to \mathbb{R}^n : \)

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\[ \dot{x}(t) = \tilde{A}x(t) + f(t) + \tilde{L}(\tilde{C}x(t) - y(t)), \]
\[ \dot{x}(t) = \tilde{A}\tilde{x}(t) + \bar{f}(t) + \tilde{L}(\tilde{C}\tilde{x}(t) - y(t)), \]
\[ x(0) \leq \tilde{x}(0) \leq \bar{x}(0) \text{ and } \tilde{A} + \tilde{L}\tilde{C} = \text{Hurwitz and Metzler matrix.} \]
Interval observation (Gouze, Rapaport & Hadj-Sadok 2000)

\[
\dot{x}(t) = \tilde{A}\tilde{x}(t) + f(t), \quad y = \tilde{C}\tilde{x}
\]

where \(\tilde{x} \in \mathbb{R}^n\), \(y \in \mathbb{R}^k\), \(\tilde{A} \in \mathbb{R}^{n\times n}\), \(\tilde{C} \in \mathbb{R}^{k\times n}\), and \(f : \mathbb{R} \rightarrow \mathbb{R}^n\):

\[
f_u(t) \leq f(t) \leq \bar{f}(t),
\]

where \(f_u(t)\) and \(\bar{f}(t)\) are known.

\[
\begin{align*}
\dot{x}(t) &= \tilde{A}x(t) + f(t) + \tilde{L}(\tilde{C}x(t) - y(t)), \\
\dot{x}(t) &= \tilde{A}x(t) + \bar{f}(t) + \tilde{L}(\tilde{C}x(t) - y(t)), \\
x(0) &\leq \bar{x}(0) \leq x(0) \text{ and } \tilde{A} + \tilde{L}\tilde{C} \text{ is a Hurwitz and Metzler matrix.}
\end{align*}
\]

\[
\begin{align*}
\bar{e} &= \bar{x} - x \text{ and } \bar{e} = x - \bar{x} \\
\dot{e} &= (\tilde{A} + \tilde{L}\tilde{C})e + f(t) - \bar{f}(t), \\
\dot{\bar{e}} &= (\tilde{A} + \tilde{L}\tilde{C})\bar{e} + \bar{f}(t) - f(t),
\end{align*}
\]

\(\bar{e}(t) \geq 0\) and \(\bar{e}(t) \geq 0\) — positive system.
Illustration of the interval observation
Lemma

Under Assumptions 2-3 there always exist matrices $L \in \mathbb{R}^{n \times k}$ and $S \in \mathbb{R}^{n \times n}$, $\det(S) \neq 0$ such that

$$S^{-1}(A + LC)S - \text{Hurwitz and Metzler},$$

and the interval observer of the form

$$\dot{x}(t) = \tilde{A}x(t) + \min_{s \in [t-H,t-h]} \tilde{B}u(s) + \tilde{L}(\tilde{C}x(t) - y(t)),$$
$$\dot{\bar{x}}(t) = \tilde{A}\bar{x}(t) + \max_{s \in [t-H,t-h]} \tilde{B}u(s) + \tilde{L}(\tilde{C}\bar{x}(t) - y(t)),$$

$$\underline{x}(0) \leq \tilde{x}(0) \leq \overline{x}(0),$$

$$\tilde{A} = S^{-1}AS, \quad \tilde{B} = S^{-1}B, \quad \tilde{L} = S^{-1}L, \quad \tilde{C} = CS, \quad \tilde{x} = S^{-1}x,$$

guarantees

$$\underline{x}(t) \leq \tilde{x}(t) \leq \overline{x}(t) \quad \forall t > 0,$$

and $\underline{x}(t) \rightarrow \tilde{x}(t), \overline{x}(t) \rightarrow \bar{x}(t)$ if $u(t) \rightarrow 0$ for $t \rightarrow +\infty$. 

(NON-A & LAGIS)
Some remarks

I. Let Hurwitz and Metzler matrix $R$ be given and we need to find $S$, $L$:

$$S - 1(A + LC)S = R.$$  (7)

Denote $X = S - 1$ and $Y = S - 1L$. Then $XA + YC = RX$.  (7)

If the matrix $R$ has disjoint spectrum and the pair $(A, C)$ is observable then the equation (7) has a solution.

II. The condition $x(0) \leq \tilde{x}(0) \leq x(0)$ may be guaranteed, since the set of admissible initial conditions $\Omega$ is assumed to be known and bounded.

For example, if $\Omega = \{x \in \mathbb{R}^n : x^TPx < 1\}$, $P \succ 0$, then $\tilde{x}^TSTPS \tilde{x} < 1$ and $x_i(0) = -x_i(0) = -1/\lambda_{\min}(STPS)$, $i = 1, 2, \ldots, n$.  (NON-A & LAGIS)
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let us introduce the following predictor variables:

\[
    z(t) = e^{\tilde{A}h} \bar{x}(t) + \int_{-\Delta}^{0} e^{-A\theta} \min_{s \in [t+\theta-\Delta, t+\theta]} \tilde{B}u(s) \, d\theta, \quad (8)
\]

\[
    \bar{z}(t) = e^{\tilde{A}h} \bar{x}(t) + \int_{-\Delta}^{0} e^{-A\theta} \max_{s \in [t+\theta-\Delta, t+\theta]} \tilde{B}u(s) \, d\theta, \quad (9)
\]

which are correctly defined due to Assumption 3.
Let us define the control in the form

\[ u(t) = Kz(t), \quad z(t) = \frac{\overline{z}(t) + \underline{z}(t)}{2}, \tag{10} \]

where \( K \in \mathbb{R}^{m \times n} \).
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Remark

Let us mention that the control function can be selected in a more general form

\[ u(t) = K \bar{z}(t) + \bar{K} \underline{z}(t), \]

where \( K, \bar{K} \in \mathbb{R}^{m \times n} \).
Theorem

If for some given $\alpha, \beta, \gamma \in \mathbb{R}_+$ the matrices $X, Z, R_i, S_i \in \mathbb{R}^{n \times n}$, $i = 1, 2, \ldots, 2n$ and the matrix $Y \in \mathbb{R}^{m \times n}$ satisfy the following LMI system

\[
\begin{pmatrix}
W_e & W_{ez} \\
W_{ez}^T & W_z
\end{pmatrix} \preceq 0, \quad X \succ 0, \quad Z \succ 0, \quad R_i \succ 0, \quad S_i \succ 0,
\]

(11)

then the system (1) together with the control (10) for

\[K = YX^{-1}\]

is exponentially stable with the convergence rate: $r \geq \min\{\alpha, \beta, \gamma\}$. 

(NON-A & LAGIS)
\[
W_e = \begin{pmatrix}
\Pi_1 & \tilde{B}_1 Y & \ldots & \tilde{B}_{2n} Y \\
Y^T \tilde{B}_1^T & -e^{-\beta \Delta h} S_1 & \ldots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
Y^T \tilde{B}_{2n}^T & \ldots & \ldots & -e^{-\beta \Delta h} S_{2n}
\end{pmatrix},
\]

\[
W_z = \begin{pmatrix}
\Pi_2 & \tilde{B}_1 Y & \ldots & \tilde{B}_{2n} Y \\
X \tilde{A}^T + Y^T \tilde{B}^T & \Pi_3 & \tilde{B}_1 Y & \ldots & \tilde{B}_{2n} Y \\
Y^T \tilde{B}_1^T & \ldots & \ldots & \ldots & \ldots \\
Y^T \tilde{B}_{2n}^T & \ldots & \ldots & 0 & \ldots & -e^{-\alpha \Delta h} R_{2n}
\end{pmatrix},
\]

\[
W_{ez} = \begin{pmatrix}
Z \tilde{C}^T \hat{L}^T e^{\tilde{A}^T h} \begin{bmatrix} I_n & I_n \end{bmatrix} 0 \\
0 & 0
\end{pmatrix},
\]

\[
\Pi_1 = (\tilde{A} + \tilde{L} \tilde{C}) Z + Z (\tilde{A} + \tilde{L} \tilde{C})^T + \beta Z,
\]

\[
\Pi_2 = \tilde{A} X + \tilde{B} Y + X \tilde{A}^T + Y^T \tilde{B}^T + \alpha X, 
\]

\[
\Pi_3 = \frac{1}{4} \sum_{i=1}^{2n} (R_i + e^{\gamma h} S_i) - \frac{2}{\Delta h} X, \quad \Delta h := \bar{h} - h,
\]

where \(\tilde{B}_i \in \mathbb{R}^{n \times m}, i = 1, 2, \ldots, n\) is such that \(i\)-th row of \(\tilde{B}_i\) coincides with \(i\)-th row of \(\tilde{B}\) but all other rows of \(\tilde{B}_i\) are zero; \(\tilde{B}_{n+i} = \tilde{B}_i, i = 1, 2\ldots, n\).

For sufficiently small \(\Delta h\) the LMI is feasible.
Double integrator

\[ A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix} \]
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Solving the Silvester’s equation (7) for

\[ R = \begin{pmatrix} -3.0000 & 2.3200 \\ 0.2700 & -0.4100 \end{pmatrix} \]

we derive

\[ S = \begin{pmatrix} -3.6234 & -0.2599 \\ -1.5558 & -9.1860 \end{pmatrix}, \quad \tilde{L} = \begin{pmatrix} 0.9479 \\ -0.0948 \end{pmatrix} \]

and

\[ \tilde{A} = \begin{pmatrix} 0.4346 & 2.5663 \\ -0.0736 & -0.4346 \end{pmatrix}, \quad \tilde{B} = \begin{pmatrix} 0.0079 \\ -0.1102 \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} -3.6234 & -0.2599 \end{pmatrix} \]
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\[ h = 2, \quad \overline{h} = 4; \quad K = \begin{pmatrix} 1.1947 & 4.7560 \end{pmatrix}, \quad h(t) = 3 + \cos(100t); \]

\[ x(0) = (1, -1)^T, \quad v(t) = 0, \quad t \in [-\overline{h}, 0) \]
Figure: Controlled double integrator: Interval Predictor-based Feedback.
Figure: Controlled double integrator: Interval Predictor-based Feedback.

Figure: Controlled double integrator: Results of Choi & Lim 2010.
Linear oscillator

\[ A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}. \]

Solving the Silvester’s equation (7) for

\[ R = \begin{pmatrix} -44.5200 & 46.0040 \\ 4.4520 & -14.8400 \end{pmatrix} \]

leads to

\[ S = 10^3 \begin{pmatrix} -0.0592 & 0.0958 \\ -0.4526 & 1.5394 \end{pmatrix}, \quad \tilde{L} = \begin{pmatrix} 0.9994 \\ -0.0016 \end{pmatrix}. \]

\[ \tilde{A} = \begin{pmatrix} 14.6857 & -49.7339 \\ 4.3566 & -14.6857 \end{pmatrix}, \quad \tilde{B} = 10^{-3} \begin{pmatrix} 2.0026 \\ 1.2384 \end{pmatrix}, \]

\[ \tilde{C} = \begin{pmatrix} -59.2393 & 95.7922 \end{pmatrix}. \]

Finally, using Sedumi-1.3 for MATLAB we solve LMI system (11) for \( \alpha = \beta = \gamma = 0.2, \ h = 1, \ \bar{h} = 2 \) and obtain

\[ K = \begin{pmatrix} 137.1771 & -469.0443 \end{pmatrix}. \]
Figure: Linear oscillator

\[ h(t) = 1 + 0.5(1 - \text{sign}(\cos(0.5t))), \quad x(0) = (0, 1)^T \quad \text{and} \quad \nu(t) = 0. \]
Unstable system

\[ A = \begin{pmatrix} 0 & 1 \\ 0.1 & 0.2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad h = 1, \text{ and } \bar{h} = 2. \]

\[ h(t) = 1.5 + 0.5 \sin(t), \quad x(0) = (1, 0)^T \text{ and } \nu(t) = 0. \]
The interval observer and interval predictor for linear systems with an unknown time-varying input delay are introduced. The interval predictor-based output control algorithm is presented. Control problems with another delays (output or/and state) and system uncertainties can be also tackled by this technique.
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THANK YOU FOR YOUR ATTENTION